NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SOME REMARKS ON AN APPROXIMATE METHOD OF ESTIMATING THE WAVE DRAG DUE TO THICKNESS AT SUPERSONIC SPEEDS OF THREE-DIMENSIONAL WINGS WITH ARBITRARY PROFILE

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SUMMARY

A semiempirical profile-correction factor is discussed which enables the estimation of the wave drag due to thickness at supersonic speeds for three-dimensional wings with arbitrary airfoil section (subject to the restrictions of thin-airfoil theory) through use of previously calculated drag coefficients. Application of the proposed correction factor to the known drag coefficients of some rhombic-profile wings yielded estimates for the drag coefficients of parabolic-arc-profile wings that were in good agreement with theoretically calculated values. It is expected that satisfactory estimates can be obtained for many combinations of plan form and profile by judicious use of the proposed profile correction, especially at speeds for which the wing leading edge is supersonic.

INTRODUCTION

A number of papers dealing with the linearized-theory calculations of supersonic wave drag at zero lift (that is, drag due to thickness) for three-dimensional wings have been published. (See references 1 to 12, for example.) As a result, extensive theoretical data are available for the wave-drag coefficient at supersonic flight speeds for wing plan forms that have arbitrary sweepback (or sweepforward), taper ratio, and aspect ratio. The profiles of these wings, however, are in general restricted to symmetrical double wedges, rhombuses (symmetrical double wedges with maximum thickness located at 50 percent chord), and biconvex sections composed of two symmetrical parabolic arcs. Other types of thin small-slope airfoil sections require a great deal of mathematical and computational labor and have been avoided in the theoretical analyses.

Inasmuch as these other profiles may prove to be of interest because of structural advantages, low-drag characteristics, and other reasons, it would be desirable to be able to obtain estimates of the drag for these cases without having to resort to further theoretical investigations. The purpose of the present paper, therefore, is to discuss the application and limitations of some semiempirical profile corrections which will enable the estimation of the wave-drag coefficient for three-dimensional wings with various profiles by utilizing previously calculated drag coefficients.

SYMBOLS

M	free-stream Mach number
β	Mach number parameter $\left(\sqrt{M^2 - 1}\right)$
A	aspect ratio $\left((\text{Wing span})^2/\text{Wing area}\right)$
λ	taper ratio (ratio of tip chord to root chord)
Λ	sweep angle of leading edge, degrees
t/c	thickness ratio of section in free-stream direction (maximum thickness of given section profile divided by its chord)
c_d	wave-drag coefficient of airfoil section $\left(\dfrac{\text{Drag per unit span}}{\text{Dynamic pressure} \times \text{Chord}}\right)$
C_D	wave-drag coefficient of three-dimensional wing $\left(\dfrac{\text{Drag}}{\text{Dynamic pressure} \times \text{Wing area}}\right)$
s	airfoil section

Use of subscripts is indicated or explained in text.

ANALYSIS AND DISCUSSION

As mentioned in the introduction, the purpose of the present paper is to discuss some semiempirical profile corrections which will enable the estimation of the wave-drag coefficient for three-dimensional wings with small-slope profiles by utilizing previously calculated drag coefficients. It appears that profile-correction factors that are especially suitable for supersonic-leading-edge conditions can be obtained from available data and formulas without much difficulty.

For example, suppose the drag coefficient C_{D_1} is desired at a free-stream Mach number M_1 for a wing with aspect ratio A_1, taper ratio λ_1, sweepback or sweepforward of leading edge Λ_1, section profile s_1, and a constant thickness ratio $(t/c)_1$ along the span. If the drag coefficient C_{D_0} of a geometrically similar wing at the Mach number M_1 with profile s_0 is known (that is, if the drag coefficient is known for a wing with all characteristics other than the profile s identical to those of the desired wing), then the following expression may be written:

$$\left(C_{D_1}\right)_{\substack{M_1\\A_1\\\lambda_1\\\Lambda_1\\(t/c)_1\\s_1}} = \left(C_{D_0}\right)_{\substack{M_1\\A_1\\\lambda_1\\\Lambda_1\\(t/c)_1\\s_0}} \times \text{Profile-correction factor}$$

where the subscripts outside the parentheses indicate wing characteristics and Mach number. In addition to the leading-edge-sweepback requirement, the sweepback of the maximum-thickness line for the two wings should be as nearly the same as available calculations permit. Two simple correction factors suggest themselves; these are stated and discussed in the following paragraphs.

