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THE USE OF PURE TWIST FOR DRAG REDUCTION ON ARROW WINGS WITH SUBSONIC LEADING EDGES

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Langley Field, Va.

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THE USE OF PURE TWIST FOR DRAG REDUCTION ON ARROW WINGS WITH SUBSONIC LEADING EDGES

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SUMMARY

Linearized-theory calculations of the drag reduction achieved by applying the first three terms of a power series for twist to flat delta wings are presented. In addition, the reductions due to applying linear twist to a family of flat arrow wings are presented. The results cover the speed range of subsonic leading edges.

The results show a 6-percent drag reduction due to twisting a flat delta wing with sonic leading edges and a steady decrease in the gains as sweepback increases.

For the family of linearly twisted arrow wings investigated (that with sonic trailing edges), the maximum drag reduction is 2 percent in the medium sweepback range with a steady diminution in both directions. A beneficial effect of increasing aspect ratio obscures the twist effects in this case. The convergence to the optimum-power-series twist appears to be rapid.

INTRODUCTION

Numerous examples of drag reduction by warping sweptback supersonic wings have been calculated. (For example, see refs. 1 to 3.) In a recent paper (ref. 4), as many as 10 types of loading have been combined on a delta wing including power-series twist terms. In the present paper two cases of an arrow-wing plan form subjected to pure twist (spanwise slope variation) are considered. The twist is applied symmetrically from the root as a power of the distance from the root. Powers up to three are considered for a delta plan form. For an arrow plan form with sonic trailing edges the effects of linear twist are shown.

The Lagrange method of reference 3 is used to compute both the optimum twist and the drag decrease over a flat wing for a given total lift. Leading-edge thrust is included in the drag calculations.

SYMBOLS

a	proportionality constant
b,f	parameters defining pressure distribution
C_D	drag coefficient
$C_{D,ij}$	interference drag coefficient excluding leading-edge thrust
$\left(C_{D,ij}\right)_T$	interference drag coefficient of leading-edge thrust
$\bar{C}_{D,ij}$	interference drag coefficient including leading-edge thrust
$C_{L,i}$	lift coefficient of a wing with ith power twist
$\bar{C}_L$	lift coefficient of a combination of twisted wings
$\bar{C}_{L,i}$	part of lift coefficient due to ith-power component wing when $\bar{C}_L = 1$
C_p	lifting pressure coefficient (pressure on the lower surface minus pressure on the upper surface, divided by free-stream dynamic pressure)
E	complete elliptic integral of second kind with modulus k
K	complete elliptic integral of first kind with modulus k
$k = \sqrt{1 - \bar{k}^2}$	
$\bar{k} = \theta_0 = \beta m$	speed-sweepback parameter (see fig. 1)
M	free-stream Mach number
m	tangent of semiapex angle (see fig. 1)
S	wing area
$t = \beta y/x$, $\theta = \beta y/x$, $\eta = \beta y/x$	basic variables of generalized conical flow

V	free-stream velocity
v_n	velocity of flow normal to leading edge, near leading edge
w	velocity of flow in z-direction
x,y,z	right-handed Cartesian coordinate system (see fig. 1)
α	local angle of attack
$\beta = \sqrt{M^2 - 1}$	
ΔC_D	flat-plate drag coefficient minus optimum drag coefficient
$1 - \mu$	root chord of arrow wing (see fig. 1)
ϕ	velocity potential

Subscripts:

f	flat
i,j	power of twist variation
r	dummy index or root
t	tip
τ	plan form

ANALYSIS

Pressure Computation

For a wing subjected to a symmetrical pure twist which varies as a power of the distance from the root (fig. 1), the downwash variation is

$$\frac{w}{V} = -a_i |y|^i \tag{1a}$$

By introducing the variable $\eta = \beta y/x$, equation (1a) becomes

$$\left(\frac{\beta}{x}\right)^i \frac{w}{V} = -a_i |\eta|^i \tag{1b}$$

The downwash variation (eq. (1b)) is of the type treated in references 5 and 6, and the general formula for the pressure distribution is given in appendix A. If i is no greater than 3, the lifting pressure coefficient is

$$C_{p,i} = \left(\frac{x}{\beta}\right)^{i} \frac{b_{i0} + b_{i2}\theta^2 + b_{i4}\theta^4}{\sqrt{\theta_0^2 - \theta^2}} \qquad (\theta = \beta y/x) \tag{2}$$

The coefficients b_{ir} in equation (2) can be determined from the following matrix equation:

$$CB = A \tag{3}$$

A general case of equation (3) is discussed in appendix A. For the case at hand, for linear twist,

$$C_1 = \begin{bmatrix} I(-1,2,0) & I(-1,2,2) \\ I(0,2,0) & I(0,2,2) \end{bmatrix} \qquad B_1 = \begin{bmatrix} b_{10} \\ b_{12} \end{bmatrix} \qquad A_1 = \begin{bmatrix} -\frac{4}{\beta} a_1 \\ 0 \end{bmatrix} \tag{4a}$$

for quadratic twist,

$$C_2 = \begin{bmatrix} I(-1,3,0) & I(-1,3,2) \\ I(1,3,0) & I(1,3,2) \end{bmatrix} \qquad B_2 = \begin{bmatrix} b_{20} \\ b_{22} \end{bmatrix} \qquad A_2 = \begin{bmatrix} -\frac{8}{\beta} a_2 \\ 0 \end{bmatrix} \tag{4b}$$

and for cubic twist,

$$C_3 = \begin{bmatrix} I(-1,4,0) & I(-1,4,2) & I(-1,4,4) \\ I(0,4,0) & I(0,4,2) & I(0,4,4) \\ I(2,4,0) & I(2,4,2) & I(2,4,4) \end{bmatrix} \qquad B_3 = \begin{bmatrix} b_{30} \\ b_{32} \\ b_{34} \end{bmatrix} \qquad A_3 = \begin{bmatrix} -\frac{24}{\beta} a_3 \\ 0 \\ 0 \end{bmatrix} \tag{4c}$$

The $I(a,b,c)$ functions of equations (4) are evaluated (appendix A) from the following relations:

$$I(a+1,b+1,c) = (c - b - 1)I(a,b,c) - \theta_0 \frac{\partial}{\partial\theta_0} I(a,b,c) \tag{5a}$$

$$I(a-b,0,c) = J_{a-b+c} - J_{a-b+c+2} \tag{5b}$$

$$J_n = \int_1^{\theta_0} \frac{t^n dt}{\sqrt{1 - t^2}\sqrt{t^2 - \theta_0^2}} \tag{5c}$$

If, for convenience, the following new variables are introduced:

$$\left.\begin{aligned} \bar{k} &= \theta_0 \\ \psi &= \frac{\theta}{\theta_0} \\ \alpha_{it} &= a_i m^i \\ \bar{k}^{(1+i-r)} f_{ir} &= \frac{b_{ir}}{4ma_i(i!)} \end{aligned}\right\} \tag{6}$$

equation (2) may be rewritten as follows:

$$C_{p,i} = 4m\alpha_{it}(i!)x^i \frac{f_{i0} + f_{i2}\psi^2 + f_{i4}\psi^4}{\sqrt{1 - \psi^2}} \tag{7}$$

By evaluating the elements of equations (4) and substituting in terms of the new variables (eqs. (6)), the coefficients f_{ij} are obtained as given in table I.

