NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2514

RELATIVE STRUCTURAL EFFICIENCIES OF FLAT BALSA-CORE SANDWICH AND STIFFENED-PANEL CONSTRUCTION

By Ralph E. Hubka, Norris F. Dow, and Paul Seide

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
October 1951

1

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2514

RELATIVE STRUCTURAL EFFICIENCIES OF FLAT BALSA-CORE SANDWICH AND STIFFENED-PANEL CONSTRUCTION

By Ralph E. Hubka, Norris F. Dow, and Paul Seide

SUMMARY

An analysis is made and charts are presented for the determination of regions of efficient application of flat balsa-core sandwich and stiffened-panel construction for a large range of design requirements. Optimum sandwiches were found to have relatively low values of the ratio of core thickness to face thickness.

INTRODUCTION

The choice of the proper type of construction for the compression-carrying upper skin of an airplane wing is affected by many factors such as cost, ease of production, and structural efficiency. Although numerous types of construction have been proposed, the most common in the past was the longitudinally stiffened compression panel. Extensive studies (references 1 to 10) have been made of the compressive strength of stiffened panels with the result that the proportions of such panels can readily be chosen to give the maximum structural efficiency that can be obtained with this type of construction.

Efficiency studies of the type used for the stiffened panels lead logically to the conclusion that no one type of construction is universally the most efficient. For example, in a comparison of stiffened-panel with multiweb wing construction, Gerard showed in reference 11 that for thin wings carrying high bending moments, the multiweb wing construction is more efficient.

One type of construction, as yet not evaluated, is the end-grain balsa-core sandwich. Simple physical reasoning does not indicate in advance the range of loading conditions for which the sandwich is most efficient. Consequently, some quantitative studies must be made if the region is to be determined for which this type of construction is more efficient.

In the present paper an analysis is made to determine the respective regions of application in which the sandwich and the stiffened panel represent the more structurally efficient construction when used as a compression cover for a wing structure. The effects of transverse air load and shear stress due to torsion in the wing on the strength of the structure are not considered. It is assumed that the curvature of the wing surface is slight; therefore, for the purpose of the analysis, the wing structure is idealized into a long rectangular box. The maximum strength and buckling strength of the sandwich compression cover are assumed to be equal and the side support provided for the stiffened-panel compression cover is assumed to contribute nothing to its strength. The effects of these assumptions are discussed in the evaluation of the results of the analysis.

SYMBOLS

b	width of box beam, inches
B	flexural stiffness per unit width of beam cut from sandwich, inch-kips $\left(\frac{E_f t_f (h_c + t_f)^2}{2}\right)$
d	depth of box beam, inches
E_f	Young's modulus of elasticity of sandwich face material, taken as 10,500 ksi in analysis
G_c	shear modulus of elasticity of sandwich core material, taken as 20.1 ksi in analysis
h_c	thickness of sandwich core, inches
$\bar{h}$	distance from outside skin surface to axis of center of gravity of stiffened panel, inches
L	rib spacing of box beam with stiffened-panel compression cover, inches
M	bending moment carried by box beam, inch-kips
P_i	compressive failing load per inch of stiffened-panel width, kips per inch
t_f	actual face thickness of sandwich panel, inches $\left(t_{f_{min}} + \Delta t_f\right)$

$t_{f_{min}}$ minimum face thickness t_f of sandwich required for adequate torsional stiffness of box beam, inches

Δt_f difference between actual thickness t_f and minimum required thickness $t_{f_{min}}$ of sandwich, inches

t_S actual skin thickness of stiffened panel, inches $\left(t_{S_{min}} + \Delta t_S\right)$

$t_{S_{min}}$ minimum skin thickness t_S of stiffened panel required for adequate torsional stiffness of box beam, inches

Δt_S difference between actual thickness t_S and minimum required thickness $t_{S_{min}}$ of stiffened panel, inches

$\bar{t}$ average thickness of cross-sectional area per inch of stiffened-panel width, inches

$\bar{t}_r$ average thickness of ribs, inches

W weight per square foot of compression cover (including ribs in the case of stiffened-panel compression cover), pounds per square foot

σ_{cr} buckling stress of sandwich (assumed equal to buckling compressive load divided by cross-sectional area of faces), ksi

IDEALIZED STRUCTURES COMPARED

In the present study the wing was idealized into a long rectangular box beam (see fig. 1) of depth d and width b, having at least a skin thickness $t_{S_{min}}$ established by requirements of torsional stiffness, and subjected to an end moment M. The tension (bottom) cover was assumed to have zero thickness but had a cross-sectional area equal to the area of the compression (top) cover. Transverse air load on the compression cover and shear stress due to torsion of the box were not considered for the present analysis. The two types of compression cover compared for this beam were a Y-stiffened panel supported by ribs and a balsa-core sandwich.

The stiffened panel cover of the box beam was considered to be made of longitudinal 75S-T6 aluminum-alloy extruded straight-web Y-section stiffeners strongly riveted to flat alclad 75S-T6 sheet; the ribs were spaced at equal intervals L along the box. The Y-stiffened panel was chosen because it was found to be the most efficient of the various types of stiffened panels tested by the Langley Structures Research Laboratory of the National Advisory Committee for Aeronautics. Each rib was assumed to have an average thickness $\bar{t}_r$ of 0.4 of the skin thickness (fairly heavy ribs), and in all cases the weight of the compression cover was calculated as the sum of the weight of the ribs and of the stiffened panel. The rib stiffness was assumed to be such that the coefficient of end fixity c for the panel would be equal to unity.

The sandwich cover for the box beam was considered to be made of a 7-pound-per-cubic-foot end-grain balsa core with alclad 75S-T6 faces of equal thickness. The length and construction were assumed to be such that the sandwich would act like an infinitely long flat plate simply supported along the unloaded edges.

RANGE OF PROPORTIONS INVESTIGATED

The range of proportions investigated was chosen in an effort to cover the region in which most designs utilizing sandwich or stiffened-panel construction might be expected to fall. For the box, a depth-width ratio $\frac{d}{b}$ of 0.12 to 0.60 was selected to correspond (with not less than 40 percent of the wing chord in the box) to a wing thickness of approximately 5 to 25 percent. A ratio of depth to minimum skin thickness $\frac{d}{t_{s_{min}}}$ of 25 to 800 was selected (to correspond, for example, to a 25-inch-thick wing with a skin thickness of 1 inch down to 0.032 inch). For the sandwich a range of ratios of core thickness to face thickness $\frac{h_c}{t_f}$ of 10 to 100 was investigated. For the stiffened panel the proportions investigated were those covered by the design charts of reference 12.

RESULTS AND DISCUSSION

Determination of regions of efficient application of sandwich and stiffened panel.- The efficient regions of application of sandwich and stiffened panel were determined by essentially calculating, for each set of assumed proportions $\left(\frac{d}{t_{s_{min}}}\right.$ and $\left.\frac{d}{b}\right)$ and for each type of construction, the maximum moment-carrying capacity M corresponding to a given weight W of compression cover. The results of these calculations, which were carried out as described in appendix A for the sandwich and in appendix B for the stiffened panel, are given in figure 2. In this figure the weights of sandwich and of stiffened panel are plotted against the applied bending moments. The range of values of the bending-moment parameter of the curves for minimum values of $\frac{d}{t_{S_{min}}}$ and $\frac{d}{b}$ is limited to that covered by the extensive stiffened-panel data of reference 12. Figure 2 shows that, generally, as the bending-moment parameter $\frac{M}{\left(t_{S_{min}}\right)^3}$ increases, each sandwich curve for a given value of $\frac{d}{t_{S_{min}}}$ crosses to the left of the corresponding stiffened-panel curve in the lower range of values of $\frac{M}{\left(t_{S_{min}}\right)^3}$. Hence, the sandwich is more suitable for applications in which the intensity of loading is relatively low and the stiffened-panel construction, for cases in which the intensity of loading is high.

