NATIONAL ADVISORY COMMITTEE
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TECHNICAL NOTE 3934

EXPERIMENTAL STUDY OF HEAT TRANSFER TO SMALL CYLINDERS

IN A SUBSONIC, HIGH-TEMPERATURE GAS STREAM

By George E. Glawe and Robert C. Johnson

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SUMMARY

A Nusselt-Reynolds number relation for cylindrical thermocouple wires in crossflow was obtained from the experimental determination of time constants. Tests were conducted in exhaust gas over a temperature range of 2000° to 3400° R, a Mach number range of 0.3 to 0.8, and a static-pressure range from $2/3$ to $1\frac{1}{3}$ atmospheres. Combinations of these conditions yielded a Reynolds number range of 450 to 3000, based on wire diameter.

From these data, the correlation obtained between Nusselt and Reynolds numbers with average deviations of a single observation of 8.5 percent is

$$\mathrm{Nu} = (0.428 \pm 0.003) \sqrt{\mathrm{Re}^*}$$

where Re^* is the Reynolds number based on evaluation of gas density and viscosity at total temperature, and Nu is the Nusselt number with gas thermal conductivity computed at total temperature. This correlation equation agrees with the one previously reported in Technical Note 2599 for room-temperature conditions.

INTRODUCTION

The temperature indicated by a thermocouple immersed in a gas stream can be represented by a balance between various modes of heat transfer, and at higher velocities also includes the effect of aerodynamic recovery. Theoretical and empirical correlations can be made and applied to the thermocouple indicated temperature to account for radiation, conduction, and recovery errors (refs. 1 and 2).

In reference 1 a heat-transfer relation was obtained by the experimental determination of time constants for bare-wire crossflow thermocouples at approximately room temperature. Reference 1 also includes an

analytic approximation of conduction and radiation errors for high-temperature application, in which case it was assumed that the heat-transfer relation obtained at room temperature also applied at higher temperatures and that gas properties could be accurately established for these higher temperatures.

The work herein reported was done primarily to establish experimentally the heat-transfer relations at the higher temperatures and thereby to check on the assumptions of the analysis postulated in reference 1. Although the primary purpose was the application of the relation to thermocouples, it is felt that the correlations may also be of interest in other fields of heat transfer involving cylinders in crossflow. The method for computing viscosity and thermal conductivity of the combustion gas mixture was devised by Richard S. Brokaw and Robert C. Johnson in appendix C.

This work is part of a program of research in high-temperature measurements being conducted at the NACA Lewis laboratory.

THEORY

In order to establish a heat-transfer correlation in terms of the Nusselt and Reynolds numbers, it is necessary to evaluate the following conditions:

(a) Convective heat-transfer coefficient h

(b) Gas viscosity and thermal conductivity

(c) Free-stream conditions such as density, velocity, and temperature

A method previously described in reference 1 forms the basis for evaluating the heat-transfer coefficient. If the exposed thermocouple wire is treated as a cylinder in crossflow, a heat balance per unit length can be represented by

$$q_c + q_k + q_r = q_p \tag{1}$$

where q_c is the rate of convective heat transfer between the gas and the wire; q_k , the rate at which heat is conducted out of the junction through the thermocouple wire; q_r , the rate of net radiant heat exchange between the wire and its surroundings; and q_p , the rate of heat storage in the wire.

The terms in equation (1) with symbols defined in appendix A are as follows:

$$q_c = h(T_{ad} - T_w)\pi D \tag{2}$$

$$q_k = k_w \frac{d^2 T_w}{dx^2} \frac{\pi D^2}{4} \tag{3}$$

$$q_p = \rho_w c_w \frac{dT_w}{dt} \frac{\pi D^2}{4} \tag{4}$$

and, for the case where emissivity and absorptivity of the gas are negligible,

$$q_r = \overline{\sigma} \varepsilon_w (T_d^4 - T_w^4)\pi D \tag{5}$$

In the elementary case where radiation and conduction losses are negligible, the heat balance is found by equating q_c and q_p, which yields

$$\frac{\rho_w c_w D}{4h} \frac{dT_w}{dt} + T_w = T_{ad} \tag{6}$$

The coefficient of dT_w/dt is the time constant τ_1 of the cylinder. Thus, it is possible to solve equation (6) for h in terms of τ_1:

$$h = \frac{\rho_w c_w D}{4\tau_1} \tag{7}$$

In the present investigation, the effect of conduction heat loss on the measurements of τ was reduced to less than 1 percent by the deliberate choice of a large ratio of wire immersion length to diameter (l/D of 50). However, the radiation effect was not negligible, and a correction for this effect was included in the correlation between the measured time constant τ and the heat-transfer coefficient h. The corrected solution, derived in appendix B, is

$$h = \frac{\rho_w c_w D}{4\tau} - 4\overline{\sigma}\varepsilon_w T_{w,f}^3 \tag{8}$$

which involves the assumption that the step change in temperature is small compared with the temperature level.

With the heat-transfer coefficient established, the Nusselt number can be computed from

$$Nu = \frac{hD}{k_g} \tag{9}$$

and the Reynolds number from

$$Re^* = \frac{\rho_g V D}{\mu_g} \qquad (10)$$

where the thermal conductivity, density, and viscosity of the gas are
evaluated at total temperature (ref. 1).