Drag Computation

Integration over the plan form yields the following equations for normalized lift and interference drag coefficients ($4m\alpha_{it}(i!) = 1$):

$$C_{L,i} = \frac{1}{S}\int_{\tau} C_{p,i}\,dS = \frac{2(1 - \mu)^{i+1}}{i + 2}\int_0^1 \frac{\sum_{r=0}^{4} f_{ir}\psi^r}{(1 - \mu\psi)^{i+2}\sqrt{1 - \psi^2}}\,d\psi \tag{8}$$

$$mC_{D,ij} = -\frac{m}{S}\int_{\tau}\left[C_{p,i}\left(\frac{w}{V}\right)_j + C_{p,j}\left(\frac{w}{V}\right)_i\right]dS$$

$$= \frac{1}{2}\,\frac{(1 - \mu)^{i+j+1}}{i + j + 2}\int_0^1 \frac{1}{(1 - \mu\psi)^{i+j+2}}\left(\frac{\psi^j}{j!}\,\frac{\sum_{r=0}^{4} f_{ir}\psi^r}{\sqrt{1 - \psi^2}} + \frac{\psi^i}{i!}\,\frac{\sum_{r=0}^{4} f_{jr}\psi^r}{\sqrt{1 - \psi^2}}\right)d\psi \tag{9}$$

It is understood in equations (8) and (9) that only even values of r occur.

The interference drag given by equation (9) does not include leading-edge thrust which is calculated separately in appendix B where it is shown that

$$m\left(C_{D,ij}\right)_T = -\frac{\pi}{4(1-\mu)}\frac{\sqrt{1-\bar{k}^2}}{i+j+2}\left(f_{i0}+f_{i2}+f_{i4}\right)\left(f_{j0}+f_{j2}+f_{j4}\right) \quad (10)$$

The total interference drag may be written as the sum of equations (9) and (10)

$$m\bar{C}_{D,ij} = mC_{D,ij} + m\left(C_{D,ij}\right)_T \quad (11)$$

Calculated values of the drag-lift coefficients $\epsilon_{ij} = \frac{\bar{C}_{D,ij}}{C_{L,i}C_{L,j}}$ derived from equations (8) to (11) are given in table II. The quantities ϵ_{ij} and the analysis given in reference 3 allow the calculation of the optimum partition of a given lift among the twisted wings of this report for least drag of the combination. The optimum partitions of a unit lift coefficient among the twisted wings of this report, as well as the drag of these partitions, are calculated by the method of reference 3 and are given in table III.

RESULTS

Delta Wings

The total drag of the optimum twisted delta wings, together with the total drag of the component loadings taken alone, is shown in figure 2(a); all are at unit lift coefficient. Unit lift coefficient on the twisted wings taken alone is attained by increasing the tip angle of attack α_{it}, the root angle of attack remaining zero. These values all include leading-edge thrust. A lower bound for the drag calculated as in reference 7 is shown for comparison. It is pointed out in reference 4 that this bound is a poor approximation to the minimum in the lower sweepback range.

The small overall improvement in delta-wing drag level indicated by figure 2(a) is made by lowering the flat-plate wave drag at the expense of increasing the vortex drag. The vortex drag is a minimum for the flat delta wing and is equal to the total drag at $\bar{k} = 0$. The net result of this process in shown in figure 2(b) for a wing with optimum linear twist, as well as for the wing with optimum combined linear, quadratic, and cubic twist which is plotted in figure 2(a). Figure 2(b) shows that the greater part of the small gains obtained are contributed by the linear twist throughout the speed range. This indicates a rapid convergence to the optimum-power-series twist. At $\bar{k} = 1.0$ the six additional cambered surfaces combined with the flat delta wing in reference 4 gave about another 4-percent reduction in the total drag, or a total of nearly 10 percent.

Arrow Wings

Figure 3 is analogous to figure 2, except that in figure 3 only linear twist is combined with the flat plate. The family of arrow wings considered in figure 3 have sonic trailing edges.

Figure 3(a) indicates that the possible gains due to twist are small. The increase in aspect ratio with increasing $\bar{k}$ has a beneficial effect in reducing the total drag which obscures the effect of the twist. The small twist effect is shown more clearly in figure 3(b) with a maximum drag reduction of 2 percent.

Optimum Settings

Figure 4 shows the settings required on linearly twisted wings for maximum drag reduction; α_f is the angle of attack required for unit lift coefficient on a flat wing, whereas α_r and α_t are the root and tip angles of attack on the wing with optimum linear twist, also at unit lift coefficient.

If a 60° delta wing at $M = \sqrt{2}$ $\left(\bar{k} = \frac{1}{\sqrt{3}}\right)$ and a lift coefficient of 0.2 are considered, figure 4(a) indicates that about 2° of washout is required. The drag reduction of 2.4 percent in this example compares with 3.3-percent drag reduction that could have been obtained if quadratic and cubic twist had been included (fig. 2(b)).

CONCLUDING REMARKS

Calculations of the maximum drag reduction achieved by applying the first three terms of a power series for twist have shown that the drag of flat delta wings with sonic leading edges can be reduced approximately 6 percent. The possible gains steadily diminish as sweepback is increased. Similar drag-reduction calculations for a family of linearly twisted arrow wings with sonic trailing edges show a maximum drag reduction of about 2 percent in the medium sweepback range with steadily decreasing gains in both directions. In this case, however, the beneficial effects of high aspect ratio obscure the effects of twist in the lower sweepback range. The convergence to the optimum-power-series twist appears to be rapid.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 13, 1957.

