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RESEARCH MEMORANDUM

HEAT TRANSFER AND SKIN FRICTION FOR TURBULENT BOUNDARY LAYERS ON HEATED OR COOLED SURFACES AT HIGH SPEEDS

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HEAT TRANSFER AND SKIN FRICTION FOR TURBULENT BOUNDARY LAYERS ON HEATED OR COOLED SURFACES AT HIGH SPEEDS

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SUMMARY

The method presented in NACA TN 2692 for evaluating the skin friction of a turbulent boundary layer in compressible flow on an insulated surface is extended to evaluate the turbulent skin friction and heat transfer in compressible flow on a surface which is heated or cooled.

The results of this analysis are in good agreement with the heat transfers measured in flight on the NACA RM-10 missile up to Mach numbers of 3.7.

INTRODUCTION

Within the last decade the tremendous increase in flight Mach numbers that has been achieved has placed the problem of aerodynamic heating among the most important of the present time. In particular the problem of the heat transfer through turbulent boundary layers at high speeds has received considerable attention. It has long been known that the heat transfer through turbulent boundary layers is so intimately connected with the skin-friction force exerted by the boundary layer that the two problems are almost one.

The analysis of the turbulent boundary layer presented in NACA TN 2692 (ref. 1) permits an easy extension of the turbulent skin-friction law from incompressible to compressible flow. The analysis led to expressions for the extent of the effective laminar sublayer δ_L and for the velocity u_L at the point δ_L. These results permitted the skin-friction law to be derived. If use is made of the quadratic dependence of temperature on velocity derived by Crocco in reference 2, the results of reference 1 may be extended to obtain the heat transfer through the turbulent boundary layer in both incompressible and compressible flows. It is the purpose of the present short analysis to make this extension.

SYMBOLS

A parameter, $\left[\frac{n(r - 1)}{k^2} \frac{\nu_o}{u_o \delta}\right]^{\frac{1}{n+1}}$

c_f skin-friction coefficient, $\frac{2\tau_w}{\rho_o {u_o}^2}$

$$F = \left\{1 + \frac{r.f.(\gamma - 1)M^2}{2}\left[1 - \left(\frac{u_L}{u_o}\right)^2\right] + \frac{T_w - T_{adw}}{T_o}\left(1 - \frac{u_L}{u_o}\right)\right\}^{0.56}$$

$$G = \left(\frac{\delta}{\theta}\right)^{\frac{1}{5}}\left\{1 + \frac{r.f.(\gamma - 1)M^2}{2}\left[1 - \left(\frac{u_L}{u_o}\right)^2\right] + \frac{T_w - T_{adw}}{T_o}\left(1 - \frac{u_L}{u_o}\right)\right\}^{0.448}$$

K constant (0.045)

k,m,n,r constants

M Mach number

N_x Nusselt's number, $\frac{qx}{\kappa \Delta T}$

N_δ Nusselt's number, $\frac{q\delta}{\kappa \Delta T}$

q local rate of heat transfer

r.f. recovery factor

R_x Reynolds number, $\frac{u_o x}{\nu_o}$

R_δ Reynolds number, $\frac{u_o \delta}{\nu_o}$

T absolute temperature

u	velocity in x-direction
x	distance along surface
y	distance normal to surface
γ	ratio of specific heats
δ	boundary-layer thickness
θ	boundary-layer momentum thickness
κ	thermal conductivity
μ	viscosity
ν	kinematic viscosity
ρ	density
τ	shear stress

Subscripts:

av	average value
adw	adiabatic wall conditions
L	conditions at edge of laminar sublayer
o	free-stream conditions
w	wall conditions

DERIVATION OF THE HEAT-TRANSFER LAW FOR TURBULENT BOUNDARY LAYERS

Before the present analysis is presented, a summary of the findings of reference 1 is in order. It was found that for boundary layers of the type

$$\frac{u}{u_o} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}} \tag{1}$$

the dimensionless effective extent of the laminar sublayer $\frac{\delta_L}{\delta}$ was given by

$$\frac{\delta_L}{\delta} = \left[\frac{n(r - 1)}{k^2} \frac{\nu_L}{u_o\delta}\right]^{\frac{n}{n+1}} \tag{2}$$

and the velocity at the point δ_L was given by

$$\frac{u_L}{u_o} = \left[\frac{n(r - 1)}{k^2} \frac{\nu_L}{u_o\delta}\right]^{\frac{1}{n+1}} \tag{3}$$

