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EFFECT OF FREQUENCY AND TEMPERATURE ON FATIGUE OF METALS

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SUMMARY

Some of the properties of ductile metals which are exhibited under fatigue loading may be described in a qualitative manner on the basis of a physical model that is familiarly called the standard linear solid. Its importance lies in the fact that it shows a relation that must exist between temperature and frequency. On the basis of this model it seems possible to conclude from phenomenological considerations that the critical temperatures observed by Daniels and Dorn and by Valluri are not real critical values but are simply temperatures associated with corresponding frequencies of fatigue stressing above which the fatigue behavior changes. It is suggested that the reason why one does not observe the effect of frequency at room temperature in normal engineering practice is that the critical frequency associated with room temperature differs substantially from those frequencies customarily used in fatigue testing.

INTRODUCTION

While it has been felt for a long time that frequency has no effect on fatigue failures, a paper by Daniels and Dorn (ref. 1) shows experimental evidence that under certain conditions frequency does have an effect on fatigue and, in fact, that above a particular temperature, which was associated by the authors with the recovery temperature for the material (99.995 percent pure aluminum), a higher frequency should give rise to a higher fatigue life. The authors suggested that this is a critical temperature for this material and as corroborative evidence cited some of their work on creep in which the same temperature played a prominent part. On the other hand, some work done by Valluri (ref. 2) on commercially pure (99.18 percent) aluminum while investigating the relation between internal friction and fatigue behavior suggested that the temperature at which the internal friction of a well-annealed specimen reaches a maximum may have a definite effect on fatigue. In fact, preliminary results showed a tendency toward the fatigue life reaching a minimum in the immediate vicinity of this temperature. It was found, however, that the temperatures observed in the two cases were quite different (150° C versus 236° C) and it did not seem possible to explain this difference satisfactorily on the bases of purity, of the variations of recovery temperature as a function of pre-anneal cold-work, or of other variables that affect recovery temperature.

It is also not clear why one should expect a substantial change in fatigue life at the recovery temperature or at the grain-boundary critical temperature. There is, however, one basic difference between the two tests and this is the difference between the frequencies employed in the two cases. While Daniels and Dorn used 25 and 1,440 cpm in their work, Valluri used 21,000 cpm, and it appears that it may be this difference in frequency that gave rise to the two different temperatures at which the fatigue behavior shows substantial changes. A qualitative approach to the problem based on a two-component system of which one is a viscous phase suggests that there is really nothing unique about these two temperatures but what is of real consequence in fatigue problems is a frequency and an associated temperature. This report outlines a qualitative approach to the problem of the effect of frequency and temperature on fatigue.

SYMBOLS

a_1,a_2,b_1,b_2	constants
H	heat of activation
$K = \nu\tau_o e^{H/RT}$	
M_R	relaxed modulus
M_U	unrelaxed modulus
N	number of cycles to failure
R	gas constant
T	temperature of relaxation
T_1,T_2,T_3,T_4	temperatures in decreasing order of magnitude
t	time
ϵ	shear strain
$\dot{\epsilon}$	rate of change of shear strain
ϵ_o	strain corresponding to instantaneous load
ν	frequency

$\nu_1, \nu_2, \nu_3, \nu_4$ frequencies in increasing order of magnitude

σ shear stress

$\dot{\sigma}$ rate of change of shear stress

σ_o stress corresponding to instantaneous load

σ_{cr} critical resolved shear stress

τ relaxation time

τ_o constant in equation (4)