APPENDIX A

GENERAL FORMULAS FOR PRESSURE DISTRIBUTIONS

Generalized Conical Wings

For downwash distributions of the rather general type

$$\left(\frac{\beta}{x}\right)^i \frac{w}{V} = P_i(\eta) \qquad \left(\eta = \frac{\beta y}{x}\right) \tag{A1}$$

where P_i is a polynomial of degree i, Heaslet and Lomax in references 5 and 6 have derived the lifting pressure coefficient

$$C_{p,i} = \left(\frac{x}{\beta}\right)^i \sum_{r=0}^{i+1} \frac{b_{ir}\theta^r}{\sqrt{\theta_0^2 - \theta^2}} \tag{A2}$$

for the case of symmetrical sweepback. (See fig. 1.) The coefficients b_{ir} are determined in reference 6 by the following equation:

$$P_i(\eta) = \left(\frac{\beta}{x}\right)^i \frac{w}{V} = \frac{\beta}{4(i!)} \int_1^{\theta_0} \frac{(\eta - t)^i\sqrt{1 - t^2}}{t} \left(\frac{\partial}{\partial t}\right)^{i+1} \frac{\sum_{r=0}^{i+1} b_{ir}t^r}{\sqrt{t^2 - \theta_0^2}}\, dt \tag{A3}$$

Equating coefficients of η on both sides of equation (A3) yields $i + 1$ linear relations for b_{ir}. Since solutions are additive, single terms of the polynomial P_i will be considered separately.

Although equation (A3) is of concise and elegant form, it is not one suitable for calculations or convenient for discussion of solutions. In order to start a systematic reduction of the finite-part integrals of equation (A3), the following notation is introduced:

$$I(a,b,c) = \int_1^{\theta_0} t^a\sqrt{1 - t^2}\left(\frac{\partial}{\partial t}\right)^b \frac{t^c}{\sqrt{t^2 - \theta_0^2}}\, dt \tag{A4}$$

The derivative in the integrand of equation (A4) is a homogeneous function of degree $c - b - 1$. For this derivative, Euler's relation for homogeneous functions is

$$t\left(\frac{\partial}{\partial t}\right)^{b+1} \frac{t^c}{\sqrt{t^2 - \theta_0{}^2}} + \theta_0 \frac{\partial}{\partial \theta_0}\left(\frac{\partial}{\partial t}\right)^{b} \frac{t^c}{\sqrt{t^2 - \theta_0{}^2}} = (c - b - 1)\left(\frac{\partial}{\partial t}\right)^{b} \frac{t^c}{\sqrt{t^2 - \theta_0{}^2}} \tag{A5}$$

Multiplying equation (A5) by $t^a\sqrt{1 - t^2}$ and integrating yields

$$I(a+1,b+1,c) = (c - b - 1)I(a,b,c) - \theta_0 \frac{\partial}{\partial \theta_0} I(a,b,c) \tag{A6}$$

From equation (A6) it is plain that any $I(a,b,c)$ can be derived by b applications of equation (A6) to

$$I(a-b,0,c) = J_{a-b+c} - J_{a-b+c+2} \tag{A7}$$

where

$$J_n = \int_1^{\theta_0} \frac{t^n dt}{\sqrt{1 - t^2}\sqrt{t^2 - \theta_0{}^2}} \tag{A8}$$

In these equations J_n is, in general, an elliptic integral and as shown in reference 8 satisfies the following relation:

$$(n + 1)J_{n+2} - n(1 + \overline{k}^2)J_n + (n - 1)\overline{k}^2 J_{n-2} = 0 \tag{A9}$$

For n near zero, J_n may be readily put in a standard form. Any other J_n may be derived by using equation (A9). For n near zero:

$$\left.\begin{aligned}
J_{-2} &= -\frac{E}{\bar{k}^2} \\
J_{-1} &= -\frac{1}{\bar{k}}\frac{\pi}{2} \\
J_0 &= -K \\
J_1 &= -\frac{\pi}{2} \\
J_2 &= -E
\end{aligned}\right\} \tag{A10}$$

where the modulus of K and of E is equal to $\sqrt{1 - \bar{k}^2}$. Inspection of equation (A9) shows that J_n is a homogeneous linear function of E and K for even values of n. For odd values of n, J_n is a rational function of $\bar{k}$. Since the derivatives of E and K

$$\left.\begin{aligned}
\frac{dE}{d\bar{k}} &= \frac{\bar{k}}{k^2}(K - E) \\
\frac{dK}{d\bar{k}} &= \frac{\bar{k}^2 K - E}{\bar{k}k^2} \qquad \left(k^2 = 1 - \bar{k}^2\right)
\end{aligned}\right\} \tag{A11}$$

are also homogeneous linear functions of K and E, then $I(a,b,c)$ is a homogeneous linear function of K and E, when $a - b + c$ is even.

If $P_i = a_i \eta^i$, then a linear set of equations for b_{ir} may be written as follows:

$$\left.\begin{aligned}
a_i &= \frac{\beta}{4(i!)} \sum_{r=0}^{i+1} b_{ir} I(-1,i+1,r) \\
0 &= \sum_{r=0}^{i+1} b_{ir} I(j,i+1,r) \qquad (j=0,1,2,\ldots,i-1)
\end{aligned}\right\} \tag{A12a}$$

or in matrix form as

$$\begin{bmatrix} I(-1,i+1,0) & I(-1,i+1,1) & \dots & I(-1,i+1,i+1) \\ I(0,i+1,0) & I(0,i+1,1) & \dots & I(0,i+1,i+1) \\ \vdots & \vdots & \dots & \vdots \\ I(i-1,i+1,0) & I(i-1,i+1,1) & \dots & I(i-1,i+1,i+1) \end{bmatrix} \begin{bmatrix} b_{i0} \\ b_{i1} \\ \vdots \\ b_{ii} \\ b_{i,i+1} \end{bmatrix} = \begin{bmatrix} \frac{4}{\beta}(i!)a_i \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{A12b}$$

which may also be written as

$$CB = A \tag{A12c}$$

Since the single term under consideration is an odd or even function as i is odd or even, the pressure must be an odd or even function with i. Thus, every other element of the B matrix must be zero. The first element is zero when i is odd; the second element is zero when i is even. Consider now the C matrix. Equations (A7), (A9), and (A10) show that, for odd values of $\rho = a - b + c$, $I(a-b,0,c)$ is a polynomial in $\frac{1}{\theta_0}$ for $\rho < 0$ and a polynomial in θ_0 for $\rho > 0$. The polynomial degrees are $-\rho$ for $\rho < 0$ and $\rho + 1$ for $\rho > 0$. When the recursion formula (A6) is applied $i + 1$ times to $I(j-i,0,r)$, in every row except the first, the value of I is zero when ρ is odd. Thus, every other I is identically zero, except in the first row. When ρ is even, I is, in general, not zero.