With these results, it was reasoned that the skin friction was

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_w = \mu_L \frac{u_L}{\delta_L} \tag{4}$$

which, when the necessary substitutions had been made, resulted in the following skin-friction formula

$$c_f = 2\left[\frac{n(r - 1)}{k^2}\right]^{\frac{1-n}{n+1}}\left(\frac{\nu_o}{u_o\delta}\right)^{\frac{2}{n+1}} \frac{\rho_L}{\rho_o}\left(\frac{\nu_L}{\nu_o}\right)^{\frac{2}{n+1}} \tag{5}$$

Since

$$\frac{\rho_L}{\rho_o} = \frac{T_o}{T_L} \tag{6}$$

and if it is assumed that

$$\frac{\mu_L}{\mu_o} = \left(\frac{T_L}{T_o}\right)^m \tag{7}$$

then

$$c_f = 2\left[\frac{n(r - 1)}{k^2}\right]^{\frac{1-n}{n+1}} R_\delta^{-\frac{2}{n+1}} \left(\frac{T_o}{T_L}\right)^{\frac{n-2m-1}{n+1}} \tag{8}$$

For a velocity profile having $n = 7$, it was found in reference 1 (from skin-friction measurements in incompressible flows) that

$$\frac{n(r - 1)}{k^2} = 158$$

so that the local skin-friction coefficient for the compressible case became

$$c_f = 0.045\ R_\delta^{-1/4} \left(\frac{T_o}{T_L}\right)^{0.56} \tag{9}$$

With these results it is now possible to pass on to the derivation of the heat transfer through the turbulent boundary layer.

If a temperature distribution in the boundary layer similar to that used by Crocco (ref. 2) for Prandtl number = 1 and zero pressure gradient is adopted; namely:

$$T = A + B\frac{u}{u_o} + C\left(\frac{u}{u_o}\right)^2$$

the temperature in the boundary layer may then be written

$$T = T_w - \left(T_w - T_{adw}\right)\frac{u}{u_o} - \left(T_{adw} - T_o\right)\left(\frac{u}{u_o}\right)^2 \tag{10}$$

It might be noted that, although the quadratic form of the temperature dependence on velocity was derived under conditions of zero pressure gradient, this form appears also to be a very useful relationship for most supersonic-missile shapes where the pressure gradients are generally quite small.

Differentiating equation (10) and evaluating it at the wall where $\frac{u}{u_o} = 0$ gives

$$\left(\frac{\partial T}{\partial y}\right)_w = -\left(\frac{T_w - T_{adw}}{u_o}\right)\left(\frac{\partial u}{\partial y}\right)_w \tag{11}$$

Hence, the heat transfer at the wall is

$$-q_w = \kappa_w\left(\frac{\partial T}{\partial y}\right)_w = -\frac{\kappa_w\left(T_w - T_{adw}\right)}{u_o}\left(\frac{\partial u}{\partial y}\right)_w \tag{12}$$

From equation (4) it will be seen that

$$\left(\frac{\partial u}{\partial y}\right)_w = \frac{\mu_L u_L}{\mu_w \delta_L} \tag{13}$$

so that

$$q_w = \frac{\kappa_w \mu_L}{\mu_w \delta_L}\left(T_w - T_{adw}\right)\frac{u_L}{u_o} \tag{14}$$

If the values of δ_L and $\frac{u_L}{u_o}$ that are obtained from equations (2) and (3) are substituted into equation (14) and the heat transfer is expressed in terms of a Nusselt number based on the dimension δ, the result is

$$N_\delta = \frac{q\delta}{\kappa_o \Delta T} = \left[\frac{n(r - 1)}{k^2}\right]^{\frac{1-n}{n+1}}\left(\frac{u_o\delta}{\nu_L}\right)^{\frac{n-1}{n+1}}\frac{\kappa_w}{\kappa_o}\frac{\mu_L}{\mu_w} \tag{15}$$