τ_ϵ time of relaxation of stress under conditions of constant strain

τ_σ time of relaxation of strain under conditions of constant stress

SOME ASPECTS OF TWO-COMPONENT SYSTEM

It has long been recognized that a simple linear relation between stress and strain does not adequately represent the physical behavior of metals and that, in order to explain some of the features of materials characterized as anelastic, one has to include the time variations of stress and strain in the stress-strain relations. A mechanical model that is simple enough and adequately describes some of these features is the so-called standard linear solid which is shown in figure 1 and may be seen to have the form

$$a_1\sigma + a_2\dot{\sigma} = b_1\epsilon + b_2\dot{\epsilon} \tag{1}$$

It will be noticed that this manifests the well-known feature of elastic aftereffect commonly observed in engineering metals. In order to facilitate clear understanding Zener in his book (ref. 3) introduces three constants τ_ϵ, τ_σ, and M_R and writes the above equation in the form

$$\sigma + \tau_\epsilon\dot{\sigma} = M_R(\epsilon + \tau_\sigma\dot{\epsilon}) \tag{2}$$

where τ_ϵ is the time of relaxation of stress under conditions of constant strain, τ_σ is the time of relaxation of strain under constant stress and M_R is called the relaxed modulus, which has in general a value less than that of the unrelaxed modulus M_U which is defined simply as $\frac{\Delta\sigma}{\Delta\epsilon}$. It will be observed that under an instantaneous load a standard linear solid suffers an instantaneous deformation and as the load is maintained, the viscous phase represented by the dashpot in figure 1 starts relaxing and the deformation continues to increase with time, reaching the equilibrium value $M_R^{-1}\sigma_o$ where σ_o is the stress corresponding to the instantaneous load. In fact, it may be easily seen that the deformation is of the form

$$\epsilon(t) = M_R^{-1}\sigma_o + \left(\epsilon_o - M_R^{-1}\sigma_o\right)e^{-t/\tau_\sigma} \tag{3}$$

The essential feature of this system is the capacity of its viscous component to relax and in so doing increase the strain in the purely elastic component. The relaxation process is largely characterized by the relaxation time τ_o. In effect, it means that the rate at which the strain in the elastic component increases is dependent on the value of τ_σ. If the value of τ_σ approaches infinity one has then what amounts to a purely elastic medium and, for extremely small values, the system is substantially relaxed at all times.

It has been recognized for a long time that the grain boundaries and newly formed slip planes in a polycrystalline aggregate may be considered to have a viscous phase. Experimental evidence by many workers does show that a metal can indeed display viscous behavior. At this stage it is not clear what kind of atomic model is necessary in order that a part of a metal manifest this behavior. If, then, the existence of a component in a metal which behaves in a viscous manner is accepted, it is found in accordance with Zener (ref. 3) that there are three important characteristic features displayed by such a system. One is the large magnitude of anelastic effects that can be produced by very small amounts of the viscous component; the second is the building up of high stress concentrations within the elastic matrix as the viscous component starts to relax; and the third is the wide variety of relaxation spectra which it may have.

Of special interest in problems of fatigue is the second feature, namely, creation of sources of stress concentration within the elastic matrix. That this is possible may be seen by the following simple example. Consider the hypothetical case of a single interface in an aggregate which is subjected to a state of shear. This may, for example, be a slip surface or a grain boundary. The instantaneous application of shear stress to such a model will produce an instantaneous strain. Without loss of generality one may assume that the shear strain in the viscous component is the same as that in the elastic matrix. As the viscous component starts to relax, there is a reduction of the load sustained by it and the elastic matrix readjusts itself, suffering an increase of strain in some places and decrease in other places. It is evident, however, that there is a net increase in strain energy in the elastic phase of the system and the readjustment gives rise to an increase in stress in the elastic matrix at the corners of the interface and a reduction in a volume which will roughly be equal to a sphere with the interface as the diameter. This is the so-called sphere of relaxation. In figure 2, the number of lines per unit area is a measure of the stress. In an annealed polycrystalline material, there exists a large number of these interfaces in which relaxation takes place whenever a load is applied. In addition, however, an external load of sufficient magnitude will generate slipbands which are regions of inhomogeneous plastic deformation. It has been shown by various workers that temporarily at least the slipbands manifest viscous behavior and seem gradually to acquire an elastic nature. It is apparent then that, given suitable times of relaxation, the slipbands also can give rise to sources of stress concentration at their corners.