The crossing points of the curves of figure 2 have been summarized in figure 3 for a range of values of $\frac{M}{\left(t_{S_{min}}\right)^3}$ from 1×10^6 ksi to 14×10^6 ksi. The curves of figure 3 then represent combinations of values of $\frac{d}{b}$, $\frac{d}{t_{S_{min}}}$, and $\frac{M}{\left(t_{S_{min}}\right)^3}$ for which the most efficient sandwich and most efficient stiffened-panel construction have equal weight. If the design value of $\frac{M}{\left(t_{S_{min}}\right)^3}$ is less than the value given by a curve passing through the point represented by given values of $\frac{d}{b}$ and $\frac{d}{t_{S_{min}}}$, then no stiffened-panel structure lighter than a sandwich structure of the type considered can be designed from the charts of reference 12. If $\frac{M}{\left(t_{S_{min}}\right)^3}$ is greater than the value given by a curve at given values of $\frac{d}{b}$ and $\frac{d}{t_{S_{min}}}$ and if a stiffened-panel design to carry that moment can be obtained from the charts of reference 12, then that panel design will be lighter than the corresponding sandwich construction. Below and to the right of the region of the curves of figure 3, for all values of $\frac{M}{\left(t_{S_{min}}\right)^3}$ covered by the proportions considered in the design charts of reference 12, no stiffened-panel structure can be designed to be lighter than a sandwich design of the type considered. Above and to the left of the region of the curves of figure 3, again within the range of values of $\frac{M}{\left(t_{S_{min}}\right)^3}$ covered by reference 12, the stiffened-panel construction can be designed by reference 12 to be the lighter. In both of these regions, however, only a limited range of values of $\frac{M}{\left(t_{S_{min}}\right)^3}$ comes within the range of proportions of reference 12. Beyond this range of values of $\frac{M}{\left(t_{S_{min}}\right)^3}$, an analysis to determine whether sandwich or stiffened-panel construction is lighter would require extension of the design charts of reference 12; accordingly, this analysis has not been made.

Proportions and buckling stresses of sandwich.- The ratio of core thickness to face thickness required to meet the given design conditions $\frac{d}{b}$, $\frac{d}{t_{S_{min}}}$, and $\frac{M}{\left(t_{S_{min}}\right)^3}$ with minimum weight are presented in figure 4 and the corresponding critical stresses are presented in figure 5. Also shown in figures 4 and 5 are the values of $\frac{M}{\left(t_{S_{min}}\right)^3}$ at which the crossing of the sandwich and stiffened-panel curves of figure 2 occurs.

In figure 4 the part of each curve to the left of the cusp corresponds to a face thickness equal to the specified minimum value required for torsional stiffness. For the part of each curve to the right of the cusp, the face thickness required for minimum weight is greater than that specified as necessary for adequate torsional stiffness. These curves show that relatively low values of the ratio of core thickness to face thickness are required for optimum sandwiches.

With the ratio of core thickness to face thickness known, their individual values can be obtained by use of an alternate form of equation (A1) of appendix A:

$$t_f = \frac{W}{\frac{7}{12}\frac{h_c}{t_f} + 29}$$

and the equation

$$h_c = t_f \frac{h_c}{t_f}$$

where W is given by figure 2 and $\frac{h_c}{t_f}$ by figure 4.

In general, figure 5 shows that, at the proportions corresponding to the boundary between the regions of efficient application of sandwich and stiffened-panel construction, the stresses are in the plastic range.

Proportions and average failing stresses of stiffened panel.- The proportions of the stiffened panel required to meet the given design conditions $\frac{d}{t_{S_{min}}}$ and $\frac{M}{\left(t_{S_{min}}\right)^3}$ are the optimum proportions from charts and tables of reference 12. Unlike the sandwich, only a negligible weight advantage ever resulted in the stiffened-panel construction from using a skin thickness greater than the specified minimum required for torsional stiffness. The skin thickness t_S was therefore always taken equal to $t_{S_{min}}$.

The average failing stresses of the stiffened panels are given by the charts of reference 12.

Limitations of analysis.- Inasmuch as the present analysis contains simplifying assumptions, consideration should be given to the limitations or inaccuracies that they produce. Some of these simplifying assumptions should cause the sandwich to appear lighter than it actually would be relative to the stiffened panel; other assumptions cause the stiffened panel to be favored over the sandwich. A perfect balance between the two, however, and hence an entirely accurate evaluation of the relative efficiencies of sandwich and stiffened panel, is undoubtedly not achieved over the entire range of proportions investigated. Qualitatively, however, the results of the analysis cannot be expected to be changed appreciably by any refinements. It is only the exact location of the boundaries between regions of efficient application of the two types of construction that may be changed by refinements.

One assumption which is particularly unfavorable to the sandwich is that the buckling strength and the maximum strength are equal. Actually, at least for some proportions, the maximum strength of a sandwich can be substantially greater than its buckling strength; however, because most of the efficient sandwich designs (that is, the ones which determine the location of the boundary between regions in which the sandwich or the stiffened-panel construction is the lighter) buckle at high stresses in the plastic range, the maximum strength and buckling strength are probably very nearly the same.

The fact that the method of analysis used for the sandwich (reference 13) may be slightly unfavorable to the sandwich in regard to the evaluation of its transverse shear stiffness is suggested in reference 14. Again, however, for the proportions which establish the boundary between the sandwich and stiffened panel the difference in buckling strength associated with the different assumptions of references 13 and 14 is negligible. This difference is especially negligible in comparison with the empirical reduction used in appendix A to bring the theoretical results more nearly in line with experiment.

Also unfavorable to the sandwich is the assumption that the compression cover is flat, while actually in a wing a certain amount of curvature of the upper surface is to be expected. Because the sandwich is thicker than the unsupported sheet of the stiffened panel, curvature will be relatively more effective in increasing the strength of the sandwich than that of the stiffened panel. Hence, curvature would permit greater decreases in the weight of sandwich than in the weight of stiffened-panel construction.

An assumption which is favorable to the sandwich is that the accuracy of the theory of reference 13 for predicting the strength of the sandwiches considered herein is not diminished by local wrinkling failure of the face material within the range of stresses considered. This assumption is most likely to be valid for the efficient sandwich proportions which have relatively thick faces. The limited test data that are available (see reference 15) would suggest that for all proportions considered local wrinkling of the faces does not occur.

The assumption which is least favorable to the stiffened-panel construction is that the side support provided for the panel contributes nothing to its strength. Actually, sufficiently sturdy side support increases the strength of the stiffened panel in two respects: First, it raises the initial buckling strength by converting the panel from a column to a plate (particularly when the initial buckling is of an overall rather than a local type); second, it increases the margin between the buckling and ultimate strengths, because the side supports and some effective width of the panel adjacent to the supports can continue to carry load even after the panel has buckled.

The arbitrary assumption that the ribs have an average thickness of 0.4 of the skin thickness is probably unfavorable to the stiffened panel. The fact that this rib thickness is on the high side is suggested by the resulting optimum rib weights (10 to 30 percent of the cover weight) which are considered to be fairly high despite Farrar's conclusion based on elastic analysis (reference 16) that for optimum conditions the weight of the ribs should be one-half that of the stiffened-panel cover.

CONCLUDING REMARKS

Although a number of refinements and extensions of the present analysis suggest themselves, such as the investigation of the effects of curvature and of normal air load, the likelihood of substantial changes in the indicated boundaries between efficient regions of application of flat balsa-core sandwich and stiffened-panel construction appears rather remote. In general, the present analysis shows that the stiffened panel does not become relatively inefficient until rather extreme design conditions are reached, for which the structure is a lightly loaded, deep, and narrow box beam with a thick skin. Such a box (or series of boxes) can be achieved by the use of multiple shear webs; accordingly, another possibility to be investigated is the relative efficiency of multiweb construction with a sandwich cover.

The analysis of this paper should not be construed as appreciative or depreciative of a given type of construction. The choice of any type of wing construction involves the weighing of a number of factors, and the present type of analysis should be considered chiefly as an aid in the more accurate evaluation of the one rather important factor, the structural efficiency.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., July 16, 1951

APPENDIX A

MINIMUM-WEIGHT ANALYSIS OF BOX BEAMS WITH SANDWICH COMPRESSION COVERS

Derivation of Equations for Weight and Strength of Sandwich

The weight per square foot of a sandwich compression cover is given by the equation

$$W = 7\left(\frac{144}{1728}\right)h_c + 144(0.101)\left(2t_{f_{min}} + 2\Delta t_f\right) \qquad \text{(A1)}$$

where

W total weight per square foot of sandwich, pounds per square foot

h_c thickness of balsa core, inches

$t_{f_{min}}$ minimum allowable thickness of aluminum-alloy face material (equal to one-half skin thickness required to provide adequate torsional stiffness to box), inches

Δt_f difference between actual face thickness and $t_{f_{min}}$, inches

The numbers 0.101 and 7 are, respectively, the density of aluminum-alloy faces in pounds per cubic inch and the chosen density of balsa core in pounds per cubic foot.