If it is assumed that

$$\frac{\kappa_w}{\kappa_o} \approx \frac{\mu_w}{\mu_o} \tag{16}$$

equation (15) may be written

$$N_\delta = \left[\frac{n(r - 1)}{k^2}\right]^{\frac{1-n}{n+1}} R_\delta^{\frac{n-1}{n+1}} \left(\frac{\nu_o}{\nu_L}\right)^{\frac{n-1}{n+1}} \frac{\mu_L}{\mu_o} \tag{17}$$

Substituting the value of $\left(\frac{\nu_o}{\nu_L}\right)^{\frac{n-1}{n+1}}$ and $\frac{\mu_L}{\mu_o}$ obtained from the use of equations (6) and (7) results in

$$N_\delta = \left[\frac{n(r - 1)}{k^2}\right]^{\frac{1-n}{n+1}} R_\delta^{\frac{n-1}{n+1}} \left(\frac{T_o}{T_L}\right)^{\frac{n-2m-1}{n+1}} \tag{18}$$

Writing the equations for dimensionless skin friction and heat transfer together and expressing $2\left[\frac{n(r - 1)}{k^2}\right]^{\frac{1-n}{n+1}}$ as a constant yields

$$c_f F = \text{Constant} \times R_\delta^{-\frac{2}{n+1}} \tag{19}$$

$$N_\delta F = \frac{\text{Constant}}{2} R_\delta^{\frac{n-1}{n+1}} \tag{20}$$

where

$$F = \left(\frac{T_L}{T_o}\right)^{\frac{n-2m-1}{n+1}} \tag{21}$$

Since the right-hand sides of equations (19) and (20) are the incompressible expressions for c_f and N_δ, respectively, the effect of Mach number and temperature potential must be contained in the factor F. This factor may be evaluated in the following manner. From equation (10)

$$\frac{T_L}{T_o} = \frac{T_w}{T_o} - \frac{\left(T_w - T_{adw}\right)}{T_o}\frac{u_L}{u_o} - \frac{\left(T_{adw} - T_o\right)}{T_o}\left(\frac{u_L}{u_o}\right)^2 \tag{22}$$

which may be written

$$\frac{T_L}{T_o} = 1 + \frac{r.f.}{2}(\gamma - 1)M^2\left[1 - \left(\frac{u_L}{u_o}\right)^2\right] + \frac{T_w - T_{adw}}{T_o}\left(1 - \frac{u_L}{u_o}\right) \tag{23}$$

The factor F therefore becomes

$$F = \left\{1 + \frac{r.f.}{2}(\gamma - 1)M^2\left[1 - \left(\frac{u_L}{u_o}\right)^2\right] + \frac{T_w - T_{adw}}{T_o}\left(1 - \frac{u_L}{u_o}\right)\right\}^{\frac{n-2m-1}{n+1}} \tag{24}$$

It may be seen that the second term within the brace represents the effect of Mach number, and the third term the effect of temperature potential on the skin-friction and heat-transfer coefficients. The value of $\frac{u_L}{u_o}$ is usually of the magnitude 0.4 and although it depends upon M, R_δ, and $\frac{T_w - T_{adw}}{T_o}$, it generally does not have a first-order effect upon the factor F. The value of $\frac{u_L}{u_o}$, however, must be found in the following way:

Since

$$\frac{u_L}{u_o} = \left[\frac{n(r - 1)}{k^2}\frac{\nu_L}{u_o\delta}\right]^{\frac{1}{n+1}} \tag{3}$$

$$\frac{u_L}{u_o} = \left(\frac{n(r - 1)}{k^2}\frac{1}{R_\delta}\right)^{\frac{1}{n+1}}\left(\frac{\nu_L}{\nu_o}\right)^{\frac{1}{n+1}} = A^{\frac{1}{n+1}}\left(\frac{T_L}{T_o}\right)^{\frac{m+1}{n+1}} \tag{25}$$

where the value of $\left(\frac{\nu_L}{\nu_o}\right)^{\frac{1}{n+1}}$ has been expressed in terms of temperatures by means of equations (6) and (7) and where

$$A = \frac{n(r - 1)}{k^2}\frac{1}{R_\delta} \tag{26}$$

Now, if the value of $\frac{T_L}{T_o}$ given by equation (23) is substituted in equation (25) there results

$$\frac{u_L}{u_o} = A^{\frac{1}{n+1}}\left\{1 + \frac{r.f.(\gamma - 1)}{2}M^2\left[1 - \left(\frac{u_L}{u_o}\right)^2\right] + \frac{T_w - T_{adw}}{T_o}\left(1 - \frac{u_L}{u_o}\right)\right\}^{\frac{m+1}{n+1}} \tag{27}$$