If the solutions of equations (A12) are now considered, every other equation, excluding the first, disappears. This means that for odd i and odd downwash or for even i and even downwash the number of variables and equations is the same and a unique solution exists. Reference 5 points out that the form of the pressure solution (eq. (A2)) is unchanged if the natural sign of the downwash polynomial is required to change for $\eta < 0$. With respect to equations (A12), even solutions may be obtained from odd i, or odd solutions from even i, by reversing the natural choice of zero b_{ir} in the B matrix. Again considering equations (A12), for odd i and even downwash or for even i and odd downwash, it is found that the number of variables and equations is again the same and a unique solution again exists.

For a polynomial term of degree $\nu < i$, the linear equations are

$$\left.\begin{aligned} 0 &= \sum_{r=0}^{i+1} b_{ir} I(j,i+1,r) \qquad (j=-1,0,\ldots i-\nu-2,i-\nu,\ldots i-1) \\ a_\nu &= \frac{\beta}{4}\frac{1}{(i-\nu)!\nu!}(-1)^{i-\nu}\sum_{r=0}^{i+1} b_{ir} I(i-\nu-1,i+1,r) \end{aligned}\right\} \tag{A13a}$$

or as before

$$CB = A \tag{A13b}$$

where C and B are unchanged, but

$$A = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ \frac{4}{\beta} a_\nu (i-\nu)!\nu! \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{A13c}$$

For $\nu < i$, there is again a unique solution for each ν but it is not possible to demand odd solutions for even ν or even solutions for odd ν. Such a demand sets to zero the right-hand side of equation (A13a) and there are no solutions.

The underlying reason that the present analysis will not yield solutions in every case is that the νth and succeeding derivatives of the downwash terms with forced sign reversals have singularities at the origin which must be accounted for in the inversion of an integral equation. This was pointed out in reference 9 by Kafka, who shows that a logarithmic singularity in the pressure at $\eta = 0$ is required to produce the required downwash. Therefore the solution to equation (A2) is incomplete when $\nu < i$. In order to include all cases, expressions analogous to equations (A6), (A7), and (A8) may be derived by an analysis similar to that given, and the coefficients of the logarithmic terms may be determined.

Twisted Wings

The downwash variation for a symmetrically twisted wing is

$$\left(\frac{\beta}{x}\right)^{i}\frac{w}{V} = -a_i|\eta|^{i} \tag{A14}$$

and this case is treated by equations (A12). For linear twist

$$C_1B_1 = A_1 \tag{A15a}$$

where

$$C_1 = \begin{bmatrix} I(-1,2,0) & I(-1,2,2) \\ I(0,2,0) & I(0,2,2) \end{bmatrix} \qquad B_1 = \begin{bmatrix} b_{10} \\ b_{12} \end{bmatrix} \qquad A_1 = \begin{bmatrix} -\frac{4}{\beta}a_1 \\ 0 \end{bmatrix} \tag{A15b}$$

Use of equations (A6), (A7), (A9), and (A10) yields

$$\left.\begin{aligned} I(-1,2,0) &= -\frac{\pi}{2\bar{k}^3} \\ I(-1,2,2) &= -\frac{\pi}{\bar{k}} \\ I(0,2,0) &= \frac{K - E}{1 - \bar{k}^2} \\ I(0,2,2) &= \frac{\bar{k}^2K - E}{1 - \bar{k}^2} \end{aligned}\right\} \tag{A16}$$

In a similar manner, for quadratic twist

$$C_2B_2 = A_2 \tag{A17a}$$

and

$$C_2 = \begin{bmatrix} I(-1,3,0) & I(-1,3,2) \\ I(1,3,0) & I(1,3,2) \end{bmatrix} \qquad B_2 = \begin{bmatrix} b_{20} \\ b_{22} \end{bmatrix} \qquad A_2 = \begin{bmatrix} -\frac{8}{\beta} a_2 \\ 0 \end{bmatrix} \tag{A17b}$$

where

$$\left.\begin{aligned} I(-1,3,0) &= \frac{\left(-4 + 7\bar{k}^2 - \bar{k}^4\right)E + \left(2\bar{k}^2 - 4\bar{k}^4\right)K}{\bar{k}^4 k^4} \\ I(-1,3,2) &= \frac{\left(-6 + 10\bar{k}^2 - 2\bar{k}^4\right)E + \left(3\bar{k}^2 - 5\bar{k}^4\right)K}{\bar{k}^2 k^4} \\ I(1,3,0) &= \frac{\left(4 - 2\bar{k}^2\right)E + \left(-3 + \bar{k}^2\right)K}{k^4} \\ I(1,3,2) &= \frac{\left(1 + \bar{k}^2\right)E - 2\bar{k}^2 K}{k^4} \end{aligned}\right\} \tag{A18}$$

And for cubic twist

$$C_3 B_3 = A_3 \tag{A19a}$$

and

$$C_3 = \begin{bmatrix} I(-1,4,0) & I(-1,4,2) & I(-1,4,4) \\ I(0,4,0) & I(0,4,2) & I(0,4,4) \\ I(2,4,0) & I(2,4,2) & I(2,4,4) \end{bmatrix} \qquad B_3 = \begin{bmatrix} b_{30} \\ b_{32} \\ b_{34} \end{bmatrix} \qquad A_3 = \begin{bmatrix} -\frac{24}{\beta} a_3 \\ 0 \\ 0 \end{bmatrix} \tag{A19b}$$

where

$$\left.\begin{aligned} I(-1,4,0) &= -\frac{9\pi}{2\bar{k}^5} \\ I(-1,4,2) &= -\frac{6\pi}{\bar{k}^3} \\ I(-1,4,4) &= -\frac{12\pi}{\bar{k}} \end{aligned}\right\} \tag{A20}$$