In order to expedite the computational work, this equation was divided by $t_{S_{min}}$, and $2t_{f_{min}}$ and $2\Delta t_f$ were replaced by their equivalents $t_{S_{min}}$ and Δt_S to give the equation

$$\frac{W}{t_{S_{min}}} = \frac{7}{12}\frac{h_c}{t_{S_{min}}} + 14.5\left(1 + \frac{\Delta t_S}{t_{S_{min}}}\right) \qquad \text{(A2)}$$

The moment-carrying capacity of a box beam having a balsa-core sandwich compression cover is given by the equation

$$M = \sigma_{cr} b\left(t_{S_{min}} + \Delta t_S\right)\left[d - \frac{1}{2}\left(t_{S_{min}} + \Delta t_S\right) - \frac{h_c}{2}\right] \qquad (A3)$$

where

M applied bending moment, inch-kips

σ_{cr} buckling stress (assumed equal to buckling compressive load divided by area of faces of the sandwich), ksi

b width of box beam, inches

d depth of box beam, inches

$t_{S_{min}}$ minimum skin thickness required for adequate torsional stiffness, inches

Δt_S difference between actual skin thickness and minimum required skin thickness $t_{S_{min}}$, inches

In order to make this equation more readily usable, both sides were divided by $\left(t_{S_{min}}\right)^3$ to give the equation

$$\frac{M}{\left(t_{S_{min}}\right)^3} = \sigma_{cr} \frac{b}{t_{S_{min}}}\left[\frac{d}{t_{S_{min}}} - \frac{1}{2}\left(1 + \frac{\Delta t_S}{t_{S_{min}}}\right) - \frac{1}{2} \frac{h_c}{t_{S_{min}}}\right]\left(1 + \frac{\Delta t_S}{t_{S_{min}}}\right) \qquad (A4)$$

Replacing $\frac{b}{t_{S_{min}}}$ by its equivalent $\frac{\frac{d}{t_{S_{min}}}}{\frac{d}{b}}$ yields the following equation:

$$\frac{M}{\left(t_{S_{min}}\right)^3} = \sigma_{cr} \frac{\frac{d}{t_{S_{min}}}}{\frac{d}{b}}\left[\frac{d}{t_{S_{min}}} - \frac{1}{2}\left(1 + \frac{\Delta t_S}{t_{S_{min}}}\right) - \frac{1}{2} \frac{h_c}{t_{S_{min}}}\right]\left(1 + \frac{\Delta t_S}{t_{S_{min}}}\right) \qquad (A5)$$

Development of Sandwich Curves of Figure 2

Design chart for the sandwich compression covers.- A design chart based on theory of reference 13 (similar to fig. 6 of this reference but covering a wider range of proportions) was prepared. In the preparation of this new design chart, the parameters $\frac{\pi^2 B}{b^2 G_c h_c}$ and $\frac{\pi^2 B}{b^2 t_f}$ of the design chart of reference 13 were replaced, respectively, by their equivalents

$$\frac{\pi^2}{4}\frac{E_f}{G_c}\frac{t_{S_{min}}}{h_c}\left(\frac{d}{b}\right)^2\left[\frac{\frac{h_c}{t_{S_{min}}} + \frac{1}{2}\left(1 + \frac{\Delta t_S}{t_{S_{min}}}\right)}{\frac{d}{t_{S_{min}}}}\right]^2\left(1 + \frac{\Delta t_S}{t_{S_{min}}}\right)$$

and

$$\frac{\pi^2}{4} E_f\left(\frac{d}{b}\right)^2\left[\frac{\frac{h_c}{t_{S_{min}}} + \frac{1}{2}\left(1 + \frac{\Delta t_S}{t_{S_{min}}}\right)}{\frac{d}{t_{S_{min}}}}\right]^2$$

obtained by substituting for B its equivalent $E_f t_f\left(h_c + t_f\right)^2$ and for t_f its equivalent $\frac{1}{2} t_{S_{min}}\left(1 + \frac{\Delta t_S}{t_{S_{min}}}\right)$ and rearranging terms. In these parameters G_c and E_f are, respectively, the shear modulus of elasticity of the sandwich core material in kips per square inch and Young's modulus of elasticity of the sandwich face material in kips per square inch. The buckling stresses along the ordinate of this chart were multiplied by an empirical reduction factor of 0.86 justified by a comparison (fig. 6 of this paper) of the original buckling-stress values given by the chart with the extensive experimental buckling-stress values given in reference 15. In making this comparison of theory and experiment, values of E_f of 10,700 ksi, G_c of 19.0 ksi, and Poisson's ratio μ_f of 0.33 were used.

<u>Optimum proportions of the sandwich compression covers</u>.- For a series of chosen values of $\frac{\Delta t_S}{t_{S_{min}}}$ from zero to some estimated value greater than zero, related values of the proportions $\frac{h_c}{t_{S_{min}}}$ and $\frac{\Delta t_S}{t_{S_{min}}}$

were obtained by use of equation (A2) for a series of chosen values of the weight parameter $\frac{W}{t_{S_{min}}}$. Buckling stresses σ_{cr} of sandwich compression covers of these proportions were obtained from the previously described design chart for chosen values of $\frac{d}{t_{S_{min}}}$ and $\frac{d}{b}$. The optimum proportions of $\frac{\Delta t_S}{t_{S_{min}}}$ and $\frac{h_c}{t_{S_{min}}}$ were then determined by use of equation (A5) for each value of $\frac{W}{t_{S_{min}}}$ by graphically maximizing the bending-moment parameter with respect to $\frac{\Delta t_S}{t_S}$.

Parameters in terms of which the sandwich curves of figure 2 were plotted.- The maximum values of the bending-moment parameter $\frac{M}{\left(t_{S_{min}}\right)^3}$ and the corresponding values of the weight parameter $\frac{W}{t_{S_{min}}}$ were used as coordinates to plot points on the stiffened-panel curve of figure 2.

APPENDIX B

MINIMUM-WEIGHT ANALYSIS OF BOX BEAMS WITH STIFFENED-PANEL COMPRESSION COVERS

Derivation of Equations for Weight and Strength of Stiffened Panel

In the following derivation of equations for the stiffened panel, the skin thickness t_S of the stiffened panel is always assumed to be equal to the minimum skin thickness $t_{S_{min}}$ required for torsional stiffness. This assumption is justified by an investigation (the details of which are omitted here) which showed that, over almost the entire range of proportions covered in this analysis, increases in skin thickness over that required for torsional stiffness caused increases in the panel weight rather than decreases. Moreover, in the limited regions in which decreases in panel weight resulted, these decreases were negligible.

The weight per square foot of a longitudinally stiffened compression cover of a box beam (including the weight of ribs) is given by the equation

$$W = 144(0.101)\left(\bar{t} + \bar{t}_r \frac{d}{L}\right) \tag{B1}$$

where

W total weight per square foot of stiffened panel plus ribs, pounds per square foot

$\bar{t}$ average thickness of panel, or cross-sectional area per inch of panel width, inches

$\bar{t}_r$ average thickness of ribs (rib is assumed to extend over full depth of box), inches

L rib spacing, inches

The number 0.101 is the density of the aluminum alloy faces in pounds per cubic inch. In order to facilitate handling of the equation, both sides were divided by $t_{S_{min}}$ to give

$$\frac{W}{t_{S_{min}}} = 14.5\left(\frac{\bar{t}}{t_{S_{min}}} + \frac{\bar{t}_r}{t_{S_{min}}}\frac{d}{L}\right) \tag{B2}$$

and this equation was rewritten

$$\frac{W}{t_{S_{min}}} = 14.5\left(\frac{\bar{t}}{t_{S_{min}}} + \frac{\bar{t}_r}{t_{S_{min}}}\frac{d}{t_{S_{min}}}\frac{t_{S_{min}}}{P_i}\frac{P_i}{L}\right) \tag{B3}$$

where

P_i compressive failing load per inch of panel width, kips per inch

It is assumed that the rib stiffness is such that the coefficient of end fixity c is unity; therefore, in this equation $\frac{P_i}{L}$ as well as $\frac{P_i}{t_{S_{min}}}$ now represent the well-known structural loading parameters for a compression panel. The ribs were selected to have an average thickness of 0.4 times the minimum skin thickness and the equation became

$$\frac{W}{t_{S_{min}}} = 14.5\left(\frac{\bar{t}}{t_{S_{min}}} + 0.4\frac{d}{t_{S_{min}}}\frac{t_{S_{min}}}{P_i}\frac{P_i}{L}\right) \tag{B4}$$

The moment-carrying capacity of a box beam having a longitudinally stiffened compression cover is given by the equation

$$M = P_i b(d - \bar{h}) \tag{B5}$$

where

M bending moment carried at failure of panel, inch-kips

$\bar{h}$ distance from outside skin surface to axis of center of gravity of panel, inches

