RESEARCH MEMORANDUM

INVESTIGATION OF EQUILIBRIUM
TEMPERATURES AND AVERAGE LAMINAR HEAT-TRANSFER
COEFFICIENTS FOR THE FRONT HALF OF SWEPT CIRCULAR
CYLINDERS AT A MACH NUMBER OF 6.9

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INVESTIGATION OF EQUILIBRIUM TEMPERATURES AND AVERAGE LAMINAR HEAT-TRANSFER COEFFICIENTS FOR THE FRONT HALF OF SWEPt CIRCULAR CYLINDERS AT A MACH NUMBER OF 6.9

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SUMMARY

Average heat-transfer coefficients and equilibrium temperatures for the front half of an isothermal cylinder with a laminar boundary layer were determined from wind-tunnel tests at a Mach number of 6.9, free-stream Reynolds numbers based on diameter of $1.3 \times 10^5$ and $1.8 \times 10^5$, and sweep angles from $0^\circ$ to $75^\circ$. The equilibrium temperature for the isothermal cylinder with no net heat transfer was found to decrease with increasing sweep angle and could be closely predicted by the simple theory presented. The average heat-transfer coefficient varied approximately as the cosine of the sweep angle. A simple analysis, based on the normal component of the flow only, shows good agreement with this trend. The laminar correlating parameter, Nusselt number divided by the square root of the Reynolds number, when the air conductivity and viscosity were evaluated at the temperature just behind the bow shock, was found to vary only slightly with sweep angle.

INTRODUCTION

As flight Mach numbers increase, the aerodynamic heating at leading edges of wings and control surfaces becomes increasingly serious. High local heat-input rates can induce large thermal stresses, and in the case of hypersonic flight where the boundary-layer recovery temperatures become very high, the heat-input rate will determine the flight time available before the temperature limits of the structure are exceeded. It becomes important then to investigate the possibility of reducing the heat-transfer rates by changes in the wing geometry, for example, by changing the leading-edge radius or by sweeping the leading edge.
A leading edge designed for high Mach number operation will probably be blunted to some extent in order to provide some heat capacity at the stagnation line. The boundary layer can be expected to be laminar for some distance. Thus, the results of investigation into the laminar heat transfer to circular cylinders are directly applicable to the nose of a wing, the region where the heat transfer is highest.

A number of analytical studies of the heat transfer to cylinders normal to an airstream have been made (e.g., refs. 1 to 3). Much of the analytical work, however, is restricted to incompressible flow or to a Prandtl number of 1. The experimental work has been limited to very low speeds (e.g., refs. 4 and 5) or to the very low Reynolds numbers of hot-wire anemometry (ref. 6). The results of these studies show that local heat-transfer rates can be correlated using Nusselt and Reynolds numbers based on cylinder diameter, which means that increasing the diameter reduces the local heat-transfer coefficients.

Since sweeping a blunt leading edge is desirable for reducing the drag, the question arises whether it might not be beneficial from the standpoint of heat transfer also. This question has, however, been studied very little. Theoretical analyses have been made for incompressible flow with a Prandtl number of 1 (refs. 7 and 8). Beckwith has recently extended the integral method of reference 2 for compressible flow with arbitrary Prandtl number to the case of a swept cylinder in reference 9, and has compared the theoretical results with the experimental values from the present investigation. King (ref. 10) in 1914 published experimental results for heat loss from hot wires at very low speeds, which showed a considerable reduction in overall heat-transfer coefficient as the sweep angle increased. Eggers, Hansen, and Cunningham of the Ames Aeronautical Laboratory have recently presented results of tests on swept wires at \( M = 9.8 \), but at the very low Reynolds number of 315 (ref. 11).

In view of the lack of information applicable to the Mach and Reynolds numbers of interest to designers of supersonic vehicles, an experimental investigation was undertaken to study the effect of sweep angle on the average heat-transfer rates to the front half of a solid cylinder with a laminar boundary layer at \( M = 6.9 \) and Reynolds numbers based on diameter and free-stream air properties of \( 1.3 \times 10^5 \) and \( 1.8 \times 10^5 \). These conditions correspond to the case of a 1-inch-diameter semicircular leading edge flying at \( M = 6.9 \) at altitudes of 163,000 and 154,000 feet.