APPENDIX B

CALCULATION OF LEADING-EDGE THRUST

In order to sustain a finite downwash at the subsonic leading edge of a wing, an infinite discontinuity in the pressure is required at the edge. Such an edge experiences a suction force which depends on the nature of the pressure singularity (ref. 6). When the singularity in velocity normal to the leading edge has the form

$$\frac{v_n}{V} = \frac{G_n}{\beta} \frac{1}{\sqrt{x_n}} \qquad \text{(B1)}$$

the drag coefficient due to suction is given as follows:

$$C_D = -\frac{2\pi}{S} \int_{\text{Span}} \frac{{G_n}^2}{\beta_n} dy \qquad \text{(B2)}$$

In these equations, G_n is a constant, x_n is the perpendicular distance from the leading edge, and $\beta_n = \sqrt{1 - {M_n}^2}$ where $M_n = Mm/\sqrt{1 + m^2}$. For the twisted wings of this paper (setting $4m(i!)\alpha_{it} = 1$)

$$C_{p,i} = \frac{4}{V} \frac{\partial}{\partial x} \phi_i = x^i \frac{f_{i0} + f_{i2}\psi^2 + f_{i4}\psi^4}{\sqrt{1 - \psi^2}} \qquad \text{(B3)}$$

Integration of equation (B3) from the leading edge in the manner of reference 6 yields

$$\frac{4}{V} \phi_i = f_{i0}\sigma_{i+1} + a^2 f_{i2}\sigma_{i-1} + a^4 f_{i4}\sigma_{i-3} \qquad \text{(B4)}$$

where

$$\sigma_r = \int_a^x \frac{x^r dx}{\sqrt{x^2 - a^2}} \qquad \left(a = \frac{y}{m}\right)$$

Since the following relation holds:

$$\sigma_r = \frac{x^{r-1}\sqrt{x^2 - a^2}}{r} + \frac{r - 1}{r} a^2\sigma_{r-2}$$

any σ_r can be reduced finally to the case $r = 1$ or $r = 0$. Performing this reduction in equation (B4) and differentiating with respect to x_n gives

$$\frac{G_n}{\beta_n} = \lim_{x_n \to 0}\left(\sqrt{x_n}\,\frac{v_{n,i}}{V}\right) = \frac{a^{i+1}}{\sqrt{2y \cos \delta}}\,\frac{\left(f_{i0} + f_{i2} + f_{i4}\right)}{4} \tag{B5}$$

where

$$\delta = \tan^{-1}m$$

Substitution of equation (B5) into equation (B2) yields

$$(1 - \mu)mC_{D,i} = -\frac{\pi}{16}\,\frac{\sqrt{1 - \bar{k}^2}}{i + 1}\left(f_{i0} + f_{i2} + f_{i4}\right)^2 \tag{B6}$$

In the superposition of two solutions, $v_n = v_{n,i} + v_{n,j}$ in equation (B5) and the interference drag is given by

$$(1 - \mu)m\left(C_{D,ij}\right)_T = -\frac{\pi}{4}\,\frac{\sqrt{1 - \bar{k}^2}}{i + j + 2}\left(f_{i0} + f_{i2} + f_{i4}\right)\left(f_{j0} + f_{j2} + f_{j4}\right) \tag{B7}$$

Equation (B7) is easily extended for quartic, quintic, and higher degrees of twist by adding values of f inside the parentheses as required.

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TABLE I

VALUES OF f_{ij} FOR TWISTED WINGS

$\bar{k}$	f_{00}	f_{10}	f_{12}	f_{20}	f_{22}	f_{30}	f_{32}	f_{34}
0	1	0.63662	0	0.25000	0	0.07074	0	0
.2	.95193	.54453	.04604	.18650	.04215	.04524	.01880	.00016
.4	.86907	.42435	.10613	.12271	.08350	.02463	.03253	.00103
.6	.78348	.33026	.15318	.08282	.10773	.01480	.03700	.00248
.8	.70518	.26189	.18736	.05822	.12080	.00924	.03766	.00423
1.0	.63662	.21221	.21221	.04244	.12732	.00606	.03638	.00606

TABLE II

INTERFERENCE DRAG-LIFT COEFFICIENTS FOR TWISTED WINGS

[m = 1]

(a) Delta wings, $\mu = 0$

$\bar{k}$	ϵ_{00}	ϵ_{01}	ϵ_{02}	ϵ_{03}	ϵ_{11}	ϵ_{12}	ϵ_{13}	ϵ_{22}	ϵ_{23}	ϵ_{33}
0	0.1592	0.1592	0.1592	0.1592	0.1790	0.1910	0.1989	0.2122	0.2274	0.2487
.2	.1784	.1834	.1871	.1900	.2171	.2417	.2615	.2848	.3215	.3744
.4	.2204	.2333	.2422	.2503	.2926	.3388	.3792	.4213	.4965	.6075
.6	.2790	.3009	.3167	.3301	.3932	.4672	.5286	.5993	.7147	.8840
.8	.3559	.3900	.4170	.4407	.5259	.6370	.7291	.8322	1.0014	1.2473
1.0	.5000	.5683	.6250	.6744	.7958	.9834	1.1399	1.3000	1.5746	1.9662

(b) Arrow wings, $\mu = \bar{k}$

$\bar{k}$	ϵ_{00}	ϵ_{01}	ϵ_{11}
0	0.1592	0.1592	0.1790
.2	.1421	.1483	.1892
.4	.1353	.1467	.2104
.6	.1259	.1383	.2123
.8	.1022	.1116	.1744
1.0	0	0	0

TABLE III

OPTIMUM COEFFICIENTS OF LIFT AND DRAG FOR TWISTED WINGS

$\left[m = \bar{C}_L = 1\right]$

(a) Delta wing with 0, 1, 2, and 3rd degree twist

$\bar{k}$	$\bar{C}_{L,0}$	$\bar{C}_{L,1}$	$\bar{C}_{L,2}$	$\bar{C}_{L,3}$	$C_{D,0}$
0	1.0000	[a]-0.0001	[a]0.0001	0	0.0796
.2	1.0797	.2592	-.5956	.2567	.0888
.4	2.0980	-3.3817	3.7405	-1.4569	.1074
.6	2.0803	-3.0406	3.2043	-1.2440	.1348
.8	1.9886	-2.7857	3.0310	-1.2339	.1708
1.0	1.8384	-2.0685	2.1257	-.8956	.2341

(b) Delta wing with 0 and 1st degree twist

$\bar{k}$	$\bar{C}_{L,0}$	$\bar{C}_{L,1}$	$C_{D,0}$
0	1.0000	0	0.0796
.2	1.1744	-.1744	.0888
.4	1.2774	-.2774	.1084
.6	1.3114	-.3114	.1361
.8	1.3347	-.3347	.1722
1.0	1.4292	-.4292	.2353

(c) Arrow wing with 0 and 1st degree twist

$\bar{k}$	$\bar{C}_{L,0}$	$\bar{C}_{L,1}$	$C_{D,0}$
0	1.0000	0	0.0796
.2	1.1782	-.1782	.0705
.4	1.2187	-.2187	.0664
.6	1.2012	-.2012	.0617
.8	1.1759	-.1759	.0503
1.0	[b]X	[b]1 - X	0

[a]Exact calculation would show these values to be zero. The values of $\bar{C}_L$ are in general less accurately computed than the values of $C_{D,0}$.