In order to facilitate handling of this equation, both sides were divided by $\left(t_{S_{min}}\right)^3$ to give

$$\frac{M}{\left(t_{S_{min}}\right)^3} = \frac{P_i}{t_{S_{min}}} \frac{b}{t_{S_{min}}}\left(\frac{d}{t_{S_{min}}} - \frac{\bar{h}}{t_{S_{min}}}\right) \tag{B6}$$

which, by use of the equality

$$\frac{b}{t_{S_{min}}} = \frac{\frac{d}{t_{S_{min}}}}{\frac{d}{b}} \tag{B7}$$

was converted to

$$\frac{M}{\left(t_{S_{min}}\right)^3} = \frac{P_i}{t_{S_{min}}} \frac{\frac{d}{t_{S_{min}}}}{\frac{d}{b}}\left(\frac{d}{t_{S_{min}}} - \frac{\bar{h}}{t_{S_{min}}}\right) \tag{B8}$$

Development of Stiffened-Panel Curves of Figure 2

Optimum proportions of the stiffened-panel compression covers.- Optimum dimensional proportions of the stiffened-panel compression covers were obtained from design charts of reference 12 for a series of chosen values of the loading parameters $\frac{P_i}{t_{S_{min}}}$ and $\frac{P_i}{L}$ within the range of the charts. For these proportions the required values of $\frac{\bar{t}}{t_{S_{min}}}$ and $\frac{\bar{h}}{t_{S_{min}}}$ were then obtained from tables of reference 12.

Optimum rib spacings of the box beams with stiffened-panel compression covers.- The optimum rib spacings were determined by use of equation (B4) by graphically minimizing the weight parameter $\frac{W}{t_{S_{min}}}$ with respect to $\frac{P_i}{L}$ for each of the selected values of $\frac{P_i}{t_{S_{min}}}$ for a chosen number of values of $\frac{d}{t_{S_{min}}}$.

Parameters in terms of which the stiffened-panel curves of figure 2 were plotted.- Values of the bending-moment parameter $\frac{M}{\left(t_{S_{min}}\right)^3}$ obtained by use of equation (B8) and the corresponding minimum values of the weight parameter $\frac{W}{t_{S_{min}}}$ were used as coordinates to plot points on the stiffened-panel curves of figure 2.

REFERENCES

1. Rossman, Carl A., Bartone, Leonard M., and Dobrowski, Charles V.: Compressive Strength of Flat Panels with Z-Section Stiffeners. NACA ARR 4B03, 1944.

2. Weinberger, Robert A., Sperry, William C., and Dobrowski, Charles V.: Compressive Strength of Corrugated-Sheet-Stiffened Panels for the Consolidated XB-36 Airplane. NACA MR, Jan. 28, 1944.

3. Kotanchik, Joseph N., Weinberger, Robert A., Zender, George W., and Neff, John, Jr.: Compressive Strength of Flat Panels with Z- and Hat-Section Stiffeners. NACA ARR L4F01, 1944.

4. Dow, Norris F., Hickman, William A., and McCracken, Howard L.: Compressive-Strength Comparisons of Panels Having Aluminum-Alloy Sheet and Stiffeners with Panels Having Magnesium-Alloy Sheet and Aluminum-Alloy Stiffeners. NACA TN 1274, 1947.

5. Hickman, William A., and Dow, Norris F.: Compressive Strength of 24S-T Aluminum-Alloy Flat Panels with Longitudinal Formed Hat-Section Stiffeners Having Four Ratios of Stiffener Thickness to Skin Thickness. NACA TN 1553, 1948.

6. Schuette, Evan H., Barab, Saul, and McCracken, Howard L.: Compressive Strength of 24S-T Aluminum-Alloy Flat Panels with Longitudinal Formed Hat-Section Stiffeners. NACA TN 1157, 1946.

7. Hickman, William A., and Dow, Norris F.: Compressive Strength of 24S-T Aluminum-Alloy Flat Panels with Longitudinal Formed Hat-Section Stiffeners Having a Ratio of Stiffener Thickness to Skin Thickness Equal to 1.00. NACA TN 1439, 1947.

8. Dow, Norris F., and Hickman, William A.: Design Charts for Flat Compression Panels Having Longitudinal Extruded Y-Section Stiffeners and Comparison with Panels Having Formed Z-Section Stiffeners. NACA TN 1389, 1947.

9. Hickman, William A., and Dow, Norris F.: Data on the Compressive Strength of 75S-T6 Aluminum-Alloy Flat Panels with Longitudinal Extruded Z-Section Stiffeners. NACA TN 1829, 1949.

10. Hickman, William A., and Dow, Norris F.: Data on the Compressive Strength of 75S-T6 Aluminum-Alloy Flat Panels Having Small, Thin, Widely Spaced, Longitudinal Extruded Z-Section Stiffeners. NACA TN 1978, 1949.

11. Gerard, George: Efficient Applications of Stringer Panel and Multicell Wing Construction. Jour. Aero. Sci., vol. 16, no. 1, Jan. 1949, pp. 35-40.

12. Dow, Norris F., and Hickman, William A.: Direct-Reading Design Charts for 75S-T Aluminum-Alloy Flat Compression Panels Having Longitudinal Straight-Web Y-Section Stiffeners. NACA TN 1640, 1948.

13. Seide, Paul, and Stowell, Elbridge Z.: Elastic and Plastic Buckling of Simply Supported Solid-Core Sandwich Plates in Compression. NACA Rep. 967, 1950. (Formerly NACA TN 1822.)

14. Bijlaard, P. P.: Stability of Sandwich Plates. Jour. Aero. Sci. (Readers' Forum), vol. 16, no. 9, Sept. 1949, pp. 573-574.

15. Boller, K. H.: Buckling Loads of Flat Sandwich Panels in Compression. The Buckling of Flat Sandwich Panels with Edges Simply Supported. Rep. No. 1525-A, Forest Products Lab., U. S. Dept. Agric., Feb. 1947.

16. Farrar, D. J.: The Design of Compression Structures for Minimum Weight. Jour. R.A.S., vol. 53, no. 467, Nov. 1949, pp. 1041-1052.