FATIGUE PROBLEM FROM VIEWPOINT OF TWO-COMPONENT SYSTEM

The fatigue problem may be examined on the basis of the properties of the two-component system described in the preceding section. For simplicity, consider a pure material which initially is in a well-annealed state. Such a material may in general be expected to have some internal macroscopic flaws which may be in the form of hairline cracks or inclusions that cause local weakness. The material in addition may be assumed to consist of grains which are substantially crystalline save for certain imperfections like dislocations and lattice vacancies. The grain boundaries as suggested above are potential nuclei for generating sources of stress concentration. Since the basic mechanism that leads to the initiation and subsequent propagation of a crack under reversed loading leading to fatigue failure is not yet understood from an experimental viewpoint, it seems somewhat futile at this stage to try to develop a picture of fatigue failure on the atomic scale. It appears more practical to base a phenomenological theory of failure on the probability of generating enough sources of stress concentration. This method of approach implicitly assumes that the final failure is pretty well determined within the first few reversals, if the stress, frequency, and temperature are given. The propagation of a crack during subsequent cycles may probably be explained from elasticity theory. It is now assumed that the larger the number of sources of stress concentration that are available in the material, the smaller the number of cycles necessary to cause failure. A limiting condition to this statement will be pointed out later. Since the existence of sources of stress concentration is considered a sufficient reason to cause final failure, a material that has these sources in the form of internal hairline flaws or grain boundaries that can relax at suitable frequencies of loading can eventually generate a macroscopic crack leading to final failure even under low stress levels. This in essence means that there is really no such thing as an endurance limit, but if given enough time and suitable frequencies a specimen subjected to repeated loading should eventually fail.

In general, however, because of the process of slip, a specimen generates additional sources of stress concentration, these sources being located at the corners of the slipbands and generated because of the relaxation of the viscous component contained in these bands. It is assumed that the sources of stress concentration thus generated in the elastic matrix by the relaxation of the viscous phase will give rise to enough sources of local weakness to initiate and sustain a crack leading to fatigue failure in a finite number of cycles.

Since the generation of these sources of local weakness is dependent upon relaxation, the relative values of relaxation times and frequency of fatigue stressing become variables of paramount importance, and since relaxation times are influenced in general by temperature, one thereby obtains a means of relating frequency and temperature to the fatigue problem. This line of argument suggests that any means that can inhibit this generation of sources of stress concentration should be conducive to an increase in fatigue life. Since relaxation of the viscous phase gives rise to these sources, it seems probable that one should expect an increase in fatigue life if relaxation of the viscous phase is inhibited. This may be effected in two ways. One is to use a rate of loading that is substantially larger than the rate of relaxation so that the viscous phase has no time to relax and thus generate new sources. The second is to surround an elastic network by the viscous phase so that the viscous phase is not really at liberty to relax completely. The second model is similar to that imagined by Zener in order to explain the grain-boundary peak in internal friction. The second model in fact suggests that a slightly cold-worked metal with a suitably placed array of slipbands must have a higher fatigue strength than an annealed specimen.

Some interesting consequences follow on the basis of this model. Thus, the experimentally observed fact (ref. 1) that for pure aluminum there exists a temperature above which frequency has an effect on fatigue becomes quite clear. In fact, it suggests that there is nothing unique about this temperature, and what is of consequence is a temperature and an associated frequency. It is an experimentally known fact that physical properties of metals dependent upon relaxation usually obey a relation of the type $\nu\tau$ where ν is the frequency and τ is the relaxation time. In general, the relaxation time is found to obey an Arrhenius equation $\tau = \tau_0 e^{H/RT}$, where H is the heat of activation for the process, T is the temperature of relaxation, and R is the gas constant. Therefore, the property may be expected to obey the functional relation

$$N = f\left(\nu\tau_o e^{H/RT}, \sigma\right) \tag{4}$$

where N is the property of the material that would vary in the above manner.