This equation for $\frac{u_L}{u_o}$ may be expressed as

$$\left(\frac{u_L}{u_o}\right)^{\frac{n+1}{m+1}} + A^{\frac{1}{m+1}}\frac{r.f.(\gamma - 1)}{2}M^2\left(\frac{u_L}{u_o}\right)^2 + A^{\frac{1}{m+1}}\frac{T_w - T_{adw}}{T_o}\frac{u_L}{u_o} -$$

$$A^{\frac{1}{m+1}}\left(1 + \frac{r.f.(\gamma - 1)}{2}M^2 + \frac{T_w - T_{adw}}{T_o}\right) = 0 \tag{28}$$

and solved graphically for $\frac{u_L}{u_o}$ for given values of R_δ, M, and $\frac{T_w - T_{adw}}{T_o}$.

For the particular case of a turbulent boundary layer with a one-seventh power velocity profile in air when $\gamma = 1.4$ and m may be taken as 0.76 (an approximation which is usually adequate for calculations such as these), the equations necessary for the calculation of heat transfer become

$$N_\delta F = 0.0225 R_\delta^{0.75} \tag{29}$$

where

$$F = \left\{1 + \frac{r.f.M^2}{5}\left[1 - \left(\frac{u_L}{u_o}\right)^2\right] + \frac{T_w - T_{adw}}{T_o}\left(1 - \frac{u_L}{u_o}\right)\right\}^{0.56} \tag{30}$$

and $\frac{u_L}{u_o}$ is found from the following equation

$$\left(\frac{u_L}{u_o}\right)^{4.55} + \left(\frac{158}{R_\delta}\right)^{0.568} \frac{r.f.M^2}{5}\left(\frac{u_L}{u_o}\right)^2 + \left(\frac{158}{R_\delta}\right)^{0.568} \frac{T_w - T_{adw}}{T_o}\left(\frac{u_L}{u_o}\right) -$$

$$\left(\frac{158}{R_\delta}\right)^{0.568}\left(1 + \frac{r.f.M^2}{5} + \frac{T_w - T_{adw}}{T_o}\right) = 0 \tag{31}$$

Tables I and II give values of $\frac{u_L}{u_o}$ and $\left(\frac{u_L}{u_o}\right)^2$ found by using equation (31) for Mach numbers up to 5 for a range of values of R_δ and $\frac{T_w - T_{adw}}{T_o}$ that are useful in heat-transfer calculations.

COMPARISON WITH EXPERIMENT

In order to compare the results of this analysis with experiment, it is necessary to have measurements of local rates of heat transfer for conditions of turbulent flow when the local values of R_δ, M, and

$\frac{T_w - T_{adw}}{T_o}$ are known. This type of information for a range of Mach numbers, Reynolds number, and temperature potentials can be obtained from the heat-transfer and skin-friction measurements made on the NACA RM-10 missile in flight and reported in part in references 3 and 4. For two of the body stations at which local heat transfers were measured in reference 3 (stations 85 and 122 inches from the missile nose) the boundary layer had been surveyed under similar conditions for the skin-friction study reported in reference 4, so that the boundary-layer thickness δ and hence R_δ were known. The pertinent measured quantities at these stations for several flight conditions are given in table III. The first six points are taken from data published in references 3 and 4. The last four points are computed from heat-transfer data not yet published. The correlation of these data by the present method is shown in figure 1 wherein the measured and theoretical results are plotted in the form $N_\delta F$ versus R_δ. A reasonably good correlation of the data is obtained for Mach numbers ranging from 1.6 to 3.7 and temperature potentials $\frac{T_w - T_{adw}}{T_o}$ ranging from 0.15 to -1.8. In comparing these experiments with the theory, it was assumed that the boundary-layer profile had a one-seventh power profile even though the surveys show that the power of the boundary-layer profile varied somewhat from test to test. This is not considered to be a serious matter in making a comparison between the theory and experiment, as experience has shown that small variations of profile power from the value of seven do not materially affect the magnitude of the heat transfers involved.