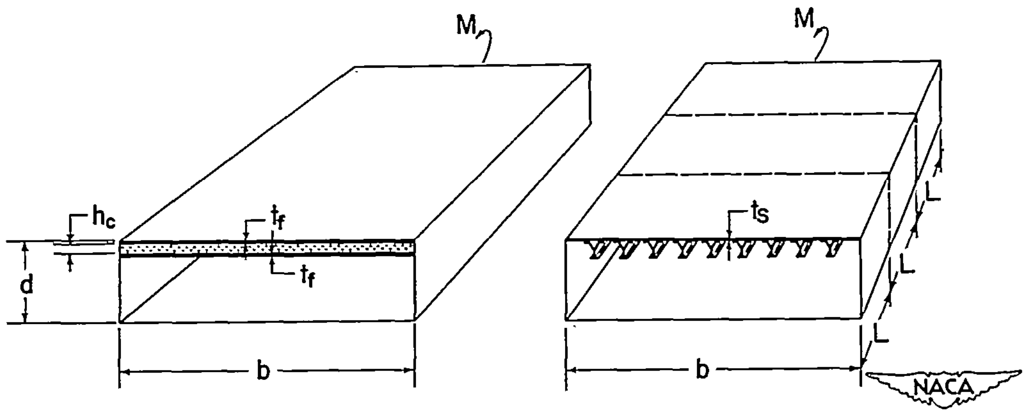
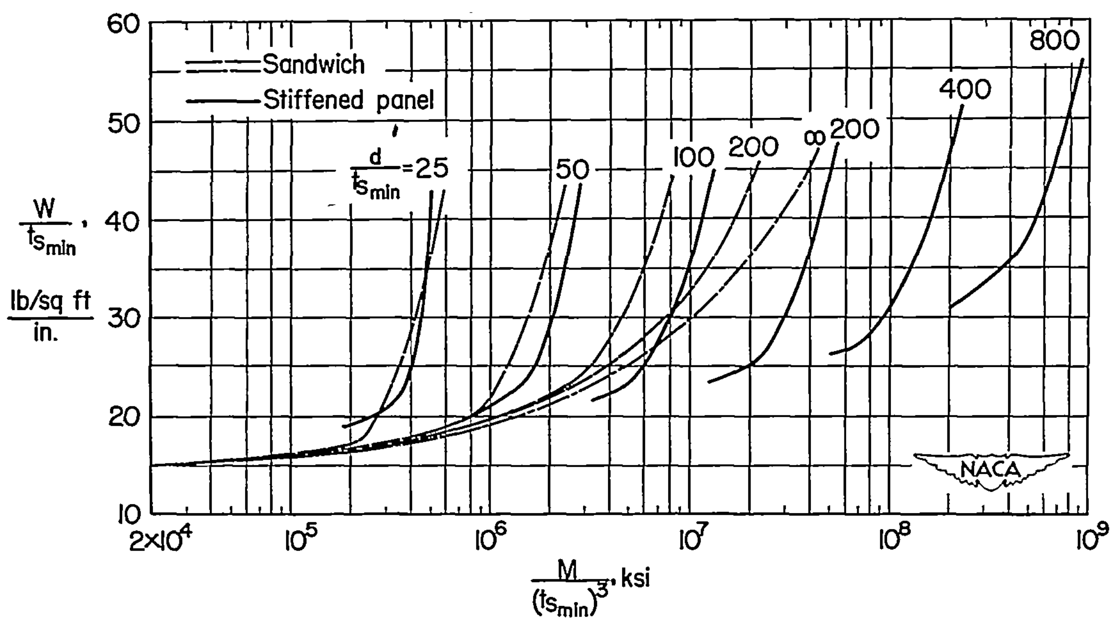
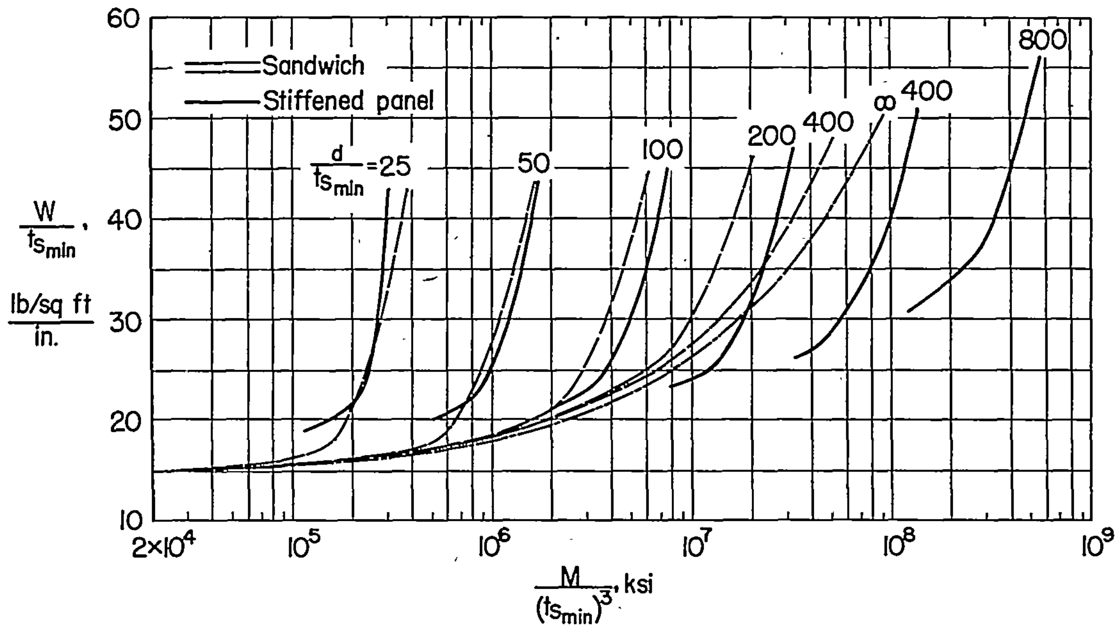


Figure 1.- Idealized structures used for comparing sandwich and stiffened-panel construction.

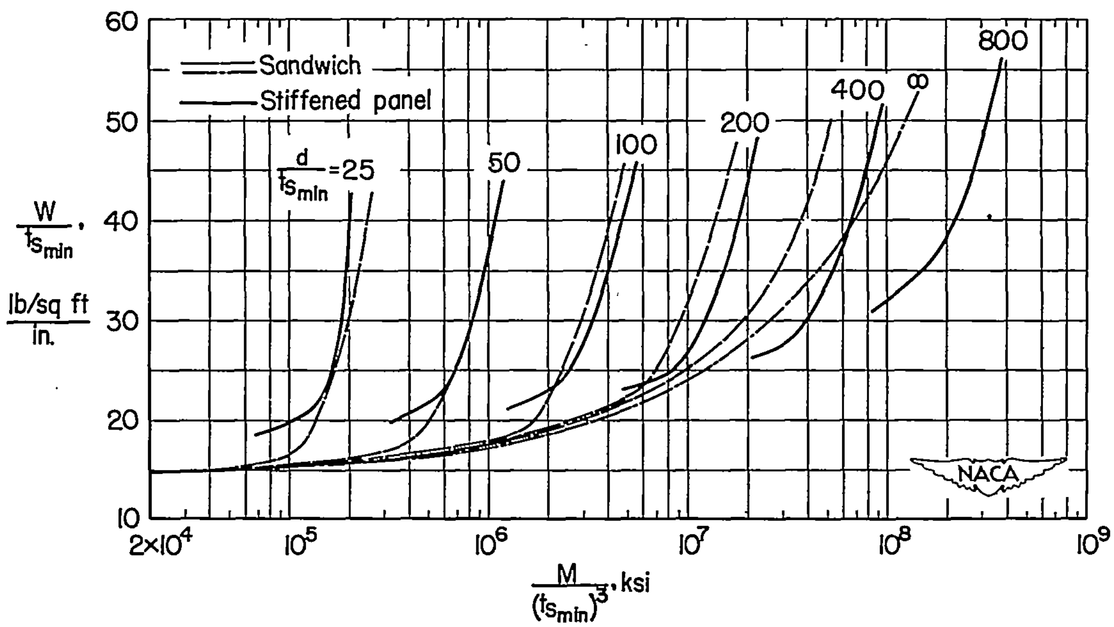


(a) $\frac{d}{b} = 0.12$.

Figure 2.- Variation with applied moment of weight of sandwich or stiffened-panel construction required for the compression cover of a box beam of given ratio of depth to width.

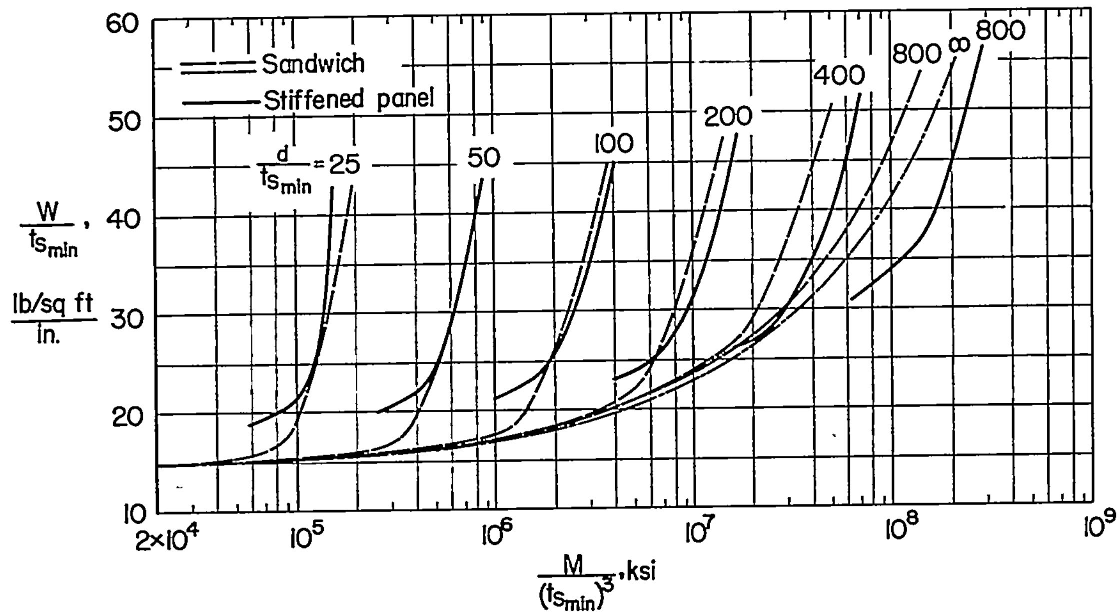


(b) $\frac{d}{b} = 0.20$.