Since the experiments were performed in hydrocarbon-combustion
exhaust gases, thermal conductivity and viscosity were calculated from
relations for mixtures derived in appendix C by using reference 3 to
provide the properties of the gas constituents. The results of these
computations are shown in figure 1, where the viscosity and thermal
conductivity are plotted as functions of total temperature at the test
section and compared with the values for air found in reference 3.

The quantities ε_w, ρ_w, c_w, and D were evaluated at wire temper-
atures, with the use of references 4 and 5.

APPARATUS AND PROCEDURE

Tests were performed in a high-temperature wind tunnel with an In-
conel combustor section, which operated with 60-octane gasoline and air
mixtures (ref. 6).

Three sizes of platinum 13 percent rhodium - platinum thermocouples
were used in the tests (fig. 2). The thermocouples were fabricated by
butt welding the junctions and then swaging the wire to a uniform diameter.
The measured wire diameters after swaging were 0.0195, 0.0336, and 0.0451
inch. The thermocouples were mounted in water-cooled, Inconel supports
having an outer diameter of 5/16 inch, and all three sizes of wire had
exposed lengths of 50 diameters.

A temperature step change was applied by placing an internally water-
cooled shell immediately ahead of the test probe and suddenly retracting
the shell by use of a pneumatic actuator (fig. 3). This method produced
step changes on the order of 5 to 10 percent of the absolute temperature
level.

The absolute temperature was recorded on a self-balancing potentiometer
and the exponential response on a recording oscillograph. A minimum of
three records for each test point was obtained to check reproducibility.

Initial tests were performed by placing a probe in the gas stream
and then retracting the cooling shell as previously described. The
oscillograph circuit contained a variable d-c voltage supply to oppose

the initial thermocouple electromotive force and maintain zero position on the oscillograph at the start of the step change. This method (method A) produced oscillograph traces that indicated high-frequency fluctuations of temperature and resulted in poor reading accuracy. The temperature fluctuations appeared to be caused by fluctuations in tunnel airflow. With these measurements, the time constant τ was determined with a probable reading error of ±6 percent.

A second method (method B) was then used in which two identical probes were placed in the test section with the exposed thermocouple wire loops in axial alinement and separated by the cooling shell. The thermocouples were connected in series opposition (bucking) so that the differential electromotive force of the two probes was obtained. The upstream probe measured tunnel temperature whether the cooling shell was retracted or extended, and only the downstream probe was subject to the step change. Such an arrangement favorably cancelled most of the effects of tunnel fluctuation and resulted in smoother and more reproducible oscillograph traces. In this case the time constant was determined with a probable reading error of ±2 percent. A record obtained by this method is shown in figure 4.

RESULTS AND DISCUSSION

The data obtained in the experiments by using test methods A and B are shown in figure 5, which is a plot of the terminal calculations of Nusselt number and Reynolds number as obtained by both procedures. Although the basic measurement of τ was obtained at slightly different degrees of accuracy for the two test methods, no attempt was made to weight the results in the final calculations. If the exponent of Re^* is fixed at a value of $1/2$, the least-squares solution for the multiplying constant leads to

$$Nu = (0.428 \pm 0.003) \ \sqrt{Re^*} \tag{11}$$

with an average deviation of any point of 8.5 percent. This relation involves the evaluation of transport properties of mixtures (appendix C). If the transport properties of air were used for the correlation, the constant (0.428) would be approximately 5 percent greater at 2500° R.

For application to other gases, the Prandtl number may be inserted explicitly in equation (11) with the exponent set equal to 0.3 (ref. 7), and a value of 0.70 chosen for the Prandtl number in the range of test temperatures. The resultant solution would then be

$$Nu = (0.476 \pm 0.003) \ \sqrt{Re^*} \ Pr^{0.3} \tag{12}$$

This result (eq. (12)) is in good agreement with the relation obtained
in reference 1 at near room temperatures. From reference 1 the following
relation is shown:

$$Nu = (0.478 \pm 0.002) \sqrt{Re^*} \, Pr^{0.3} \tag{13}$$

Figure 5 also includes, for comparison, a portion of the curve from
reference 7.

Lewis Flight Propulsion Laboratory
    National Advisory Committee for Aeronautics
        Cleveland, Ohio, March 1, 1957

APPENDIX A

SYMBOLS

C_p specific heat of gas at constant pressure

c   specific heat

D   wire diameter

h   heat-transfer coefficient

k   thermal conductivity

l total exposed length of thermocouple wire

M   molecular weight

Nu   Nusselt number with thermal conductivity evaluated at total temperature

Pr   Prandtl number

q_c rate of heat transfer by convection, per unit length

q_k rate of heat transfer by conduction, per unit length

q_p rate of heat storage, per unit length

q_r rate of heat transfer by radiation, per unit length

R   universal gas constant

Re*   Reynolds number with viscosity and density evaluated at total temperature

T_{ad} adiabatic temperature

T_d equivalent duct temperature

T_w wire temperature

t   time

V   velocity

x   variable length

ε_w emittance of wire

μ viscosity

ρ density

σ molecular diameter

$\bar{\sigma}$ Stefan-Boltzmann constant

τ measured time constant in presence of radiation

τ_1 time constant in absence of radiation and conduction

Subscripts:

calc   calculated

f   final conditions

g   gas

mix   mixture

w   wire

APPENDIX B

EVALUATION OF HEAT-TRANSFER COEFFICIENT FOR CYLINDER IN

CROSSFLOW UNDER FORCED CONVECTION AND RADIATION

To evaluate the heat-transfer coefficient h under the condition of heat transfer by forced convection and radiation, equations (2), (4), and (5) are substituted in equation (1) with the following result:

$$\frac{D}{4} \rho_w c_w \frac{dT_w}{dt} = h(T_{ad} - T_w) - \overline{\sigma}\varepsilon_w(T_w^4 - T_d^4) \tag{B1}$$