[b]Value of X may be assigned arbitrarily.

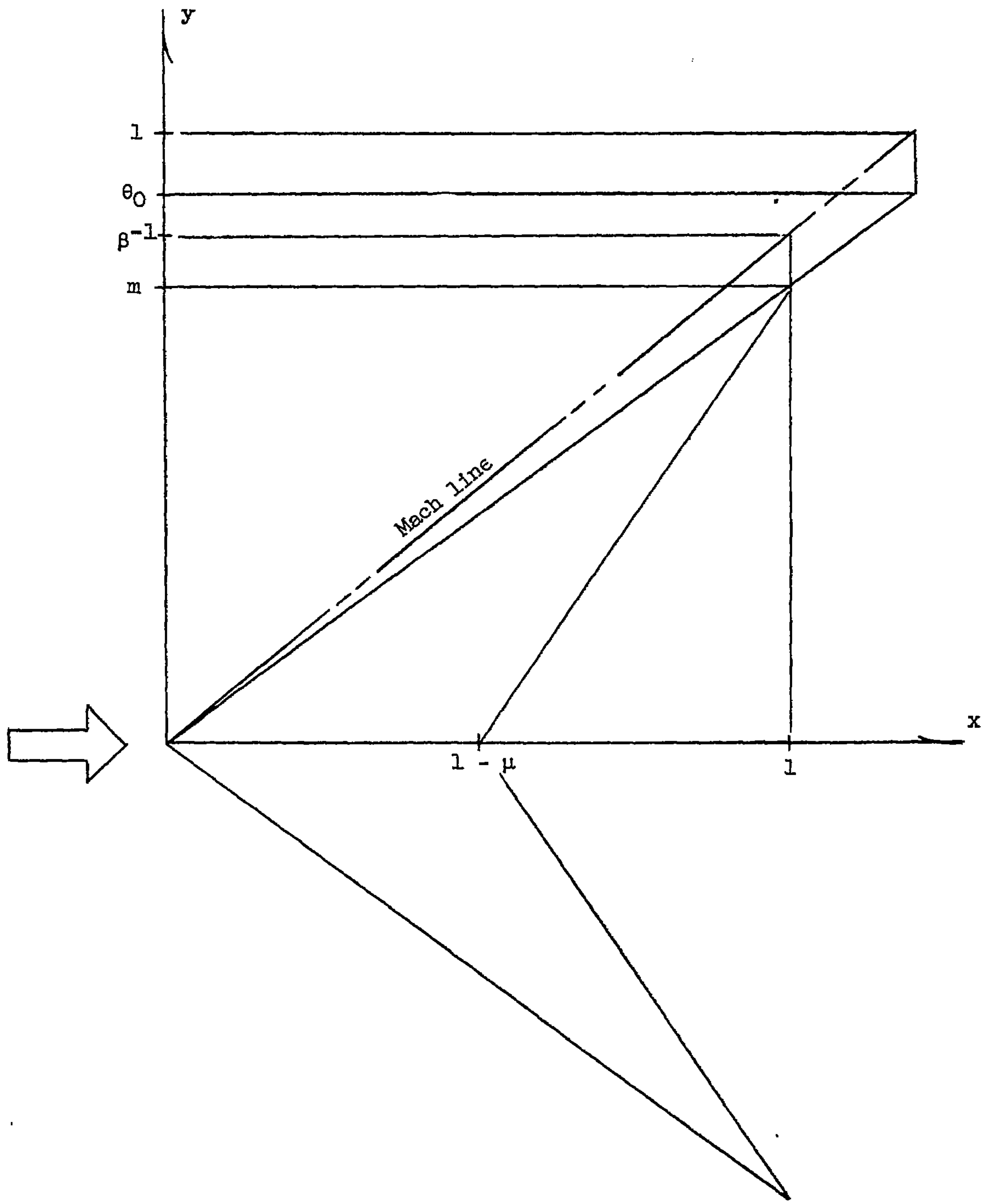
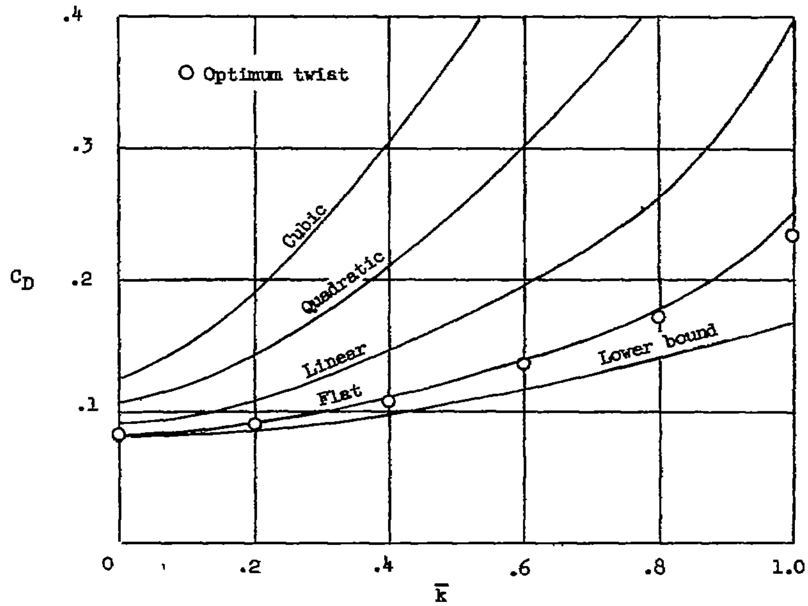
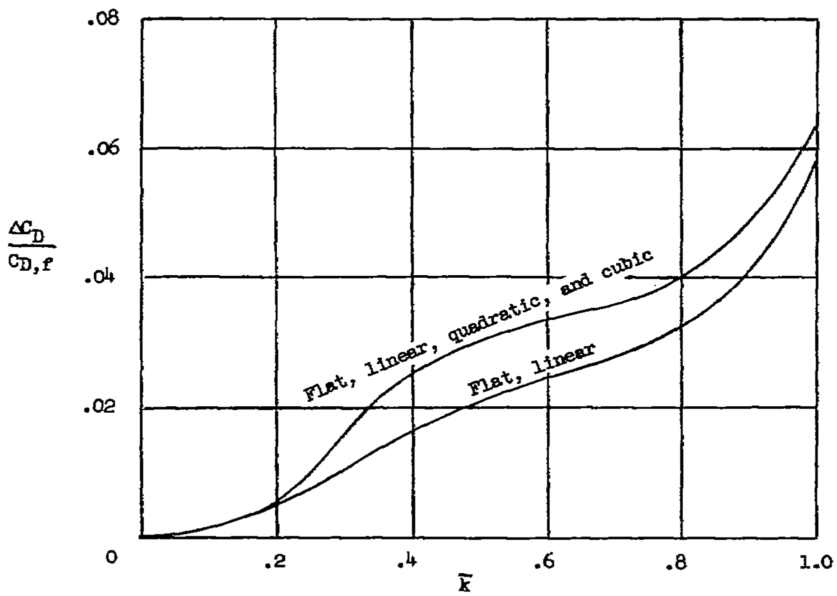