**SYMBOLS**

A area per unit length of uninsulated front half of cylinder

C specific heat of copper
\( c_p \) specific heat of air at constant pressure

\( D \) cylinder diameter

\( h \) average heat-transfer coefficient

\( k \) thermal conductivity of air

\( m \) weight of copper cylinder per unit of length

\( M \) Mach number

\( Nu \) Nusselt number, \( hD/k \)

\( P_0 \) tunnel stagnation pressure

\( Pr \) Prandtl number, \( c_p\mu/k \)

\( r \) recovery factor based on stream static temperature

\( R \) Reynolds number, \( \rho VD/\mu \)

\( t \) time

\( T_e \) equilibrium temperature of isothermal body for no net heat transfer

\( T_0 \) tunnel stagnation temperature

\( T_w \) model temperature

\( T_l \) tunnel free-stream static temperature

\( V_l \) tunnel free-stream air velocity

\( \Lambda \) sweep angle, deg

\( \rho_l \) tunnel free-stream air density

\( \mu \) dynamic viscosity of air

Subscripts:

\( e \) equilibrium

\( n \) normal component
tangential component

1 tunnel free-stream static conditions

2 behind center of shock at sweep angle

APPARATUS AND METHODS

This investigation was carried out in the Langley 11-inch hypersonic tunnel, an intermittent blowdown tunnel, at a Mach number of about 6.9. The general arrangement of the tunnel is the same as that described in reference 12 except for a new electrical heater and an invar two-dimensional nozzle, which does not show the change of Mach number with time found in the steel nozzle.

Tests were made at two tunnel stagnation pressures, 25 and 33 atmospheres, and at stagnation temperatures near 1,140° R. Calibration tests of the nozzle have shown a Mach number of 6.86 for 25 atmospheres stagnation pressure, and 6.88 for 33 atmospheres. For brevity, the Mach number for both conditions will be called 6.9. The free-stream Reynolds numbers based on the 0.5-inch diameter were about $1.3 \times 10^5$ and $1.8 \times 10^5$.

To minimize the effect of unsteady air-flow conditions on the model at the start of the run, the first 15 seconds of air flow were bypassed around the nozzle to permit the heater temperatures to steady. The nozzle air flow was started by a quick-opening valve, so that the model was exposed to flow at varying temperature and pressure for a short time only, probably about 5 seconds. The running time available was about 85 seconds.

Figure 1 shows the model tested. The cylinder was solid, machined from high-electrical-conductivity copper, and fitted with two plugs of the same material carrying the chromel-alumel thermocouple junctions. The thermocouple junctions were located very close to the cylinder axis at stations 3 and 3.75 inches from the outboard end. One end and the rear half of the cylinder were covered by a layer of glass-fiber-reinforced resin (Paraplex P-44) about 0.08 inch thick, tapered to be parallel to the airstream direction at the sides. The other end of the model was reduced in diameter for a length of 1 inch to fit inside a tubular support fastened to the tunnel wall. The interchangeable supports used gave sweep angles of 0.0°, 20.5°, 40.5°, 60.8°, and 75.0°.

Model and tunnel stagnation temperatures were read from self-balancing recording potentiometers with a rated accuracy of $2\frac{1}{2}$ F. Tunnel stagnation pressure was read from Bourdon type gages with an accuracy of 1/2 percent of full scale, or 5 inches of mercury.
REDUCTION OF DATA

Equilibrium Temperature

To calculate heat-transfer coefficients from the time histories of tunnel stagnation and model temperatures, samples of which are shown in figure 2, a reference temperature for finding the temperature difference for heat transfer had to be determined. Because of the limited tunnel running time, a direct measurement of the equilibrium temperature of the model could not be made, so an extrapolation procedure was used.

A smooth curve was faired through the printed record of model temperature with time, and the slopes of the tangents to the curve at 5-second intervals were measured. These slopes were then plotted against time, and a fair curve drawn through the points. The values from this faired curve were then integrated numerically by the trapezoidal rule with 2-second intervals, and the integral curve was compared with the original curve of temperature against time. When the discrepancy exceeded 2°F, the curve of $dT_w/dt$ was refaired and reintegrated. The slopes read at the start of the flow and at 5 seconds, in general, fitted the faired curve of $dT_w/dt$ against time poorly, and because the stagnation-temperature record was not steady during this interval, these data were not used.