When considering the steady-state solution first, let the equilibrium wire temperature (final temperature) be represented by $T_{w,f}$. Since dT_w/dt equals zero, equation (B1) can be written as follows:

$$h(T_{ad} - T_{w,f}) = \overline{\sigma}\varepsilon_w T_{w,f}^4 \left[1 - \left(\frac{T_d}{T_{w,f}} \right)^4 \right] \tag{B2}$$

Equation (B1) can then be written

$$\frac{D}{4} \rho_w c_w \frac{dT_w}{dt} = h(T_{ad} - T_{w,f}) + h(T_{w,f} - T_w) +$$

$$\overline{\sigma}\varepsilon_w(T_{w,f}^4 - T_w^4) - \overline{\sigma}\varepsilon_w(T_{w,f}^4 - T_d^4) \tag{B3}$$

If it is assumed that $\dfrac{T_{w,f} - T_w}{T_{w,f}}$ is much less than unity, then

$$(T_{w,f}^4 - T_w^4) \approx 4T_{w,f}^3(T_{w,f} - T_w) \tag{B4}$$

where ρ_w, c_w, and ε_w are taken as constant over the temperature step change and are evaluated at the final temperature. When equations (B2) and (B4) are substituted in equation (B3), the result is

$$\frac{\frac{D}{4} \rho_w c_w}{h + 4\overline{\sigma}\varepsilon_w T_{w,f}^3} \frac{dT_w}{dt} + T_w = T_{w,f} \tag{B5}$$

The coefficient of dT_w/dt in equation (B5) represents the time constant τ of the cylinder, or

$$\tau = \frac{\frac{D}{4} \rho_w c_w}{h + 4\bar{\sigma}\varepsilon_w T_{w,f}^3} \tag{B6}$$

The solution for h is then given in terms of the measured τ in the presence of radiation

$$h = \frac{\rho_w c_w D}{4\tau} - 4\bar{\sigma}\varepsilon_w T_{w,f}^3$$

(eq. (8) of the text).

APPENDIX C

METHOD USED TO COMPUTE VISCOSITY AND THERMAL CONDUCTIVITY OF

COMBUSTION GAS MIXTURES

By Richard S. Brokaw and Robert C. Johnson

Gas-Mixture Compositions

The composition of the combustion gases was computed from the measured fuel-air ratio. Combustion was assumed to be complete, and it was assumed that within the range of temperatures investigated dissociation was negligible, so that the combustion products were nitrogen, oxygen, carbon dioxide, and water vapor. Test-section total temperatures were measured and found to be functions of the fuel-air ratio.

Viscosities of Component Gases

Insofar as possible, the viscosities of air and the pure gases were obtained from reference 3. For some of the gases, extrapolations of as much as 900° R were required (extrapolations were made where necessary by taking differences).

Thermal Conductivities of Component Gases

Because of the dearth of high-temperature thermal conductivity data for gases, conductivities for air and the pure gases were computed from the Prandtl number relation

$$k_g = \frac{C_p \mu}{Pr} \tag{C1}$$

together with the theoretical expression for Prandtl number proposed in reference 8.

$$Pr = \frac{0.754}{1 + 0.325 \left(\frac{R}{C_p} \right)} \tag{C2}$$

Ideal-gas heat capacities from reference 3 were used. While equation (C2) is not strictly applicable to air (since air is a gas mixture), it was found that air thermal conductivities calculated in this fashion agreed to within 1 percent when compared with values tabulated in reference 3.

Equation (C2) is not applicable to water, a highly polar molecule. However, in the gas mixtures of interest, water was present in a relatively low concentration so that most of the collisions of the water molecules occurred with nonpolar molecules. Collisions between polar and nonpolar molecules are essentially of a nonpolar nature. Hence, it was felt that, to a fair approximation, water may be treated as a nonpolar molecule in these mixtures.

Viscosities of Combustion-Gas Mixtures

Viscosities for both the combustion gases and for air (considered as a mixture of 0.79 nitrogen and 0.21 oxygen by volume) were computed from the following equation (ref. 9):

$$\mu_{mix} = \sum_{i=1}^{n} \frac{\mu_i}{1 + \sum_{\substack{j=1 \\ j \neq i}}^{n} \varphi_{i,j} \frac{x_j}{x_i}} \tag{C3}$$

The coefficient $\varphi_{i,j}$ was taken as

$$\varphi_{i,j} = \frac{D_i}{D_{i,j}} \tag{C4}$$

which is implicit in reference 9. Here D_i is the self-diffusion coefficient for the ith species, and $D_{i,j}$ is the binary diffusion coefficient between the ith and jth species.