Figure 1.- Arrow plan form.

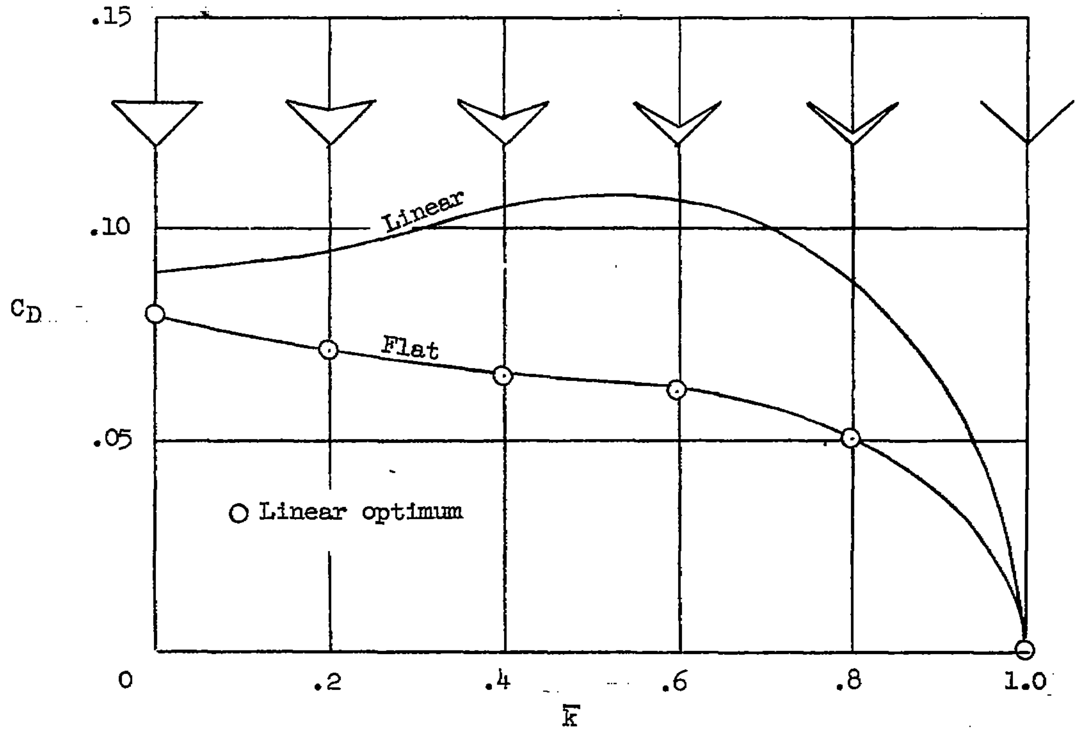


(a) Drag coefficients.

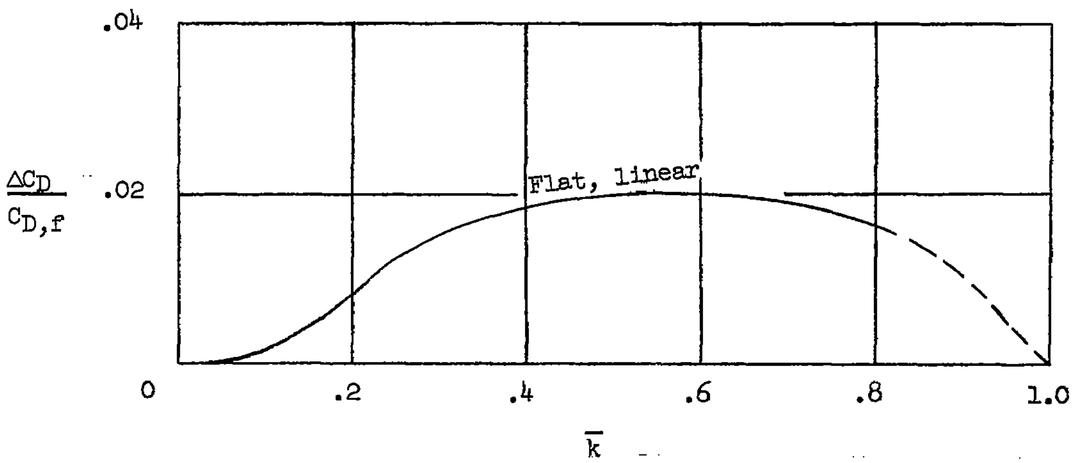


(b) Drag improvement over flat wing at optimum twist.

Figure 2.- Drag characteristics of twisted delta wings. $m = \overline{C_L} = 1$; $\mu = 0$.