Now for any given material at a particular temperature τ_o, H, and R are predetermined. If the frequency of loading is so low that at all times the material is in a substantially relaxed state, then this frequency is conducive to the generation of the maximum number of sources of stress concentration and, hence, should give rise to a low fatigue life. As the frequency is gradually increased, at a particular temperature, a state is reached where the period of fatigue cycling is smaller than the time for substantial relaxation of the viscous phase, so that generation of new sources is inhibited and therefore an increase in fatigue life should result. On the other hand, it is also evident that an increase in fatigue life due to an increase of frequency cannot go on indefinitely for two reasons. One is that there are generally other sources of stress concentration such as the hairline flaws cited above; the other reason is that sources of stress concentration leading to regions of local weakness are caused by relaxation, and, therefore, if relaxation is substantially inhibited at one frequency, a higher frequency may not be expected to add much to the process of inhibition of relaxation. On the low-frequency side, too, a similar relation must hold. Thus, if at a frequency ν_2 the material is in a substantially relaxed state at all times, a frequency $\nu_1 < \nu_2$ cannot be expected to have any appreciable influence in the fatigue life. Therefore, this model suggests that, at any particular temperature, if one starts with extremely low frequencies and gradually increases the value, in the beginning there will not be much change in fatigue life and in the vicinity of a particular frequency ν_2 the fatigue life will start to increase and keep increasing up to a particular frequency ν_3 after which the increase in fatigue life becomes negligible.

One has also to postulate the creation of an additional viscous phase during the progress of repeated loading and subsequent relaxation. It is an observed fact that in most single crystals slip takes place on certain suitably situated crystallographic planes when the resolved shear stress on these planes reaches a critical value. In a polycrystalline aggregate one may expect a similar relation to hold good with some restraints imposed by the adjacent grains. It can then be said that slip takes place in a particular crystal of an aggregate on a family of planes when the shear stress on them reaches a value $K\sigma_{cr}$, where K is a constant and σ_{cr} is the value of the critical resolved shear stress for slip if that crystal were isolated. After slip takes place, the material in the slipband according to this model assumes viscous properties and starts relaxing, thereby increasing the stress in the elastic matrix at the edges of the slipbands. Depending upon the size of the slipbands and the amount of relaxation, sooner or latter the stress in the elastic matrix within a region around the source has to exceed $K\sigma_{cr}$, which means new slipbands have to be formed, these having their origin near the corners of the existing regions. It is evident that the larger the external stress to start with, the larger is the additional number of slipbands that will be created (i.e., the larger is the additional amount of viscous phase that is generated). It is clear that there will be a limit to this process also, since as more and more viscous phase is generated, substantial relaxation becomes impossible, which means that the shear stress at the sources of stress concentration will be limited to a value at which additional slipbands cannot be generated from that source. One necessary consequence of this idea is that the effect of frequency will be far more pronounced at higher stress levels than at lower stress levels, since inhibition of relaxation at higher stress levels prevents the generation of far more sources of local weakness than at lower stress levels. It follows therefore that what is familiarly defined as endurance limit is not sensibly affected by a change of frequency. It is therefore suggested that the mean S-N curves in fatigue would vary with frequency in a manner shown in figure 3 when temperature is kept constant. Figure 4 also denotes the general trend of the effect of frequency on fatigue.

The effect of temperature on fatigue also may be argued along similar lines. As the temperature is decreased the relaxation time increases; therefore, a decrease of temperature has the opposite effect of a decrease in frequency, that is, it inhibits relaxation. Suppose at any given frequency and temperature the relaxation is substantially inhibited. Then, a further decrease in temperature, while maintaining the same frequency, cannot further inhibit the relaxation significantly. The high-temperature side may not be argued along similar lines since, in general, one can expect this effect to be covered up by some other high-temperature effects. At any particular frequency there should be, however, a definite temperature above which the material is substantially relaxed. Any further increase above this value may be expected to give rise to an additional decrease in fatigue life. What this means then is that associated with each frequency there will be a temperature above which one should get a much larger decrease in fatigue life since above this temperature a maximum number of sources of local weakness are operative at all times and hence should contribute to a much larger decrease in fatigue life. Therefore, it follows that associated with each frequency there will be a temperature above which the decrease in fatigue life should be much larger than at lower temperatures. In other words, there will be a point of inflection in a T-N curve for any particular frequency as was observed by Daniels and Dorn. The curves will therefore be expected to have the general features shown in figure 5. It is possible that at room temperature customary engineering frequencies are sufficiently greater than the relaxation times of the viscous phase so that the effect of stress concentration due to hairline flaws and natural inclusions will be more predominant and the effect of frequency may in general be expected to be substantially covered up. As the temperature is increased the relative influence of the relaxation of viscous phase becomes more prominent.