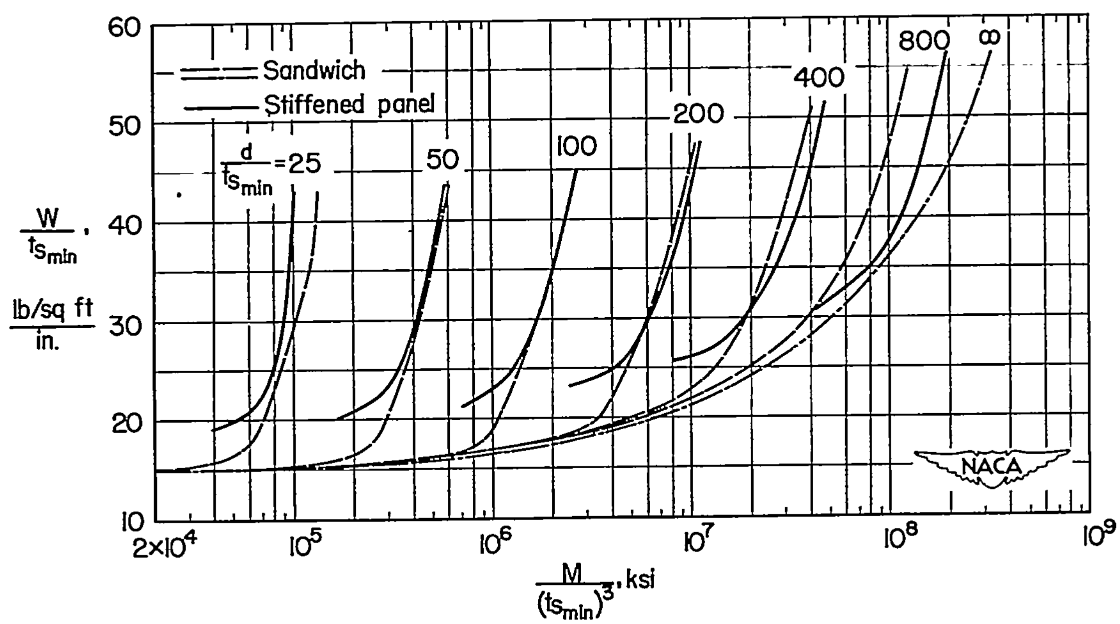


(c) $\frac{d}{b} = 0.30$.

Figure 2.- Continued.



(d) $\frac{d}{b} = 0.40$.



(e) $\frac{d}{b} = 0.60$.

Figure 2.- Concluded.

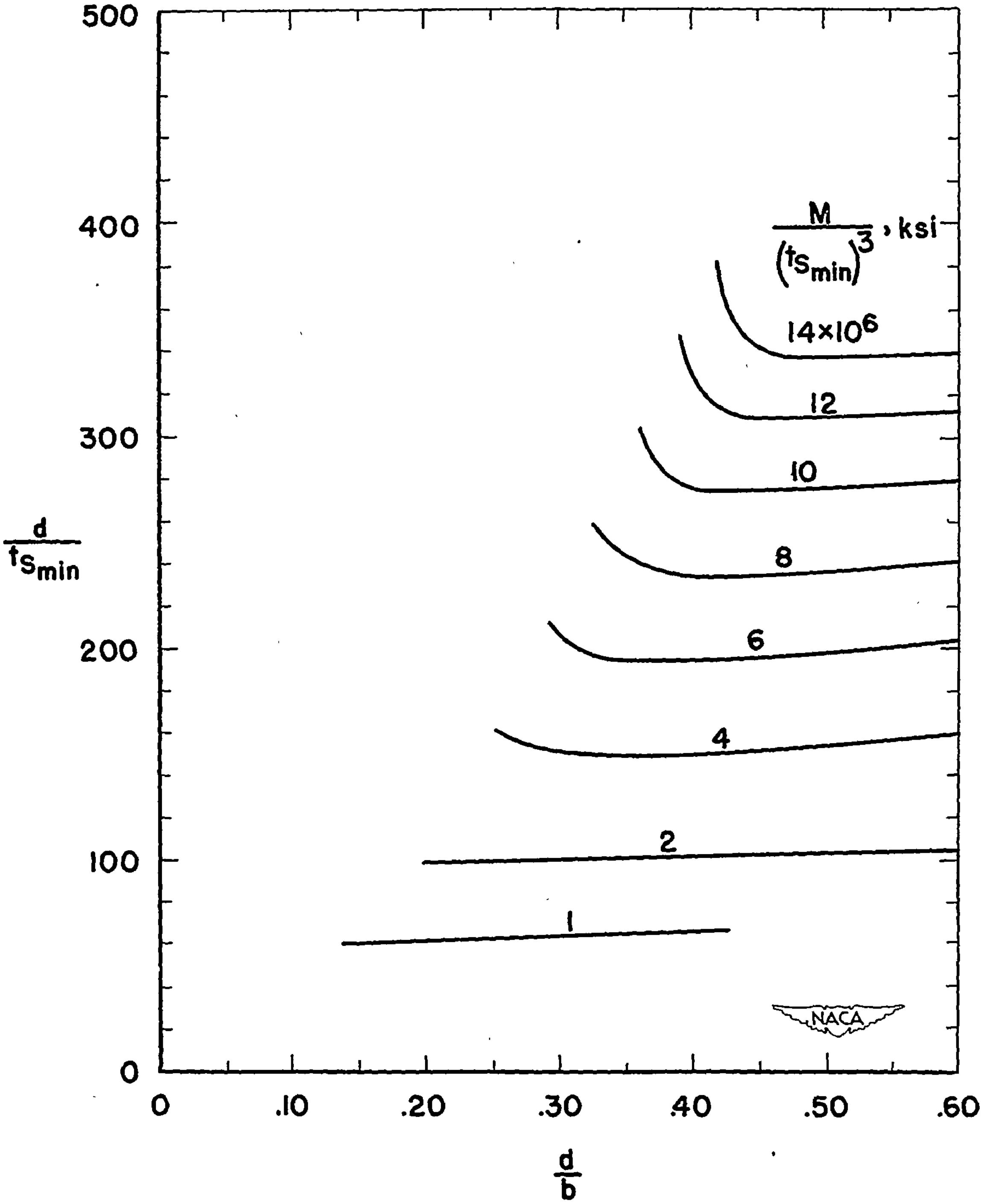
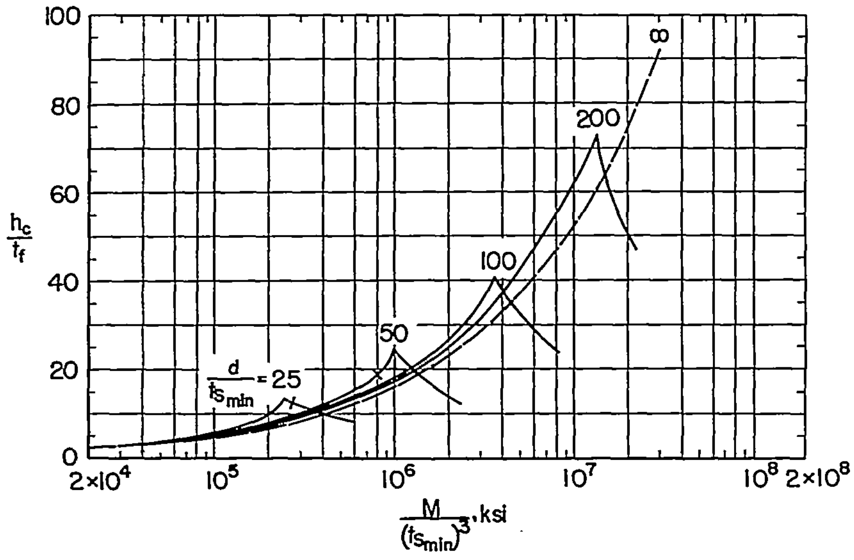
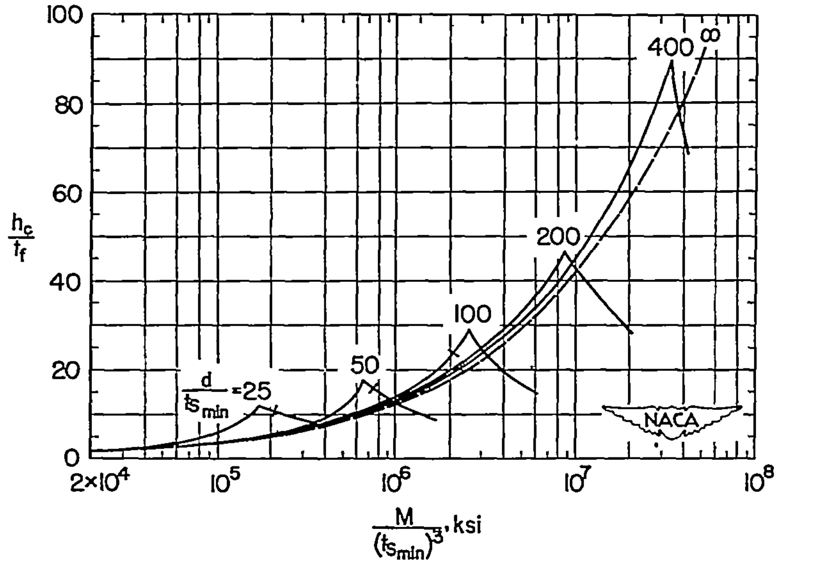


Figure 3.- Chart for determining whether the weight of balsa-core sandwich or of Y-stiffened panel plus ribs is less for a given design.