Using the formula of reference 10, equation 8.2-44, for the diffusion coefficients, changes equation (C4) to

$$\varphi_{i,j} = \frac{\sigma_{i,j}^2}{\sigma_i^2} \sqrt{\frac{2M_j}{M_i + M_j}} \frac{\Omega_{i,j}^{(1,1)*}}{\Omega_i^{(1,1)*}} \tag{C5}$$

where σ is the molecular diameter, and M is the molecular weight. The quantities $\Omega^{(1,1)*}$ are functions of the temperature and the particular molecule or molecules involved. The $\Omega^{(1,1)*}$ values for the Lennard-Jones 6-12 potential were used; they are tabulated in table I-M of reference 10.

The computed viscosities of the combustion gases were then normalized by multiplying by the ratio of the table values for air (ref. 3) to the calculated air viscosities:

$$\mu_{mix} = \left(\frac{\mu_{air,table}}{\mu_{air,calc}}\right) \mu_{mix,calc}$$

The normalization factors differed from unity by less than 1 percent. Viscosities of air and combustion gases are compared in figure 1.

Thermal Conductivities of Combustion-Gas Mixtures

Thermal conductivities for both the combustion gases and for air (considered as an oxygen-nitrogen mixture) were computed using a mixing rule analogous to equation (C3) (ref. 11).

$$k_g = \sum_{i=1}^{n} \frac{k_{g,i}}{1 + \sum_{\substack{j=1 \\ j \neq i}}^{n} A_{i,j} \frac{x_j}{x_i}} \tag{C6}$$

An expression for $A_{i,j}$, employing Sutherland constants and viscosities, is presented in reference 11. If the expression for viscosity of reference 10, equation 8.2-18, is substituted into the equation for $A_{i,j}$ of reference 11, and the quantities $1 + Si/T$ are considered equivalent to the values of Ω from reference 10, then the expression for $A_{i,j}$ is

$$A_{i,j} = \frac{1}{4}\left[1 + \left(\frac{M_j}{M_i}\right)^{1/8} \frac{\sigma_j}{\sigma_i}\right]^2 \frac{\Omega_{i,j}^{(1,1)^*}}{\Omega_i^{(1,1)^*}} \tag{C7}$$

The computed combustion-gas thermal conductivities were then normalized by multiplying by the ratio of the air thermal conductivity calculated from equations (C1) and (C2) to the air thermal conductivity calculated by equations (C6) and (C7):

$$k_{g,mix} = \left[\frac{k_{g,air,\ (C1)\ -\ (C2)}}{k_{g,air,\ (C6)\ -\ (C7)}}\right] k_{g,mix,calc}$$

The necessity of this normalization is questionable; in any event, the normalization factors differed from unity by less than 2 percent. Results are presented in figure 1 where thermal conductivities of combustion products and air are compared.

Force Constants Used

The force constants σ in Angstroms and ε/k in degrees Kelvin, together with the molecular weights of the components, are as follows:

	σ, A	ε/k, $^\circ K$	M
Nitrogen	3.749	79.8	28
Oxygen	3.541	88.0	32
Carbon dioxide	3.897	213.0	44
Water	2.824	230.9	18

Values for nitrogen, oxygen, and carbon dioxide are taken from reference 10, table I-A. Values for water were taken from reference 10, table 8.6-1 (these values are intended for use with a polar potential, but were used in the Lennard-Jones 6-12 potential).

For the binary interactions, $\sigma_{i,j}$ and $(\varepsilon/k)_{i,j}$ were computed according to reference 10, equations 8.4-8 and 8.4-9.

$$\sigma_{i,j} = \frac{1}{2}(\sigma_i + \sigma_j)$$

$$\varepsilon_{i,j} = \sqrt{\varepsilon_i \varepsilon_j}$$

Discussion

The computed viscosities and thermal conductivities of figure 1 are probably as reliable as theory permits at this time. The largest uncertainty arises as a result of the presence of water in the combustion gases; the theory of transport properties of polar gases (particularly thermal conductivity) is not well developed. Fortunately, the water concentrations did not exceed 13 percent by volume. Other uncertainties arise from equation (C2) (which is believed to be a good approximation at high temperatures) and from the extrapolation of the viscosities.

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Nonetheless, the excellent agreement between the heat-transfer correlation of this report (eq. (12)) and the previous room-temperature results (eq. (13)) suggests that the calculated viscosities and thermal conductivities are not seriously in error.