Little or no experimental evidence is available in order to check the validity of this hypothesis. A frequency range of about 10 to 50,000 cpm is considered necessary in order to investigate this idea experimentally. Some brief work (ref. 4) shows that at approximately 30,000 cpm the fatigue life may be expected to increase over that at lower frequencies. However, Daniels and Dorn offer the first experimental evidence to show the effect of frequency and temperature on fatigue. Since frequency and temperature are related in an inverse manner insofar as their effect on fatigue is concerned and since as shown by them the number of cycles to failure seems to be a function $\nu e^{H/RT}$, it seems plausible to assume that $\nu\tau_o e^{H/RT} = K$ as a condition relating the frequency and temperature at which the effect of frequency becomes prominent. This leads to the relation

$$T = \frac{H/R}{\log_e {}^{K/\nu\tau_o}}$$

Daniels and Dorn found that the T-N curve for 25 cpm has a knee at 150° C. Therefore, taking $\nu = 25$ cpm, $T = 150^\circ$ C, and $H = 34,000$ cal/g mole (as determined by them), K/τ_o may be determined. Applying this value of the constant with $\nu = 1,440$ cpm gives 197° C as the associated critical temperature. This checks favorably with the value of roughly 205° C obtained for the knee portion of the curve for $\nu = 1,440$ cpm. In the work done by Valluri while investigating the relation between internal friction and fatigue, a temperature of 236° C appeared as the value around which the fatigue life tended to be a minimum. The frequency used in this case was 21,000 cpm. This value of the frequency gives an associated critical temperature of 232° C. The experimental values and the values obtainable by the assumed relation are given in figure 6. Because of the higher impurity of the test material used by Valluri (99.18 versus 99.995) the value of H may be slightly different. Since this is a phenomenological approach to the problem the techniques of stressing are not expected to affect sensibly the above result. Consequently, the above check on this physical model based on a two-component system seems to point a way toward understanding the effect of frequency and temperature on fatigue.

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Pasadena, Calif., January 27, 1956.

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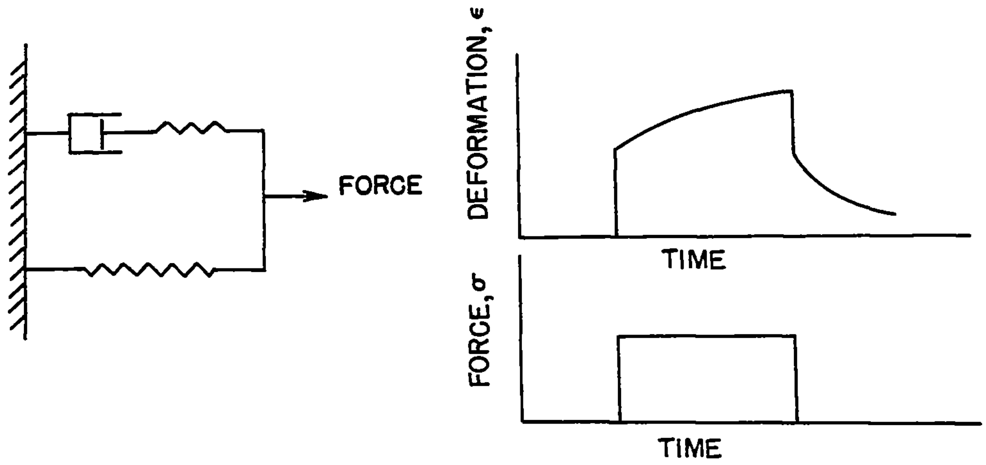

Figure 1.- Mechanical model and deformation of a standard linear solid.