In general, heat-transfer data are not plotted as has been done in figure 1, in terms of local correlations, but in terms of N_x and R_x. This usual practice is generally permissible in the case of a flat plate having no pressure gradient, but it may be of interest to see how the results of the present analysis appear when integrated along such a flat plate so as to be presented in more conventional form. The integrations necessary (carried out in detail in the appendix) result in the following relations for incompressible flow with $n = 7$

$$c_f = 0.0578{R_x}^{-1/5}$$

$$N_x = 0.0289{R_x}^{4/5}$$

Thus the normal minus one-fifth and four-fifth power variations of c_f and N_x with R_x are found.

For the case of compressible flow, the solution is not quite so simple and only in the case of a flat plate where $\frac{T_w - T_{adw}}{T_o}$ is constant can the following useful approximations be made. For n = 7,

$$c_f G = 0.0906 R_x^{-1/5}$$

$$N_x G = 0.0453 R_x^{4/5}$$

where

$$G = \left(\frac{\delta}{\theta}\right)^{1/5} \left\{1 + \frac{r.f.M^2}{5}\left[1 - \left(\frac{u_L}{u_o}\right)^2_{av}\right] + \frac{T_w - T_{adw}}{T_o}\left[1 - \left(\frac{u_L}{u_o}\right)_{av}\right]\right\}^{0.448}$$

It is, however, desirable for the sake of accuracy and generality to retain the relations given in equations (29), (30), and (31) and correlate turbulent-boundary-layer heat transfer and skin friction on the basis of the length δ rather than x.

CONCLUSIONS

The method presented in NACA TN 2692 for evaluating the skin friction of a turbulent boundary layer in compressible flow on an insulated surface is extended to evaluate the turbulent skin friction and heat transfer in compressible flow on a surface which is heated or cooled.

The results of this analysis are in good agreement with the heat transfers measured in flight on the NACA RM-10 missile up to Mach numbers of 3.8.

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APPENDIX

DERIVATION OF DEPENDENCE OF SKIN FRICTION AND HEAT TRANSFER ON R_x FOR A FLAT PLATE

The momentum equation for the boundary layer on a flat plate may be written

$$\frac{d\theta}{dx} = \frac{c_f}{2} \tag{A1}$$

Since $\frac{\theta}{\delta}$ is a constant on a flat plate at a given Mach number

$$\frac{d\delta}{dx} = \frac{\delta}{2\theta} c_f \tag{A2}$$

Substituting the expression

$$c_f = \frac{K}{F\left\{M, \frac{T_w - T_{adw}}{T_o}, R_\delta\right\}} R_\delta^{-\frac{2}{n+1}} \tag{A3}$$

into (A2) there results

$$\frac{d\delta}{dx} = \frac{\delta}{2\theta} \frac{K}{F}\left(\frac{\nu_o}{u_o\delta}\right)^{\frac{2}{n+1}} \tag{A4}$$

When $M = 0$ and $\frac{T_w - T_{adw}}{T_o} = 0$, $F = 1$ so that equation (A4) may be integrated to obtain

$$\delta = K^{\frac{n+1}{n+3}}\left(\frac{n+3}{n+1}\,\frac{\delta}{2\theta}\right)^{\frac{n+1}{n+3}}\left(\frac{\nu_o}{u_o}\right)^{\frac{2}{n+3}} x^{\frac{n+1}{n+3}} \tag{A5}$$

The local skin friction and Nusselt number may then be expressed for $M = 0$ and $T_w = T_{adw}$ as

$$c_f = K^{\frac{n+1}{n+3}}\left(\frac{n+1}{n+3}\,\frac{2\theta}{\delta}\right)^{\frac{2}{n+3}}\left(\frac{\nu_o}{u_o x}\right)^{\frac{2}{n+3}} \tag{A6}$$

$$N_x = \frac{K^{\frac{n+1}{n+3}}}{2}\left(\frac{n+1}{n+3}\,\frac{2\theta}{\delta}\right)^{\frac{2}{n+3}}\left(\frac{u_o x}{\nu_o}\right)^{\frac{n+1}{n+3}} \tag{A7}$$

For $n = 7$, $K = 0.045$, and $\frac{\theta}{\delta} = \frac{7}{72}$ so that

$$c_f = 0.0578 R_x^{-1/5} \tag{A8}$$

$$N_x = 0.0289 R_x^{4/5} \tag{A9}$$

These formulas are the more conventional expressions for local skin-friction coefficient and Nusselt numbers as a function of Reynolds number.