Consider first a "three-dimensional" correction factor obtained by forming the ratio of the wave-drag coefficients of two wings of equal taper ratio (for example, untapered wings for which $\lambda = 1$) with the same aspect ratio, sweepback, and thickness ratio and at the same Mach number as the desired wing, one untapered wing having the desired profile s_1 and the other untapered wing having the profile s_0; that is, form the ratio $\left(\frac{C_{D_1}}{C_{D_0}}\right)_{M_1,A_1,\Lambda_1,(t/c)_1,\lambda=1}$. The reason the $\lambda = 1$ case was chosen is that fairly complete theoretical data exist for both the symmetrical parabolic-arc-profile and the symmetrical double-wedge-profile untapered wings. This approximation was suggested to the author of reference 9 and was applied in that paper to compare the calculated value (based on a theoretical analysis) for the parabolic-arc-profile tapered wing with the approximate value of C_{D_1} obtained by using previously known data for C_{D_0} (from reference 6),

$\left(C_{D_1}\right)_{\lambda=1}$ (from reference 8), and $\left(C_{D_0}\right)_{\lambda=1}$ (from reference 5). Excellent agreement was obtained for the cases considered. This factor can be considered as essentially correcting for taper ratio, since the untapered-wing value for the profile under consideration must be known. By rewriting the preceding equation with the correction factor inserted, this point can be clearly illustrated:

$$\left(C_{D_1}\right)_{\lambda_1} = \left(C_{D_0}\right)_{\lambda_1} \left(\frac{C_{D_1}}{C_{D_0}}\right)_{\lambda=1} = \left(C_{D_1}\right)_{\lambda=1} \frac{\left(C_{D_0}\right)_{\lambda_1}}{\left(C_{D_0}\right)_{\lambda=1}}$$

Inasmuch as only the symmetrical double-wedge, rhombic, and symmetrical parabolic-arc sections have been treated in detail for untapered three-dimensional wings, and since even less detailed calculations are available for tapered plan forms, the use of this type of correction factor is very limited and restricted. Also, because most of the theoretical solutions are based on linearized theory, such solutions for the double-wedge- and rhombic-profile wings exhibit an additional "drag peak" (in violation of the small-perturbation assumptions of the linear theory) whenever the Mach number is such that the Mach lines parallel the maximum-thickness ridge line. (This condition is termed a sonic line of maximum thickness since the free-stream flow component normal to that line is sonic.) Hence, good agreement with experiment or with more exact calculations would not be expected at or near Mach numbers for which this condition is present. It might be added that equally erroneous drag peaks result from using linear theory whenever a line connecting the points of appreciable wing-slope discontinuities becomes sonic. For example, the biconvex-, rhombic-, and double-wedge-profile wings all exhibit drag peaks at Mach numbers for which the leading and trailing edges become sonic; hence linearized-theory calculations, or approximations involving the use of such calculations, at or near these Mach numbers should not be expected to agree with more exact calculations or experiment, although any approximations so obtained would probably be consistent with linear-theory estimates. The three-dimensional profile-correction factor is thus seen to be impractical for application to arbitrary profiles because (1) limited types of profile - plan-form combinations were treated in the previous three-dimensional drag analyses and hence there are insufficient calculations or formulas, or both, available and (2) drag peaks associated with linearized-theory calculations would lessen the accuracy of the approximation for certain ranges of Mach number.

A correction factor which is more general in application than the three-dimensional correction factor is that based on the ratio of the drag coefficients for two-dimensional airfoils. These coefficients (based on Busemann or Ackeret approximations) are readily available for many airfoil shapes and in any case are easily obtainable for thin profiles of arbitrary shape. (For example, see formulas in section V of reference 13.) Find the section wave-drag coefficient c_d for the profile s_1 and the section wave-drag coefficient for the profile s_0; then form the following profile-correction factor

$$\left[\frac{(c_d)_{s_1}}{(c_d)_{s_0}}\right]_{\text{Two-dimensional flow}}$$

An expression relating the drag coefficient of the desired wing C_{D_1} to known coefficients can then be written as follows:

$$\left(C_{D_1}\right)_{\substack{M_1\\A_1\\\lambda_1\\\Lambda_1\\(t/c)_1\\s_1}} = \left(C_{D_0}\right)_{\substack{M_1\\A_1\\\lambda_1\\\Lambda_1\\(t/c)_1\\s_0}} \times \left[\frac{(c_d)_{s_1}}{(c_d)_{s_0}}\right]_{\substack{\text{Two-dimensional flow}\\M_1\\(t/c)_1}} \qquad (1)$$

where the subscripts outside the parentheses or brackets indicate, as before, the wing or section characteristics and Mach number. (It is interesting to note that an application of the above correction factor for some wings of arrowhead plan form with biconvex profile may be found on page 21 of reference 14.) Of course, this type of correction factor cannot be expected to render as reliable an approximation for a wing with the leading edge swept behind the foremost Mach cone (subsonic leading edge) as for a wing with the leading edge protruding from the foremost Mach cone (supersonic leading edge).