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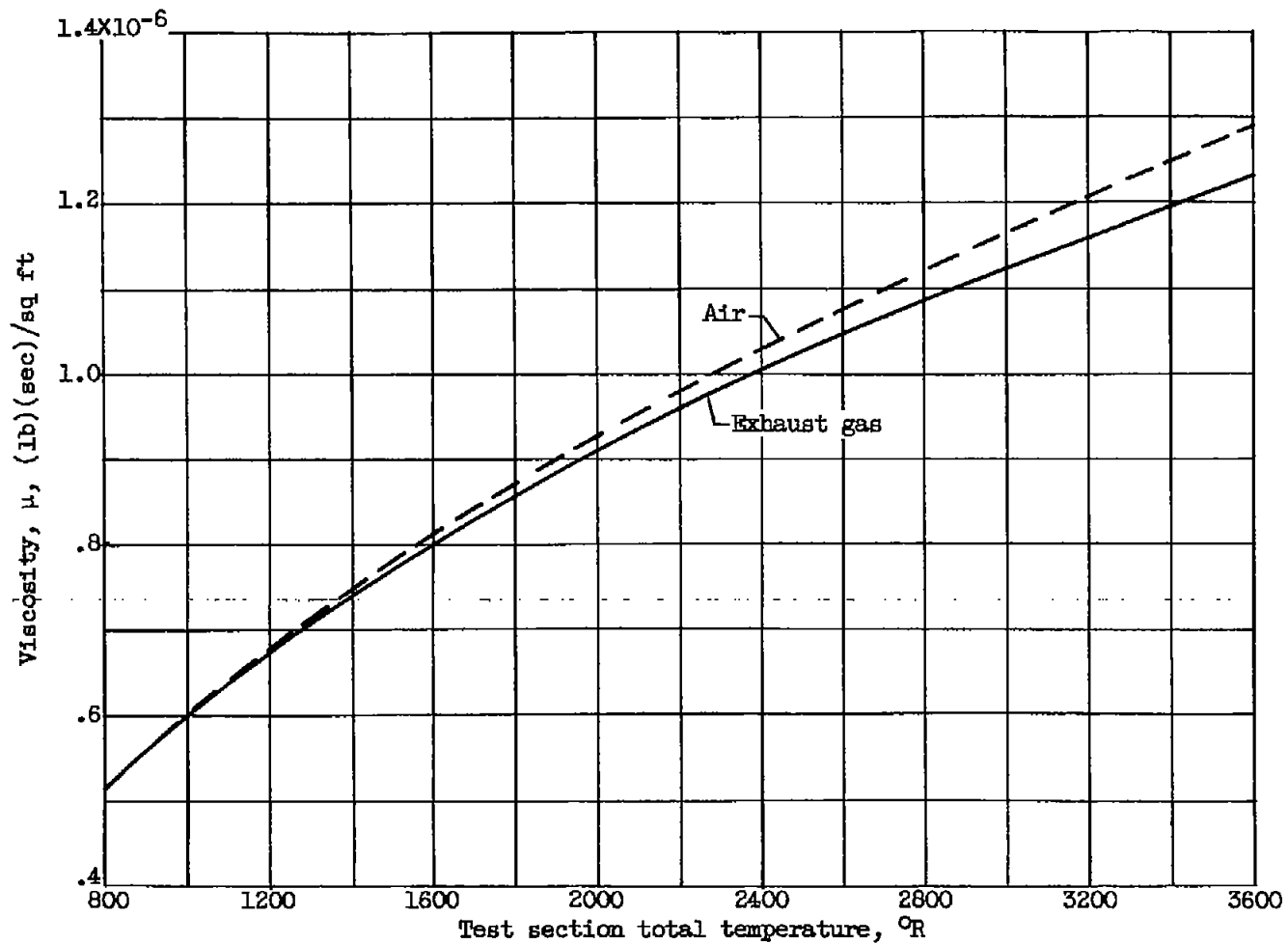
(a) Viscosity.

Figure 1. - Variation of viscosity and