It may be seen from the differential equation (A3) that the extremely simple expressions just derived are not valid for the case when M or $\frac{T_w - T_{adw}}{T_1}$ is other than zero. However, F is not a very sensitive function of R_δ and, if an average value of R_δ for a given problem is used to evaluate an $F = F_{av}$, the resulting approximate equations are extremely useful and fairly accurate. Thus, for a flat plate with a constant surface temperature

$$\delta = \left(\frac{K}{F_{av}}\right)^{\frac{n+1}{n+3}}\left(\frac{n+3}{n+1}\frac{\delta}{2\theta}\right)^{\frac{n+1}{n+3}}\left(\frac{\nu_o}{u_o}\right)^{\frac{2}{n+3}} x^{\frac{n+1}{n+3}} \tag{A10}$$

and

$$c_f = \left(\frac{K}{F_{av}}\right)^{\frac{n+1}{n+3}}\left(\frac{n+1}{n+3}\frac{2\theta}{\delta}\right)^{\frac{2}{n+3}}\left(\frac{\nu_o}{u_o x}\right)^{\frac{2}{n+3}} \tag{A11}$$

$$N_x = \frac{1}{2}\left(\frac{K}{F_{av}}\right)^{\frac{n+1}{n+3}}\left(\frac{n+1}{n+3}\frac{2\theta}{\delta}\right)^{\frac{2}{n+3}}\left(\frac{u_o x}{\nu_o}\right)^{\frac{n+1}{n+3}} \tag{A12}$$

For $n \doteq 7$ these expressions become

$$c_f\left(\frac{\delta}{\theta}\right)^{1/5}\left(F_{av}\right)^{4/5} = 0.0906 R_x^{-1/5} \tag{A13}$$

$$N_x\left(\frac{\delta}{\theta}\right)^{1/5}\left(F_{av}\right)^{4/5} = 0.0453 R_x^{4/5} \tag{A14}$$

These equations may be written

$$c_f G = 0.0906 R_x^{-1/5} \tag{A15}$$

$$N_x G = 0.0453 R_x^{4/5} \tag{A16}$$

where

$$G = \left(\frac{\delta}{\theta}\right)^{1/5}\left\{1 + \frac{r.f.M^2}{5}\left[1 - \left(\frac{u_L}{u_o}\right)^2_{av}\right] + \frac{T_w - T_{adw}}{T_o}\left[1 - \left(\frac{u_L}{u_o}\right)_{av}\right]\right\}^{0.448} \tag{A17}$$

In equation (A17) $\left(\frac{u_L}{u_o}\right)_{av}$ is an average value of $\frac{u_L}{u_o}$ along the plate.

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TABLE I

VALUES OF $\frac{u_L}{u_o}$ FOR n = 7

M	u_L/u_o for n = 7 and R_δ = —					
	2×10^4	6×10^4	1×10^5	5×10^5	1×10^6	1.5×10^6
	$\frac{T_w - T_{adw}}{T_o} = 0$					
0	0.5465	0.4900	0.4463	0.3655	0.3350	0.3192
1	.5603	.5038	.4593	.3778	.3457	.3295
2	.5943	.5365	.4911	.4060	.3752	.3555
3	.6318	.5725	.5285	.4392	.4140	.3852
4	.6702	.6108	.5660	.4734	.4352	.4171
5	.7085	.6493	.6032	.5076	.4663	.4463
	$\frac{T_w - T_{adw}}{T_o} = 0.5$					
0	0.5704	0.5140	0.4679	0.3875	0.3560	0.33985
1	.5817	.5250	.4805	.3970	.3650	.3482
2	.6100	.5524	.5067	.4200	.3870	.3691
3	.6432	.5868	.5400	.4492	.4150	.39627
4	.6787	.6220	.5742	.4800	.4430	.42418
5	.7150	.6573	.6089	.5110	.4720	.4525
	$\frac{T_w - T_{adw}}{T_o} = -0.5$					
0	0.5140	0.4544	0.4135	0.3375	0.3055	0.2896
1	.5334	.4760	.4320	.3515	.3205	.3052
2	.5759	.5180	.4725	.3885	.3532	.3383
3	.6185	.5606	.5130	.4275	.3900	.3731
4	.6630	.6045	.5554	.4650	.4283	.4095
5	.7070	.6492	.5983	.5058	.4660	.4460
	$\frac{T_w - T_{adw}}{T_o} = 1.0$					
0	0.5890	0.5330	0.4885	0.4045	0.3720	0.3561
1	.5990	.5422	.4972	.4120	.3783	.3629
2	.6230	.5651	.5200	.4330	.3970	.3808
3	.6539	.5963	.5499	.4590	.4240	.4052
4	.6860	.6281	.5815	.4870	.4500	.4308
5	.7185	.6610	.6135	.5160	.4775	.4565
	$\frac{T_w - T_{adw}}{T_o} = -1.0$					
0	0.461	0.401	0.356	0.2755	0.2465	0.232
1	.494	.435	.3905	.310	.2814	.266
2	.553	.496	.449	.3645	.3335	.3165
3	.6085	.5495	.530	.4138	.3794	.362
4	.6561	.5965	.5495	.4558	.4202	.4015
5	.6968	.6385	.5878	.4925	.4555	.435