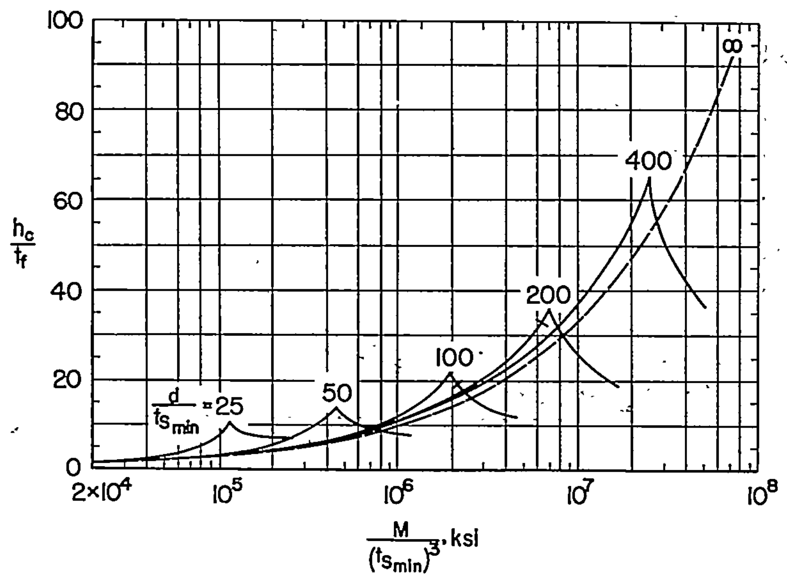


(a) $\frac{d}{b} = 0.12.$

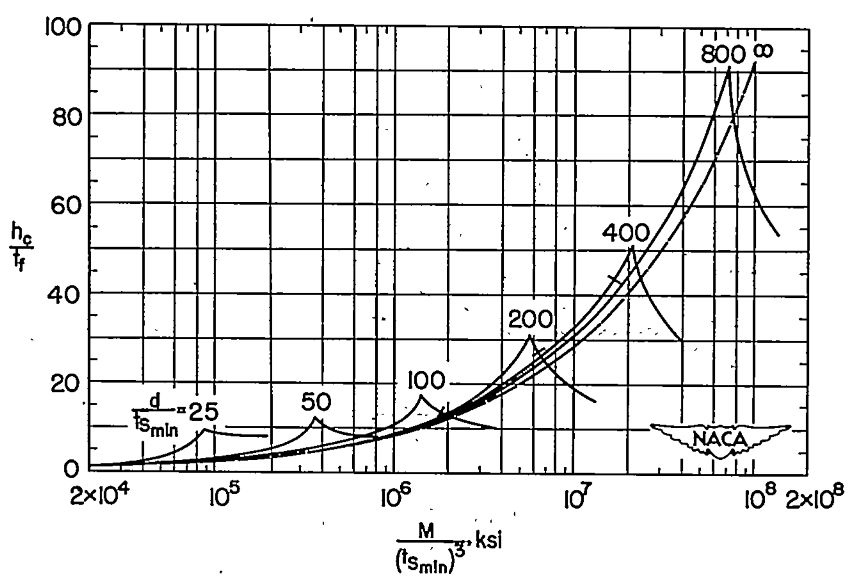


(b) $\frac{d}{b} = 0.20.$

Figure 4.- Proportions of sandwiches to meet given design conditions with minimum weight. The value of $\frac{M}{\left(t_{s_{min}}\right)^3}$ at which the crossing of the sandwich and stiffened-panel curves of figure 2 occurs is indicated by a short line cutting the curve.

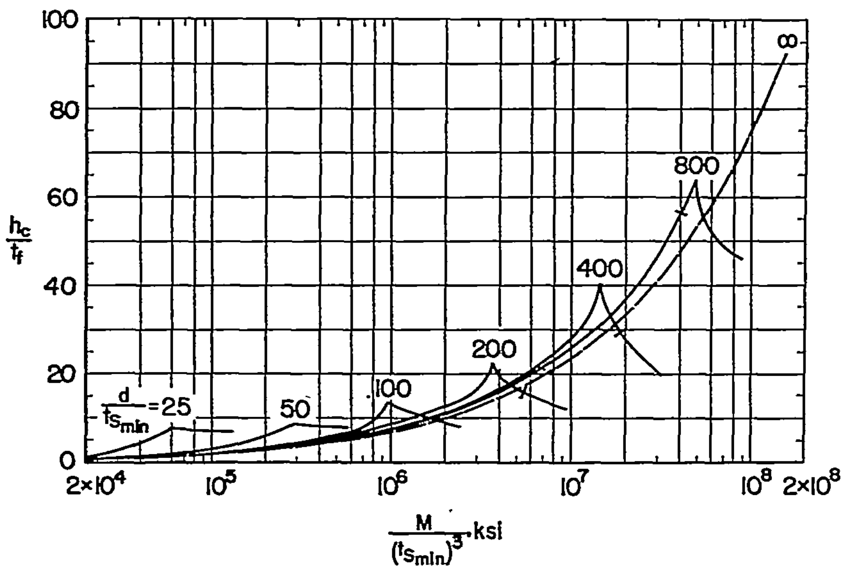


(c) $\frac{d}{b} = 0.30$.



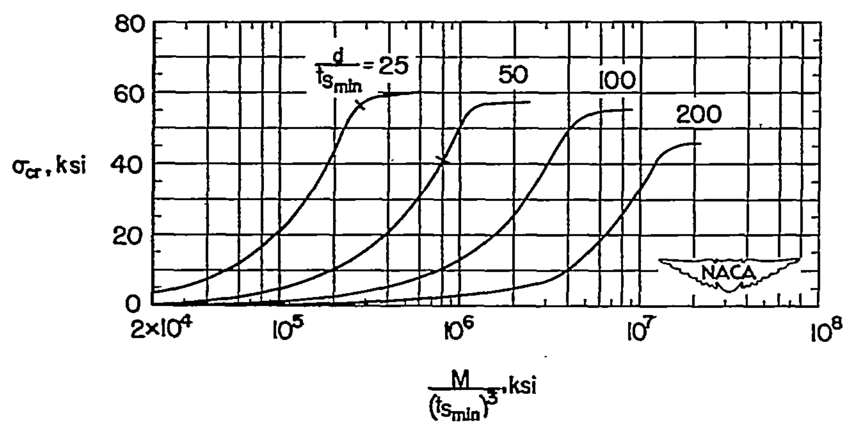
(d) $\frac{d}{b} = 0.40$.

Figure 4.- Continued.



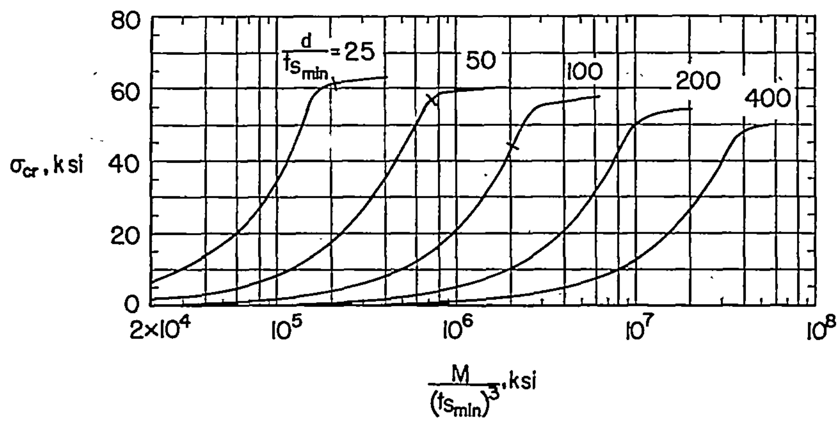
(e) $\frac{d}{b} = 0.60.$

Figure 4.- Concluded.