The faired and verified values of $dT_w/dt$ were multiplied by $C$, the specific heat of copper at the corresponding model temperature, and plotted against model temperature. A sample curve is shown in figure 2. This curve was extrapolated to the model temperature at which $C \frac{dT_w}{dt}$, and therefore the net heat transfer to the body, was zero. This temperature, which will be referred to as the equilibrium temperature $T_e$ was used as the reference temperature in forming the temperature difference for heat transfer. The values of $T_e$ determined as described above showed a correlation with the tunnel stagnation temperature. Accordingly, the ratio $T_e/T_0$ was formed and was plotted against sweep angle. A smooth curve was faired through the points obtained, weighted to favor points for which the extrapolation was believed to be most reliable. Values of $T_e/T_0$ from this curve were used with the measured stagnation temperatures to find the value of $T_e$ used in the calculation of heat-transfer coefficients.
Average Heat-Transfer Coefficients

The average heat-transfer coefficients were calculated from the model temperatures and slopes read from faired curves at 5-second intervals by using the relation

\[ hA(T_e - T_w) = mC \frac{dT_w}{dt} \]  \hspace{1cm} (1)

which assumes that all of the heat input went into raising the temperature of the body. The area A used was the surface area of the uninsulated front half of the cylinder, which was intended to simulate a semi-circular wing leading edge. Some unpublished work by Goodwin of the Ames Aeronautical Laboratory shows that at \( M = 3.94 \) the local heat-transfer coefficients on the rear half of a transverse circular cylinder are small compared to those on the front half. At \( M = 6.9 \), the same situation should prevail. The addition of a layer of insulation was expected to reduce the heat input from the rear half of the cylinder to a negligible amount.

The heat-transfer coefficients were expected to show a trend with temperature ratio \( T_w/T_o \). Such a trend could not, however, be demonstrated within the accuracy of the tests, in the range of temperatures available, so the values of \( h \) were averaged and are presented as representative values for a range of temperature ratio \( T_w/T_o \) between 0.5 and 0.8.

The lack of variation of \( h \) with \( T_w/T_o \) permitted a check on the values calculated from equation (1). If it is assumed that \( h \) and \( T_e \) are constant, equation (1) may be differentiated with respect to \( T_w \) to yield

\[ h = -\frac{m}{A} \frac{d}{dT_w} \left( C \frac{dT_w}{dt} \right) \]  \hspace{1cm} (2)

which does not depend on a choice of \( T_e \). For runs or portions of runs where the rate of change of tunnel stagnation temperature was less than about 0.2°F per second, the values of \( h \) calculated from equation (2) were in excellent agreement with those from equation (1). Unfortunately, it was impossible to hold the tunnel stagnation temperature constant to this accuracy for the entire running time.

The procedures described above have assumed that the temperature throughout a cross section of the cylinder was nearly constant, so that the temperatures read by the thermocouples on the cylinder axis represented both the surface temperature and the average temperature in the
cylinder, and that radiation and axial conduction were negligible. The first assumption was checked by calculating the temperature gradient for steady conduction in a one-dimensional body with the same heat flow per unit cross section as the calculated aerodynamic heat input per unit area. For the maximum heat-transfer rates observed in these tests, the temperature difference between the cylinder axis and the surface was calculated to be 30°F. The actual temperature difference in the model may safely be assumed to be less than this; therefore, the copper cylinder was considered isothermal.

The energy loss by radiation was calculated for a model temperature of 980°F, which was the highest value reached by the model in these tests. Assuming an emissivity for the copper model of 0.15, and assuming that the tunnel walls of 600°F acted as a complete enclosure, large compared to the model, the rate of heat loss by radiation was 190 Btu/hr/ft², which was about 2 percent of the heat-input rate measured at this temperature. For lower model temperatures, the radiation loss would be much less; therefore, no corrections were applied.

The assumption of negligible influence of axial conduction in the copper cylinder on the calculated heat-transfer coefficients was based on examination of the external air flow and on the lack of a temperature difference between the two thermocouples 3/4 inch apart on the cylinder axis.