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TABLE II

VALUES OF $\left(\frac{u_L}{u_o}\right)^2$ FOR n = 7

M	$(u_L/u_1)^2$ for n = 7 and R_δ = —					
	2×10^4	6×10^4	1×10^5	5×10^5	1×10^6	1.5×10^6
	$\frac{T_w - T_{adw}}{T_o} = 0$					
0	0.2987	0.2401	0.1992	0.1343	0.1122	0.1019
1	.3139	.2538	.2110	.1427	.1191	.1086
2	.3532	.2878	.2412	.1681	.1408	.1264
3	.4013	.3310	.2793	.1955	.1650	.1492
4	.4492	.3768	.3204	.2241	.1903	.1740
5	.5020	.4216	.3622	.2528	.2174	.1992
	$\frac{T_w - T_{adw}}{T_o} = 0.5$					
0	0.3254	0.2642	0.2189	0.1502	0.1267	0.1155
1	.3384	.2757	.2309	.1576	.1332	.1222
2	.3721	.3051	.2567	.1764	.1498	.1362
3	.4159	.3443	.2916	.2034	.1722	.1570
4	.4629	.3869	.3297	.2314	.1971	.1799
5	.5084	.4290	.3683	.2611	.2228	.2048
	$\frac{T_w - T_{adw}}{T_o} = -0.5$					
0	0.2642	0.2065	0.1710	0.1139	0.0933	0.0839
1	.2845	.2266	.1866	.1236	.1027	.0931
2	.3317	.2683	.2233	.1502	.1262	.1144
3	.3851	.3160	.2669	.1828	.1548	.1407
4	.4421	.3652	.3114	.2151	.1834	.1677
5	.4965	.4147	.3544	.2490	.2125	.1945
	$\frac{T_w - T_{adw}}{T_o} = 1.0$					
0	0.3469	0.2841	0.2386	0.1636	0.1399	0.1268
1	.3588	.2940	.2472	.1697	.1441	.1317
2	.3831	.3193	.2704	.1875	.1576	.1450
3	.4280	.3556	.3024	.2107	.1798	.1624
4	.4701	.3945	.3401	.2372	.2025	.1856
5	.5162	.4369	.3838	.2663	.2280	.2084
	$\frac{T_w - T_{adw}}{T_o} = -1.0$					
0	0.2125	0.1608	0.1267	0.0759	0.06076	0.05382
1	.2440	.1892	.1525	.0961	.07919	.07076
2	.3058	.2460	.2016	.1329	.1113	.1002
3	.3703	.3020	.2515	.1712	.1439	.1310
4	.4305	.3558	.3020	.2078	.1766	.1612
5	.4855	.4077	.3455	.2426	.2075	.1892

TABLE III

MEASURED QUANTITIES USED IN COMPARISON

Point (See fig. 1)	M	$T_w - T_{adw}$, °F	$\frac{T_w - T_{adw}}{T_o}$	R_δ	N_δ
1	1.59	67	0.145	0.385×10^6	270
2	1.61	58	.126	.697	422
3	2.15	-153	-.320	.585	388
4	2.19	-171	-.365	1.39	799
5	2.52	-376	-.776	.7	450
6	2.58	-394	-.837	1.973	1130
7	2.60	49	.135	.208	180
8	3.12	-90	-.258	.457	331
9	3.60	-509	-1.312	.921	561
10	3.69	-745	-1.842	1.07	634