This correction factor was used to estimate the drag coefficient for an infinite series of wings with the following characteristics: $\lambda = 0.531$, $A\beta = 4.65$, $\beta \cot \Lambda = 1.375$, and biconvex profile composed of two symmetrical parabolic arcs. A value of C_{D_0} for a family of wings that was geometrically similar but had rhombic airfoil sections was obtained from reference 6. Application of equation (1) yielded the following result: $C_D = 4.88(t/c)^2 \cot \Lambda$. This value compares excellently with the theoretically calculated value of $4.86(t/c)^2 \cot \Lambda$ (see reference 9). This remarkable agreement will, of course, not be true in general, but reasonable estimates should be obtained for most cases where there are no fictitious drag peaks associated with linearized-theory inaccuracies. Fairly good approximations may especially be expected at speeds for which the wing leading edge is supersonic.

In this connection, some discussion of the results obtained in reference 10 would be appropriate. Figure 14 of that paper presents the ratio of wave-drag coefficients for two wings of equal thickness ratio plotted against a generalized Mach number - sweepback parameter, one wing having the biconvex profile mentioned in the previous example and the other a rhombic profile. The curves of figure 14 show the ratio (based on three-dimensional analyses for the wings considered) to vary from approximately 1 to 1.5. On the basis of the profile-correction factor proposed in the present paper, the estimated ratio would be 1.33 for the entire Mach number - sweepback range. The estimated value for the entire range thus appears to be fairly satisfactory, especially when the magnitude of the wave-drag coefficients is considered. However, a closer examination of the results (see fig. 12 of reference 10) will show that the portions of the curves that deviate most from the value 1.33 are largely influenced by the erroneous drag peaks that are inherent in the linearized-theory treatment for the rhombic-profile wing. These regions should, of course, be excluded from consideration as previously discussed. Therefore, the estimated factor of 1.33 appears even more satisfactory. Although the types of sections considered in the previous examples are restricted to biconvex and rhombic profiles, the results are applicable to a considerable variety of wing plan forms: rectangular, triangular, arrowhead, sweptback, and sweptforward wings of arbitrary taper ratio (conventional) with streamwise tips. Reasonable estimates of the drag are obtained even for wings that are swept well behind the foremost Mach cone, although no generalization can be made regarding this agreement. However, inasmuch as the slope of the airfoil surface must be small everywhere it appears that reasonable estimates of the wave-drag coefficient should also be expected for other permissible profiles - especially at speeds for which the wing leading edge is supersonic.

In connection with the previous discussion relating to the erroneous drag peaks, it is quite interesting to note that if the value of C_{D_1} for a rhombic-profile wing is obtained by using the value of C_{D_0} for the parabolic-arc-profile wing at a speed range for which the maximum-thickness line is near-sonic (Mach lines parallel the maximum-thickness ridge line), a lower estimate for C_{D_1} is obtained than is calculated by linearized theory. The lower value would agree better with experimental results. This agreement is due, of course, to the fact that the effect of the fictitious drag peak associated with the theoretical solution for the rhombic-profile wing is eliminated by using the correction factor in this manner.

In addition to the limitations on Mach number resulting from near-sonic- or sonic-edge conditions and the degree of applicability for wings with subsonic leading edges, the usual restrictions of linearized three-dimensional drag analyses are applicable. Regarding the geometric similarity of the two plan forms, the sweepback of the maximum-thickness line should be as nearly the same for the two wings as available calculations permit. (The leading-edge sweepback is the same for both wings, as indicated in equation (1).) The proposed correction factor should, of course, be applied only to wings with section profiles that satisfy the thin-airfoil restriction that the slope be small everywhere on the surface.

CONCLUDING REMARKS

A semiempirical profile-correction factor is discussed which enables the estimation of the wave drag due to thickness at supersonic speeds for three-dimensional wings with arbitrary airfoil section (subject to the restrictions of thin-airfoil theory) through use of previously calculated drag coefficients. It is felt that judicious use of the correction factor should yield satisfactory accuracy for many plan-form - profile combinations, especially at speeds for which the wing leading edge is supersonic. Application of the proposed correction factor to experimentally determined three-dimensional drag coefficients presents itself as an interesting possibility.

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