Schlieren pictures of swept cylinders with tips similar to that used in the present investigation were available from the work of reference 13 on the forces and pressures on swept cylinders done in the same tunnel. Some of these schlieren pictures are shown in figure 3, for sweep angles of 15°, 45°, 60°, and 75°. Ahead of stations 6 and 7½ diameters from the upstream tip, which correspond to the thermocouple stations of the present investigation, the bow shock is straight and parallel to the cylinder for sweep angles up to 60°. The air flow is therefore expected to be two-dimensional in the region of the test section of the cylinder, and so the heat transfer from the air to the model is the same at the two stations for sweep angles up to 60°. At 75° sweep, the shock is not quite parallel to the cylinder, and some effects on the heat transfer, due to the three-dimensional nature of the flow, might be present in the tests at this sweep angle.

A variation in heat-transfer rates along the cylinder toward the tips could cause axial heat flow by conduction in the model, which would alter the heat balance assumed in equation (1). A large error due to this source is believed unlikely, because, while the absolute accuracy of the temperature measurements was ±2° F, the two thermocouple temperatures were read on the same potentiometer and were in agreement to less than 1° F.
under conditions of no air flow. In only a few runs at 0° sweep were
the two temperatures different by more than 1° F, and in these cases,
the difference was not regular in direction or trend with time.

Because of the several fairing and averaging procedures used in
reduction of the data, the probable errors cannot be assessed by esti-
mating the contributions due to the measuring equipment and to the nu-
merical quantities used in the calculations. For example, the calculated
values of heat-transfer coefficient are dependent on the choice of \( \frac{T_e}{T_0} \),
which could not be measured directly. The selection of values of \( \frac{T_e}{T_0} \)
was based on consideration of the fairing of the curve of \( \frac{T_e}{T_0} \) with
sweep angle as well as on the mean value obtained from extrapolations
of \( C \frac{dT}{dt} \) to zero. Some idea of the reliability of the values presented
was obtained from the scatter of individual values from the averages.
It is felt that the equilibrium temperature ratio is probably reliable
to ±2 percent and the average heat-transfer coefficients to ±5 percent.

Previous experience in the Langley 11-inch hypersonic tunnel with
various bodies and the results of reference 13 have shown that at \( M = 6.9 \)
and at the free-stream Reynolds number of these tests, the boundary layers
can be expected to be laminar for all of the configurations of this
investigation.

Calculation of the molecular-mean-free-path length at the conditions
behind the bow shock on the model gave values on the order of \( 10^{-5} \) inches,
which is small compared to a reasonable estimate of the boundary-layer
thickness on the model of \( 10^{-2} \) inches. Therefore, the tests are believed
to be well out of the region of possible slip-flow effects.

RESULTS AND DISCUSSION

Equilibrium Temperature

The change in the ratio of equilibrium temperature to stagnation
temperature with sweep angle is presented in the upper part of figure 4
by the data points and the solid faired curve. It must be emphasized
that the temperature \( T_e \) is the temperature of an assumed isothermal
surface for no net heat transfer to or from the airstream. The curve
can be extended beyond 75° sweep to meet the value 0.86 calculated from
the flat-plate laminar recovery factor equal to the square root of the
Prandtl number.
A simple analysis has been made of the effect of sweep angle on the recovery factors on the front half of a cylinder. By the law of conservation of energy, the enthalpy rise in a unit mass of air decelerated from velocity \( V_1 \) to zero with no heat loss can be written

\[
C_p(T_0 - T_1) = \frac{V_1^2}{2}
\]  

(3)

assuming constant \( C_p \). The components of the free-stream flow normal and tangential to the cylinder axis are considered to be decelerated independently and with different average recovery factors. The total enthalpy rise at the body at sweep angle \( \Lambda \) is then

\[
C_p(T_e - T_1) = r_n \left( \frac{V_1 \cos \Lambda}{2} \right)^2 + r_t \left( \frac{V_1 \sin \Lambda}{2} \right)^2
\]  

(4)

Dividing by the enthalpy rise for complete recovery (eq. (3)) gives, for constant \( C_p \),

\[
\frac{T_e - T_1}{T_0 - T_1} = r_n \cos^2 \Lambda + r_t \sin^2 \Lambda
\]  

(5)