thermal conductivity of exhaust-gas and air with test section total temperature.
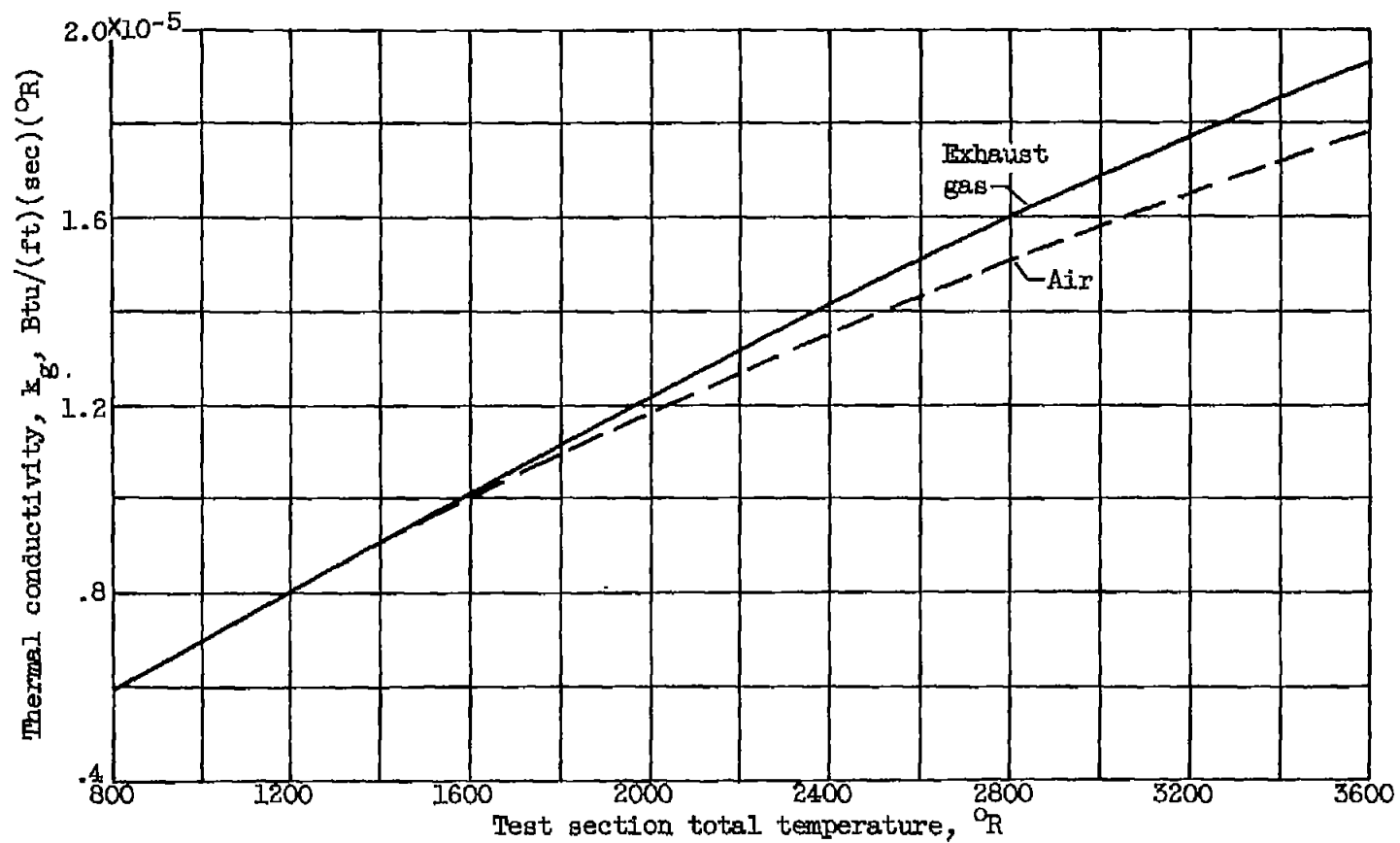
(b) Thermal conductivity.