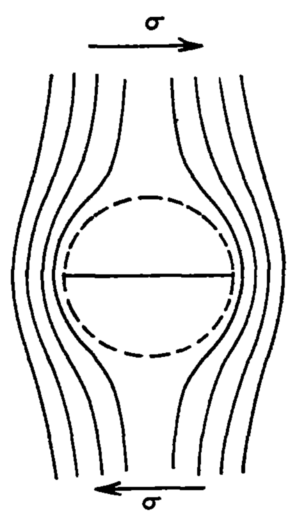

Figure 2.- Stress relaxation across an interface.

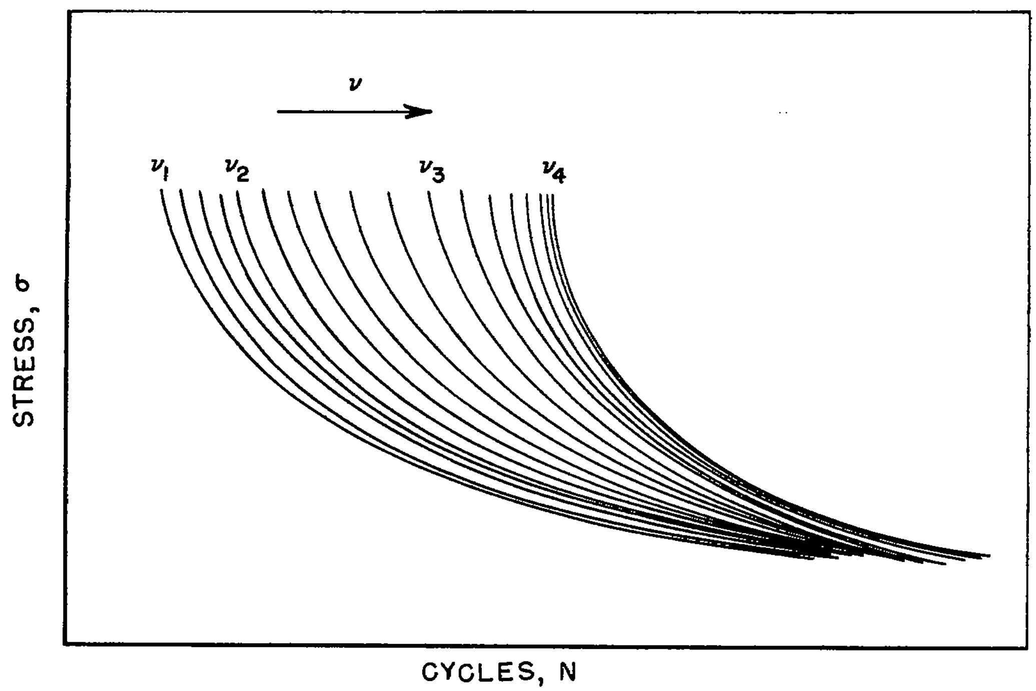

Figure 3.- Effect of increasing frequency on fatigue. T is constant.