Solving for \( T_e/T_0 \) yields

\[
\frac{T_e}{T_0} = \frac{T_1}{T_0} + \left( 1 - \frac{T_1}{T_0} \right) (r_n \cos^2 \Lambda + r_t \sin^2 \Lambda)
\]  

(6)

On the stagnation line of the cylinder, \( r_n \) can be considered 1, and using for \( r_t \), the flat-plate recovery factor \( Pr^{1/2} \), which for the test conditions is 0.85, equation (6) becomes, for the stagnation line and for Mach number 6.86 or 6.88,

\[
\frac{T_e}{T_0} = 0.096 + 0.904 (\cos^2 \Lambda + 0.85 \sin^2 \Lambda)
\]  

(7)

This relation is plotted in the upper part of figure 4.

It was next assumed that the average recovery factor over the front half of the cylinder for the normal component of the flow was independent of the normal Mach number and was equal to that calculated from the experimentally determined \( T_e/T_0 \) at zero sweep. Using again \( r_t = Pr^{1/2} = 0.85 \) gives the overall ratio
\[ \frac{T_e}{T_o} = 0.096 + 0.904 \left( 0.929 \cos^2 \Lambda + 0.85 \sin^2 \Lambda \right) \] (8)

which is also plotted in the upper part of figure 4. Even considering the fact that the curve is based on the data at 0° sweep, the agreement with the experimentally determined values is good.

**Average Heat-Transfer Coefficient**

The lower part of figure 4 shows the relative effect of sweep angle on the overall heat-transfer coefficients. The data points are the coefficients at sweep angle \( \Lambda \) divided by the average value at 0° sweep. For all these points, the Reynolds number based on diameter and free-stream air conditions was about \( 1.3 \times 10^5 \). A curve of \( \cos \Lambda \) shown dashed in the figure, seems to fit the data fairly well up to 60°. No definite conclusions can be drawn about the trend beyond 60°, since the experimental value at 75° may be influenced to some extent by end effects. The reduction in heat-input rate with sweep will be greater than the reduction in the heat-transfer coefficients because of the decrease in equilibrium temperature.

A simple analysis of the effect of sweep on the average heat-transfer coefficients to a cylinder was made. For incompressible flow, several analyses (for example, ref. 7) have shown that the temperature field, and thus the heat transfer to a swept cylinder, depends only on the normal component of the air flow. It is suggested in reference 1 that, for arbitrary bodies, the effect of compressibility on heat transfer is negligible if the local Mach numbers outside the boundary layer are somewhat below 2. On the basis of these ideas, it is assumed for the present analysis that the average heat transfer to the front half of a swept cylinder at a free-stream Mach number of 6.9 depends on the crossflow component only, and that the parameter \( \frac{Nu}{Re^{1/2}} \) for the crossflow is independent of Mach number.

These assumptions yield, for the ratio of average heat-transfer coefficient at sweep angle \( \Lambda \) to that at zero sweep,

\[ \frac{h_{\Lambda=0}}{h_{\Lambda=0}} = \frac{k}{k_{\Lambda=0}} \left( \frac{h_{\Lambda=0}}{\mu} \cos \Lambda \right)^{1/2} \] (9)

The thermal conductivity \( k \) and viscosity \( \mu \) are to be evaluated just behind the center of the bow shock, where, according to reference 14, conditions best approximate the incompressible free stream.
Values of the relative heat-transfer coefficient from equation (9) using experimental values of conductivity and viscosity (refs. 15, 16, and 17) are plotted against sweep angle in the lower part of figure 4. The theoretical curve is in fair agreement with the experimental data, except at large sweep angles, where the theory should not, of course, be expected to apply, because the contribution of the tangential component of the flow is neglected. There is some reason to believe that the parameter \( \frac{Nu}{R^{1/2}} \) is not independent of Mach number but decreases somewhat with increasing Mach number in the supersonic range. This variation would decrease the difference between the theory and the experiment.

Because of the uncertainty of the effect of three-dimensional flow on the experimental values of heat-transfer coefficient at \( \Lambda = 75^\circ \), no conclusions should be drawn about the trend beyond 60°.