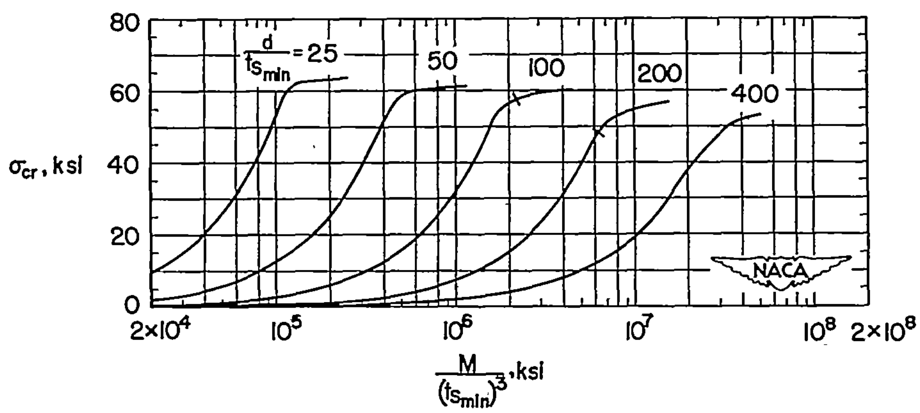


(a) $\frac{d}{b} = 0.12.$

Figure 5.- Buckling stresses of sandwiches to meet given design conditions with minimum weight. The value of $\frac{M}{\left(t_{S_{min}}\right)^3}$ at which the crossing of the sandwich and stiffened-panel curves of figure 2 occurs is indicated by a short line cutting the curve.

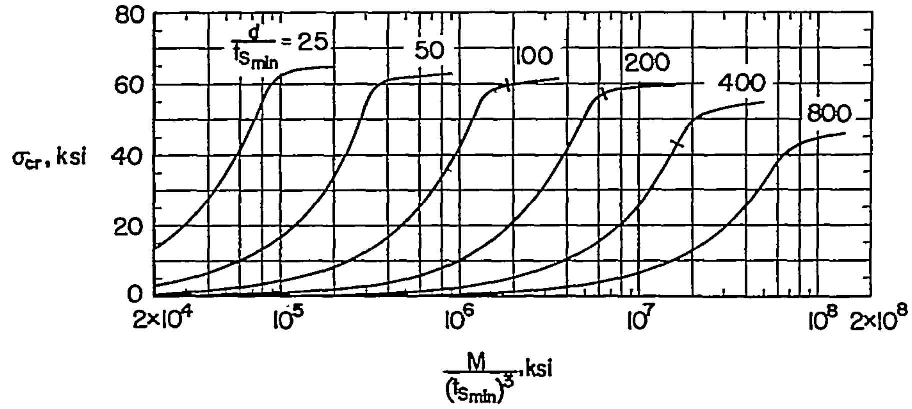


(b) $\frac{d}{b} = 0.20.$

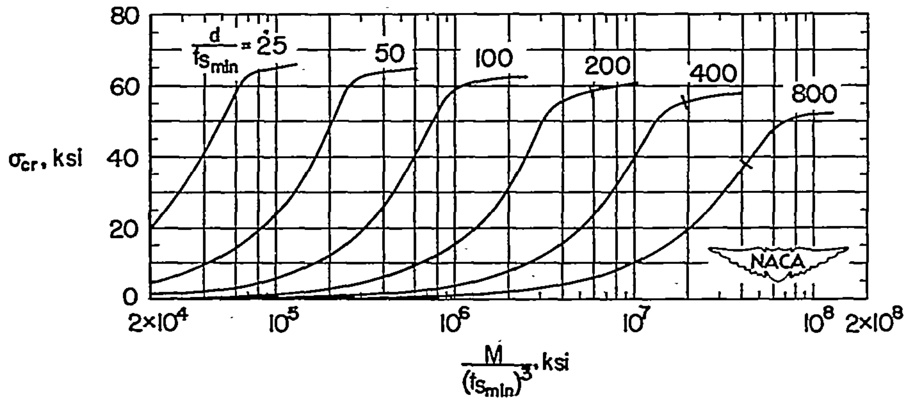


(c) $\frac{d}{b} = 0.30.$

Figure 5.- Continued.



(d) $\frac{d}{b} = 0.40.$



(e) $\frac{d}{b} = 0.60.$

Figure 5.- Concluded.

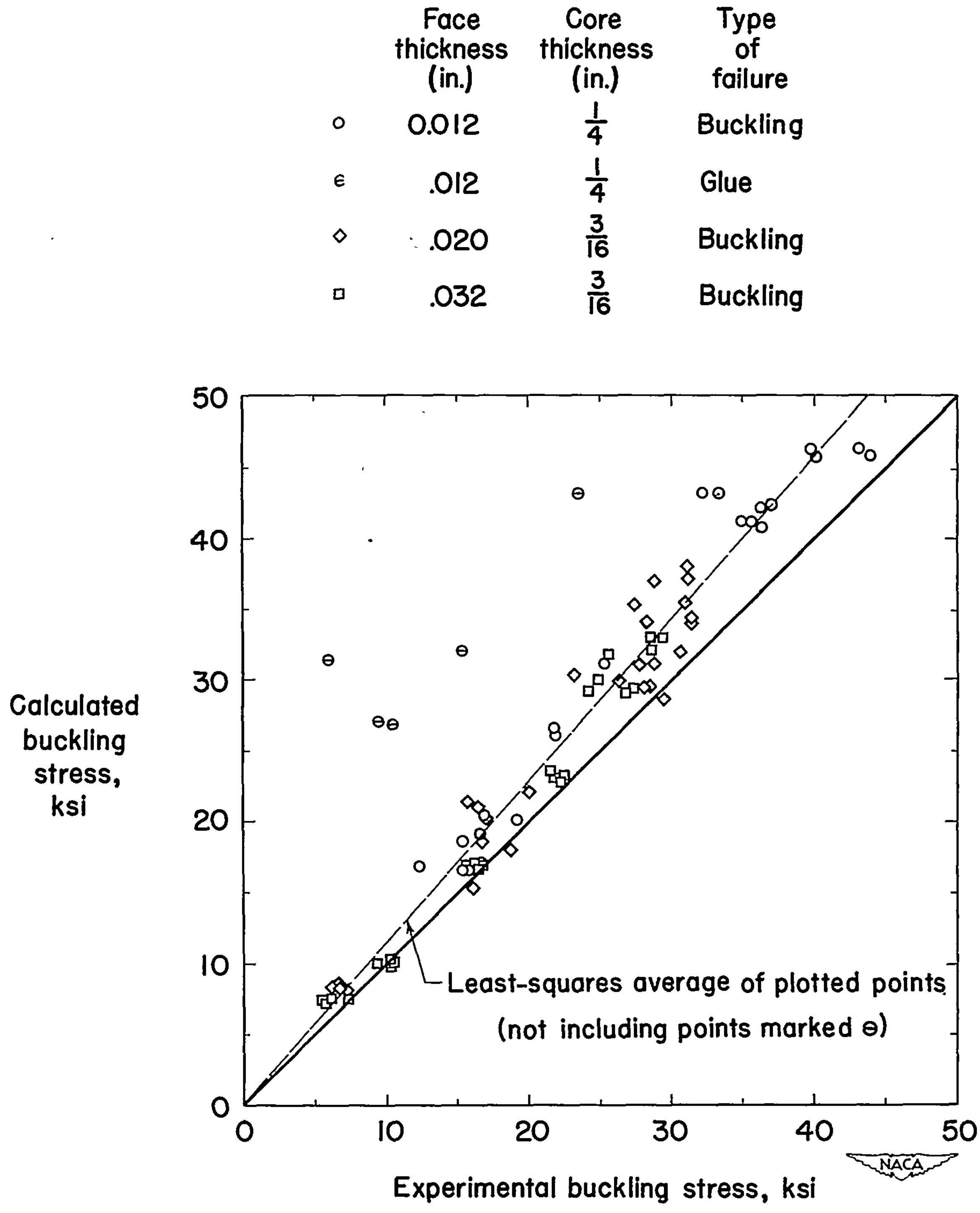


Figure 6.- Comparison of experimental buckling stresses of balsa-core sandwich plates with alclad 24S-T aluminum-alloy faces of reference 15 with calculated buckling stresses from theory of reference 13. (Dimensions shown are nominal thickness.)

NACA-Langley - 10-29-51 - 1000