Figure 1. - Concluded. Variation of viscosity and thermal conductivity of exhaust-gas and air with test section total temperature.
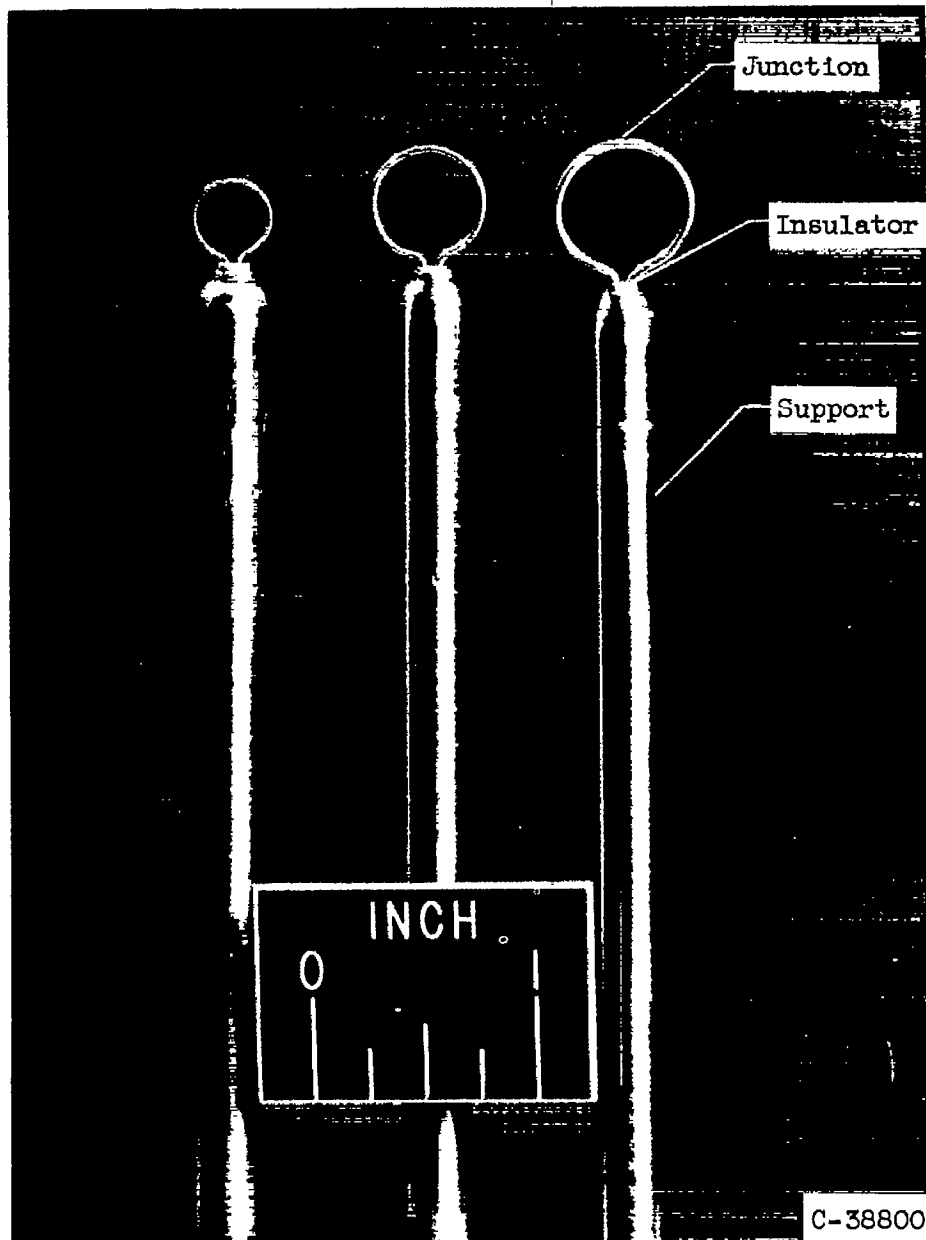
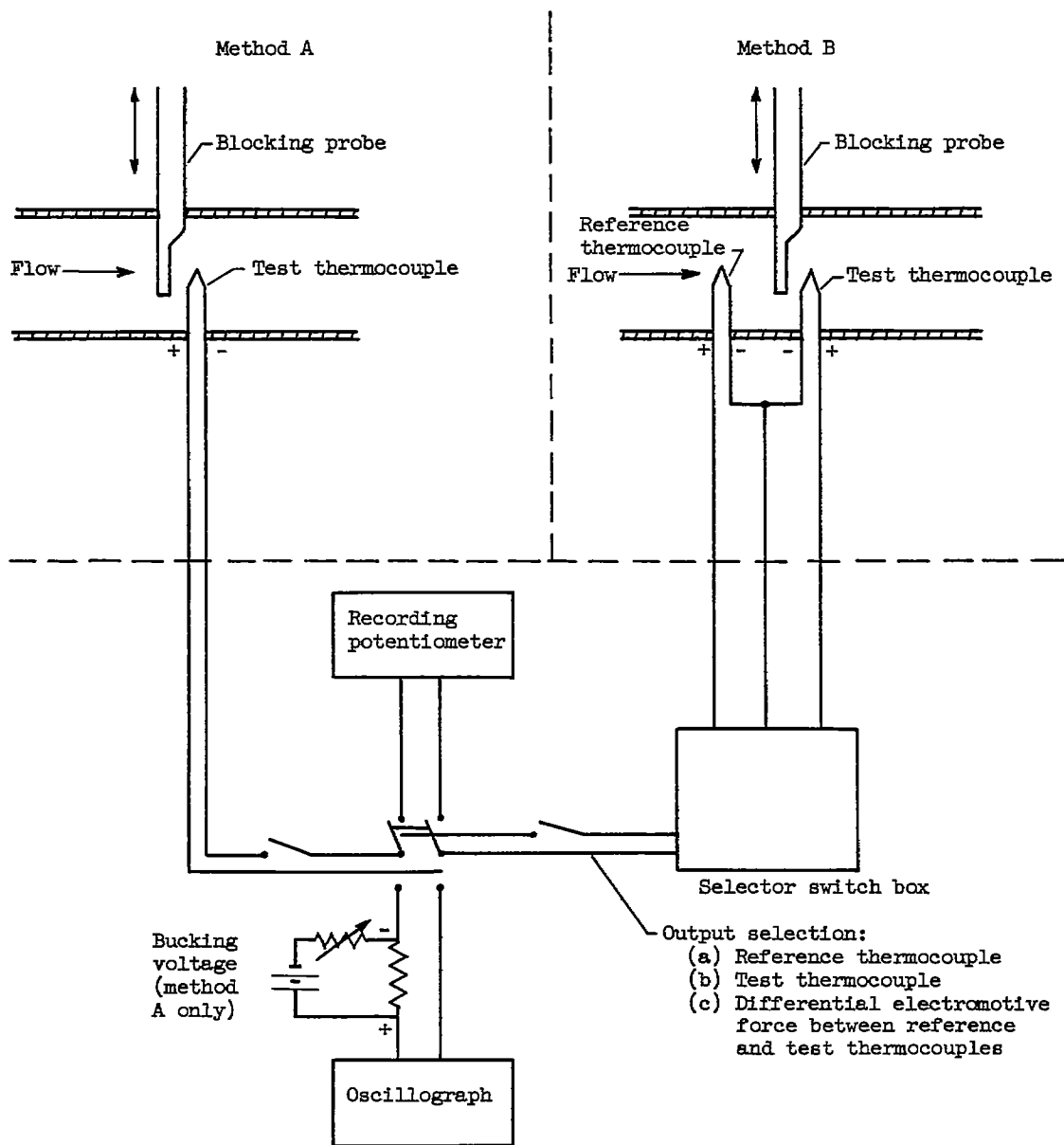Figure 2. - Platinum, 13 percent rhodium - platinum
thermocouple probes.