The average heat-transfer coefficients found at the five sweep angles and two tunnel pressure levels were formed into the dimensionless parameters \( \frac{Nu}{R^{1/2}} \) with the air properties evaluated at free-stream conditions and are presented in figure 5. The large scatter of the values, particularly evident at \( \Lambda = 0^\circ \), may be attributed partly to the fact that, at the free-stream temperature of about 110° R, the thermal conductivity and viscosity become small and could not be read to good percentage accuracy. The free-stream Reynolds numbers for the two tunnel pressure levels used in these tests were about 1.3 \times 10^5 and 1.8 \times 10^5. The results at the two pressure levels fit fairly well on a single curve, which falls rapidly with sweep angle.

To try to get a parameter independent of sweep angle also, the free-stream density and velocity in the Reynolds number were retained, but the viscosity and thermal conductivity were evaluated at the temperature just behind the center of the bow shock as suggested in reference 14. The variation of this parameter with sweep angle is shown by the lower curve in figure 5. Although this parameter is not quite independent of sweep angle, the change from 0° to 60° is only 6 percent, and an average value of 0.815 represents the data to engineering accuracy over the entire range of sweep angles.

The relation between Nusselt number and Reynolds number with air properties evaluated at the temperature behind the bow shock is shown in figure 6 in the conventional logarithmic plot. The dashed line with slope 1/2 is the approximate relation determined from figure 5:

\[
\frac{hD}{k_2} = 0.815 \left( \frac{\rho_1 V_1 D}{\mu_2} \right)^{1/2}
\]  

(10)
It can be seen that, for any sweep angle, the values of Nusselt number from tests at the two tunnel pressure levels can be joined by a line of slope 1/2, but that for either pressure level, the slope of the best-fitting straight line is slightly less, about 0.45. However, equation (10) is a useful approximation over the entire range of angles and pressures covered in the tests.

CONCLUSIONS

Wind-tunnel tests on a 1/2-inch-diameter solid copper cylinder, insulated on the rear half, at Mach number 6.9 and Reynolds numbers (based on diameter and free-stream air properties) of $1.3 \times 10^5$ and $1.8 \times 10^5$ have shown the following results, for a range of the ratio of model temperature to stagnation temperature of 0.5 to 0.8:

1. The average heat-transfer coefficient for the front half of the cylinder decreases approximately as the cosine of the sweep angle. A simple analysis, based on the normal component of the air flow alone, shows good agreement with this trend.

2. The equilibrium temperature of an isothermal cylinder, insulated on the rear half, is reduced by sweeping the cylinder. The average recovery factor based on free-stream temperature can be closely predicted by a simple analysis which assumes that the normal component of the free-stream flow decelerates with an average recovery factor equal to that found for zero sweep, and the tangential component decelerates independently with an average recovery factor equal to the flat-plate value.

3. The average heat-transfer coefficients for both tunnel pressure levels can be expressed by a single curve of Nusselt number divided by the square root of the Reynolds number against sweep angle. When the air viscosity and conductivity are evaluated at the temperature behind the bow shock, the change in $N_u/R^{1/2}$ with sweep angle is small, from about 0.84 at zero sweep to 0.79 at 60° and 0.76 at 75°. A constant
value of \( \frac{\text{Nu}}{R^{1/2}} = 0.815 \) is a good approximation of the results for the entire range of sweep angles and Reynolds number of these tests.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., May 19, 1955.
REFERENCES


Figure 1.- Model used in tests. All dimensions in inches.
Figure 2. - Samples of temperature data and extrapolation plot used to determine equilibrium temperature, for a run at 20° sweep, 25 atmospheres stagnation pressure.
Figure 3.- Schlieren pictures of swept circular-cylinder pressure models at \( M = 6.9 \) (taken during investigation reported in ref. 13).
Figure 4.- Variation of equilibrium-temperature ratio and relative overall heat-transfer coefficients with sweep angle. Data for $M = 6.86$; free-stream Reynolds number $1.3 \times 10^5$ based on diameter; wall-to-stagnation temperature ratio from 0.5 to 0.8.
Figure 5.- Variation of dimensionless heat-transfer parameters with sweep angle.
Figure 6.- Logarithmic correlation plot of Nusselt number against Reynolds number for overall heat transfer to the front half of swept circular cylinders at $M = 6.9$. Parameters based on diameter, free-stream density and velocity, and air properties at the temperature behind the bow shock.