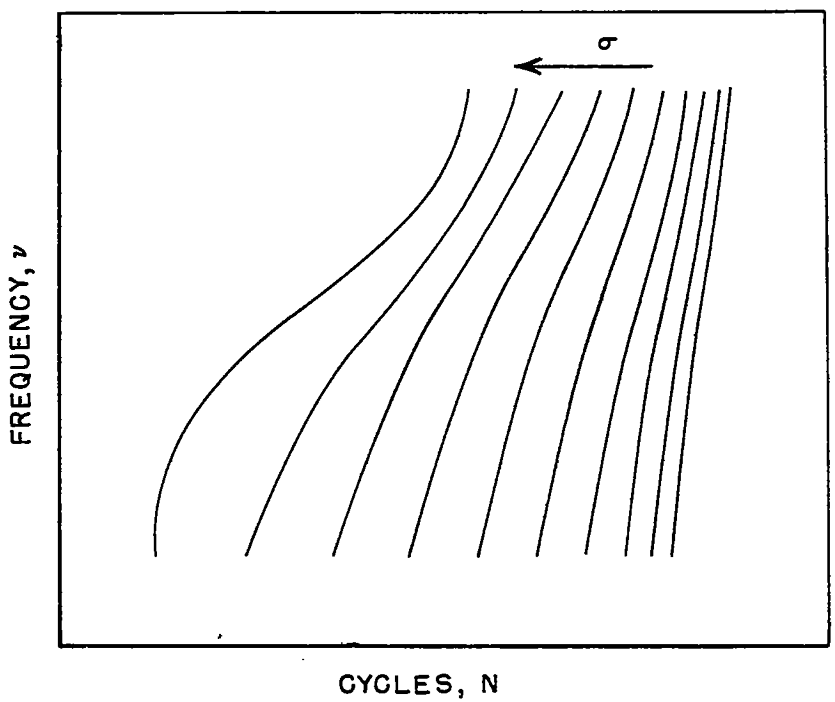

Figure 4.- Effect of frequency on fatigue. T is constant.

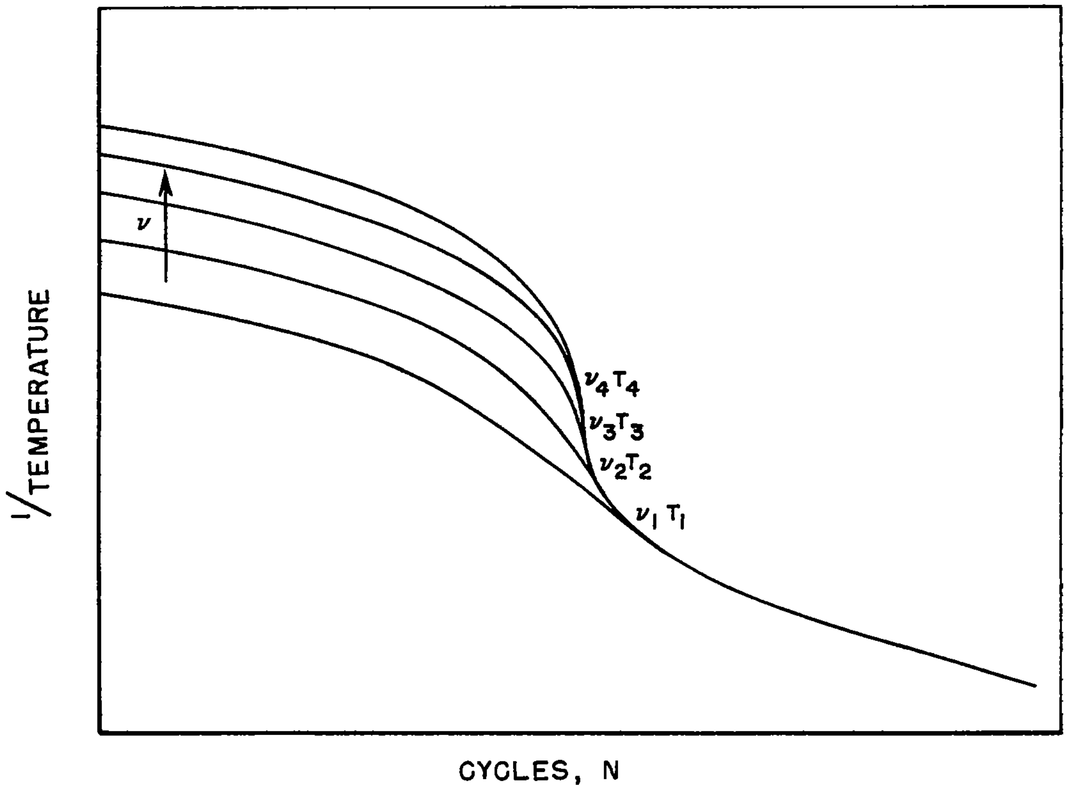

Figure 5.- Effect of temperature on fatigue.