Figure 3. - Schematic diagram of experimental setups.

0.1 Sec

Final temperature

Reference galvanometer
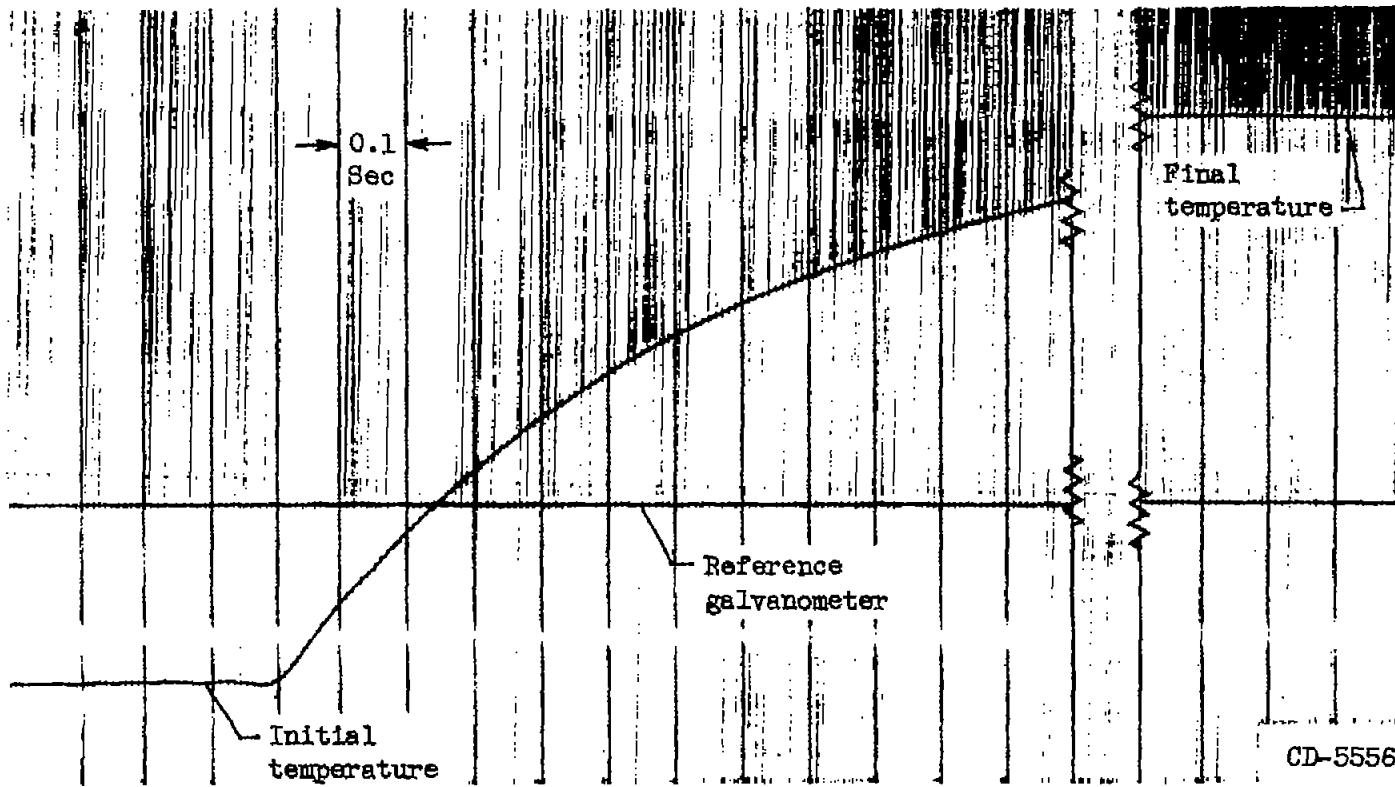
Initial temperature

CD-5556

Figure 4. - Oscillograph record of exponential response using test method B.
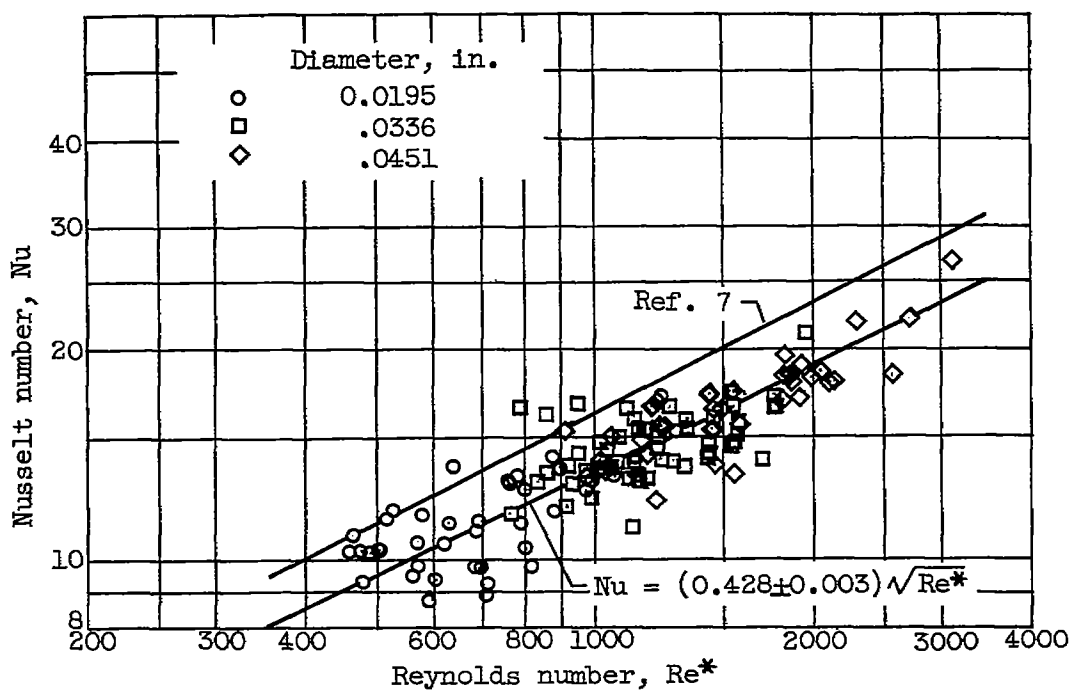
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Figure 5. - Nusselt-Reynolds number relation for
cylindrical wire transverse to airstream.