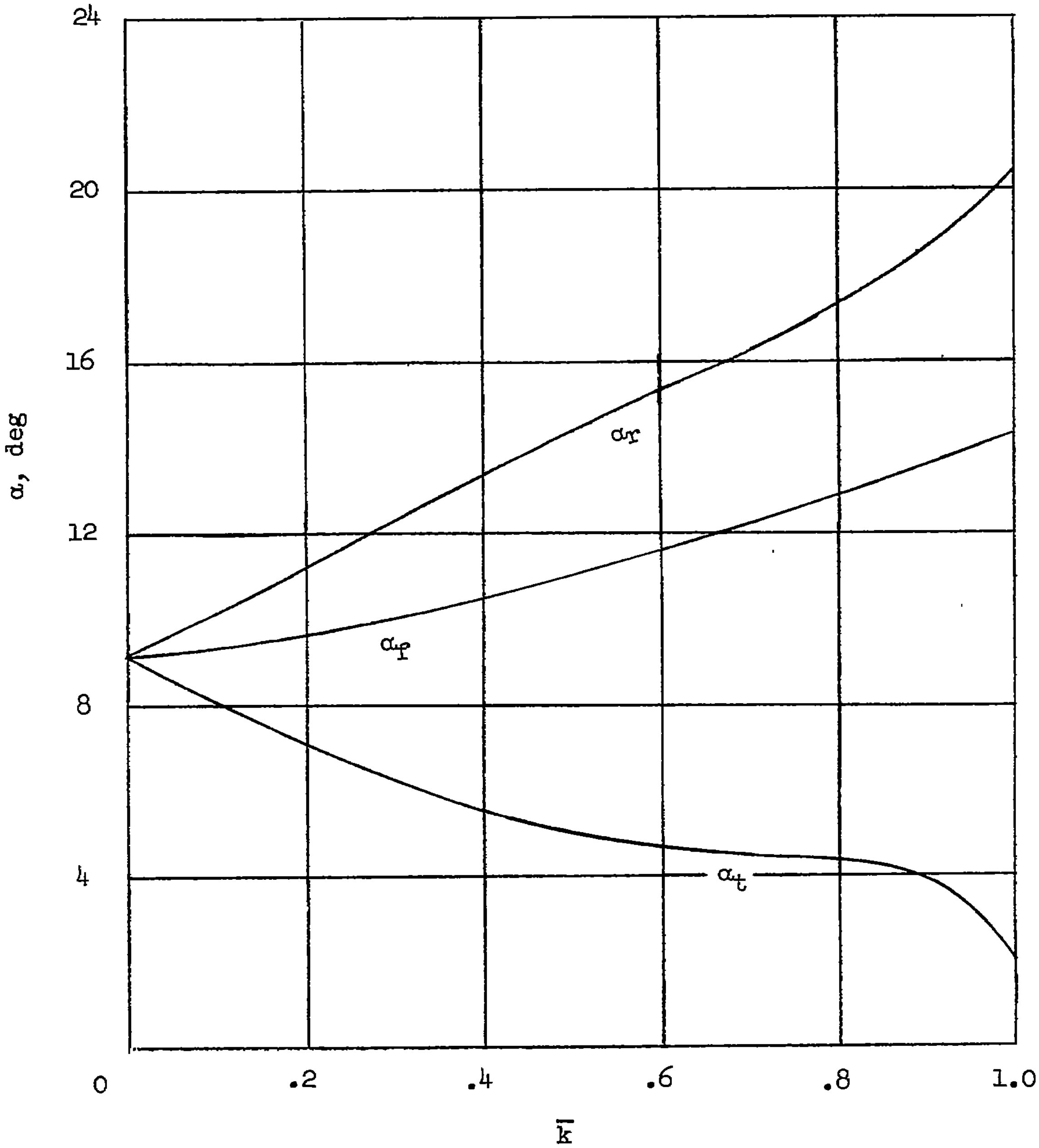


(a) Drag coefficients.



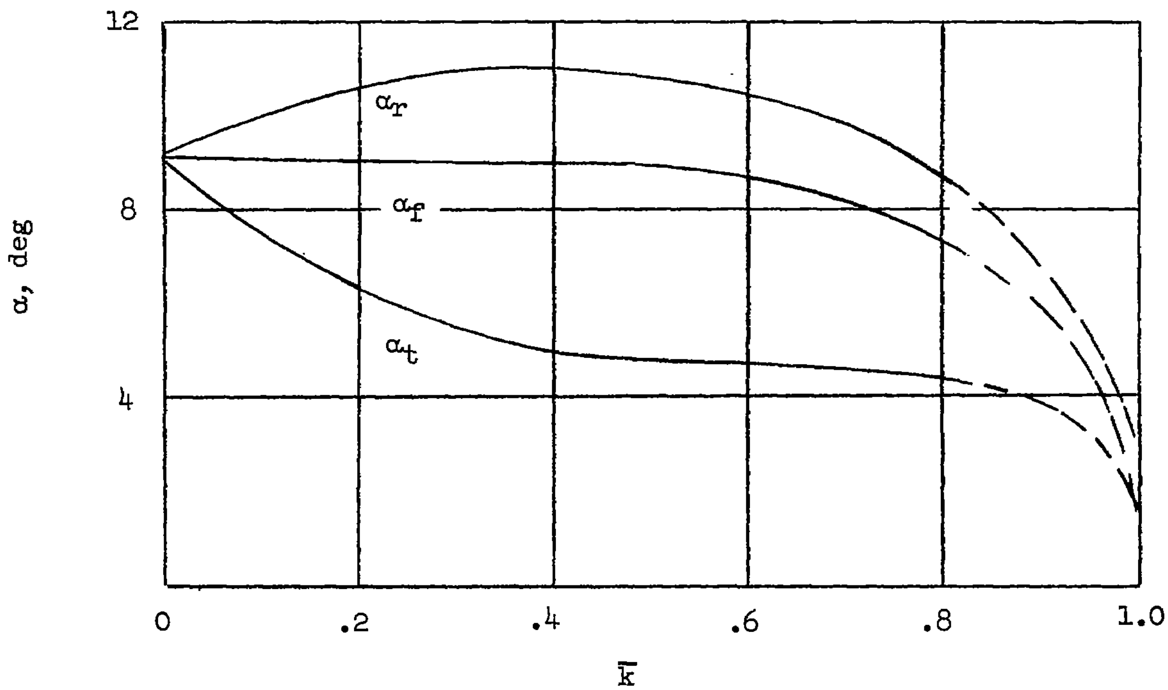
(b) Drag improvement at optimum twist.

Figure 3.- Drag characteristics of twisted arrow wings. $m = \bar{C}_L = 1$; $\mu = \bar{k}$.



(a) Delta wings. $\mu = 0$.

Figure 4.- Optimum incidence angles on linearly twisted wings. $m = \bar{C}_L = C_{L,f} = 1$.



(b) Arrow wings. $\mu = \bar{k}$.

Figure 4.- Concluded.