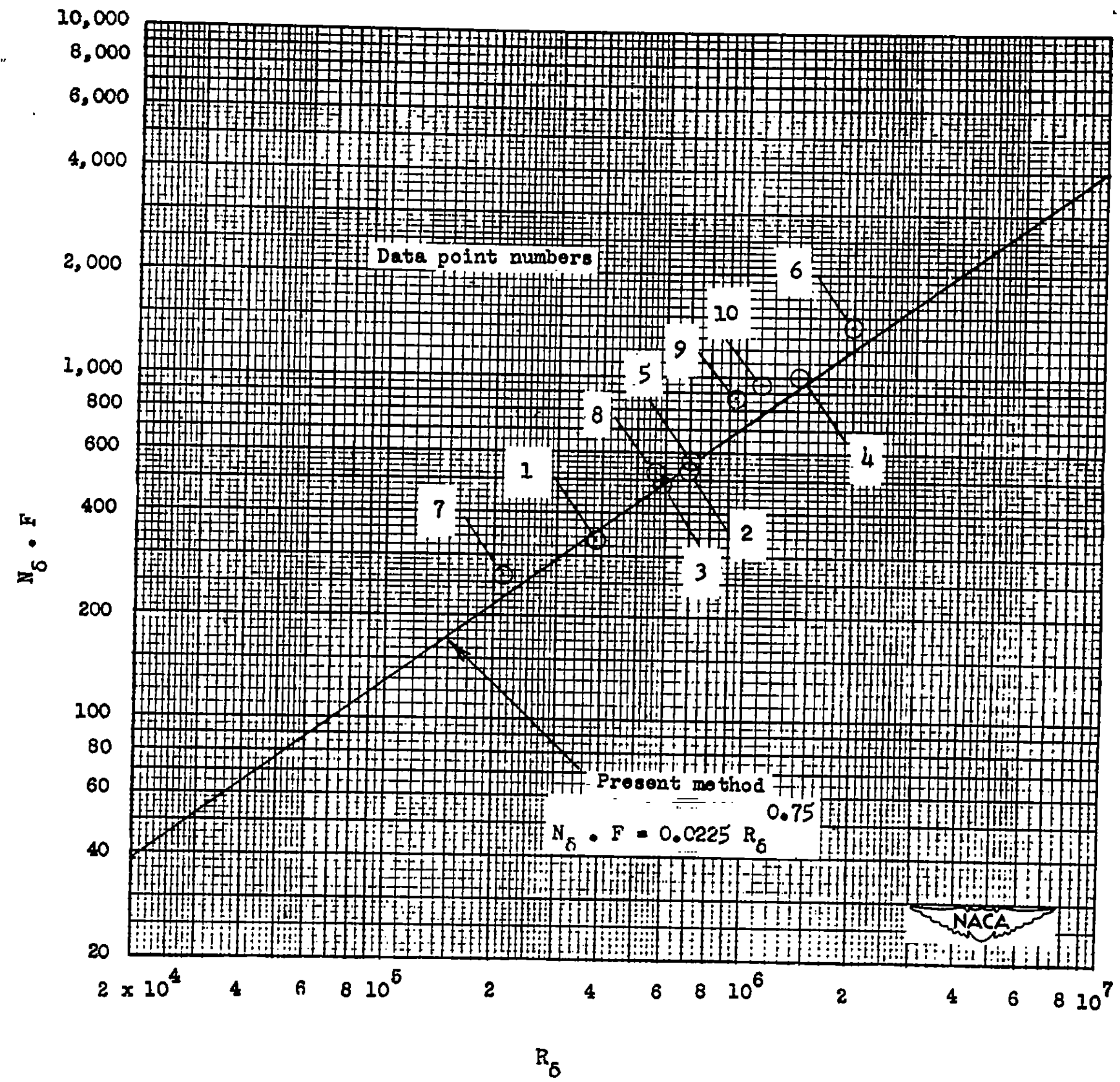

Figure 1.- Comparison of experimental results (refs. 3 and 4) and present method.

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