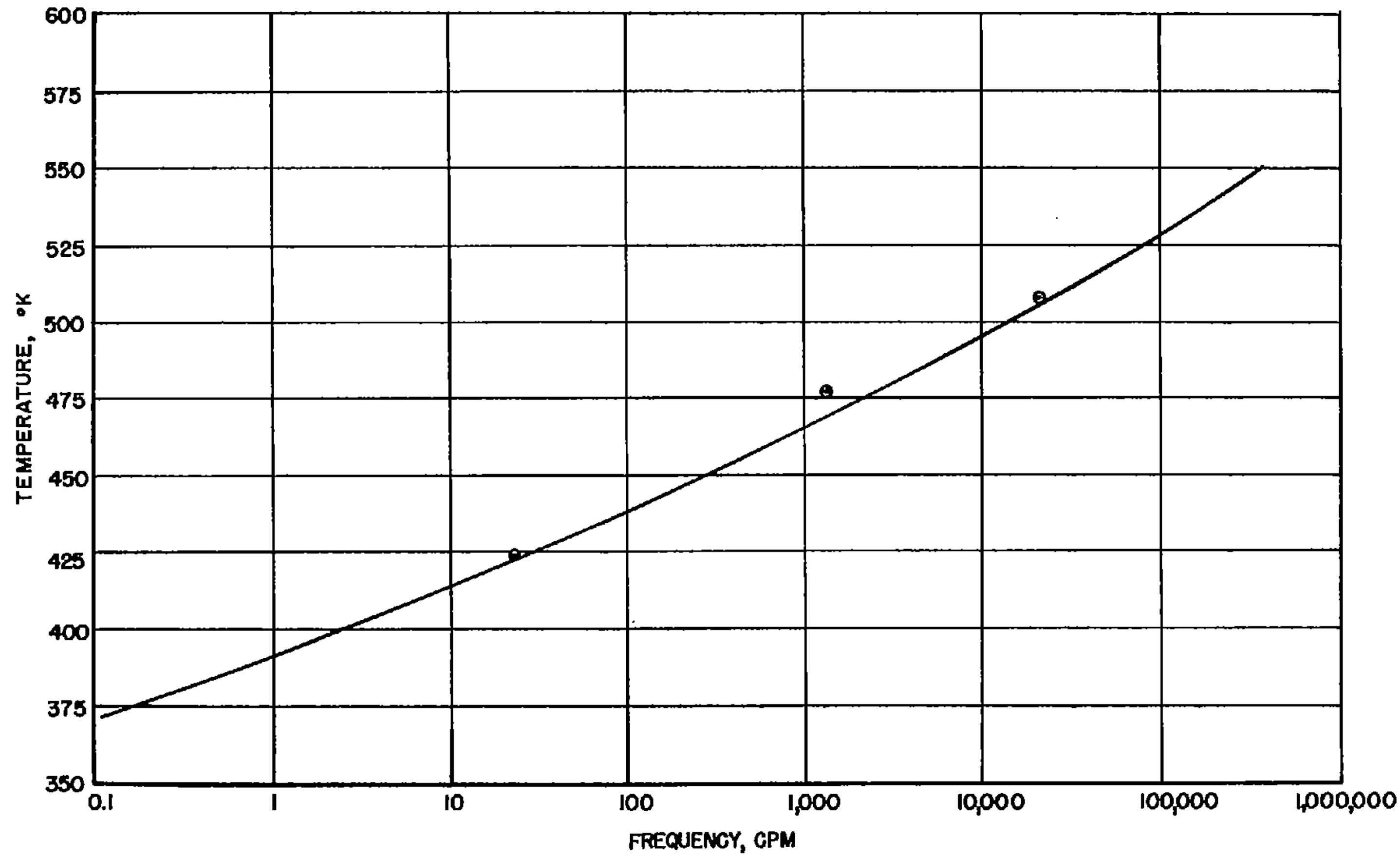

Figure 6.- Curve of frequency and associated critical temperatures.

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