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RESEARCH MEMORANDUM

EFFECT OF CENTRIFUGAL FORCE ON CRITICAL FLUTTER SPEED ON A UNIFORM CANTILEVER BEAM

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SUMMARY

Theoretical calculations were made by means of semirigid-flutter theory to determine the effect of centrifugal force on the critical flutter speed of a uniform cantilever beam. Centrifugal force can under certain conditions be detrimental, decreasing the critical flutter speed. The higher the ratio of natural torsional frequency to natural bending frequency the greater the decrease in critical flutter speed due to centrifugal effect. The conclusion is drawn that compressor and turbine blades that are flutter-free at zero rpm might become unstable at high rotative speeds. The centrifugal force effect can therefore not be ignored in any flutter calculations on compressor and turbine blades.

INTRODUCTION

The vibration problems of compressor and turbine blades have become more important as the use of axial-flow compressors and turbines has increased. These problems involve resonant vibrations due to mechanical excitation from other parts of the machine or from pulsating air flows from previous rows of blades in addition to nonresonant aerodynamic excitation. The aerodynamic excitation involves a type of vibration that is self-sustained by the continual absorption of energy from the air stream. This vibration is called flutter.

Classical flutter is more specifically defined as a self-sustained oscillation due to the coupling of inertia forces, elastic forces, damping forces, and dynamic aerodynamic forces. This type of flutter usually occurs on airplane wings at low angles of attack when the velocity reaches a certain value called the critical flutter speed. It is to be distinguished from a radically different type of flutter called stalling flutter, which occurs on profiles at high angles of attack, such as propeller blades near the stall point (reference 1). Stalling flutter can be caused by an aerodynamic hysteresis effect, the negative slope of the lift curve, or excitation by a system of Karman vortices (references 2 to 4).

Classical-flutter theory has been developed by many investigators (references 5 to 7) and the results have been found to agree with experiment. A direct application of this theory to analysis of compressor and turbine blades would seem to indicate little likelihood of such blades fluttering at zero angle of attack. This improbability of flutter is due to the stiffness of compressor and turbine blades compared to airplane wings. Two important factors, however, must be considered: the effect of centrifugal force and the effect of cascading. These effects may be either beneficial, that is, increase the critical flutter speed, or detrimental, that is, decrease the critical flutter speed.

An approximate method of solution developed at the NACA Cleveland laboratory of the effect of centrifugal force on the critical flutter speed of a uniform cantilever beam and the results for beams with various fundamental frequencies are presented herein. The effect of cascading is yet to be investigated.

SYMBOLS

The following symbols are used in the analysis:

a	coordinate of elastic axis measured from midchord in units of half chord, positive towards trailing edge
a_1, b_1, c_1, b_2, c_2	functions of reduced frequency k and fundamental bending frequency ω_b
a_y, a_y, a_θ, a_θ	aerodynamic coefficients
b	half chord, used as reference unit length
b_y, b_y, b_θ, b_θ	aerodynamic coefficients
c	μr_g^2
e	μr
F	function of k derived by Theodorsen (reference 5)
f_1, f_2	functions of ω_t/ω_b'

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G	function of k derived by Theodorsen (reference 5)
I	moment of inertia about elastic axis per unit span length
i	$\sqrt{-1}$
K	constant of proportionality
k	reduced frequency $\frac{\omega b}{v}$
L	oscillatory aerodynamic lift force per unit span
M	oscillatory aerodynamic moment per unit span
m	mass of profile per unit span
m_a	mass of cylinder of air (diameter of cylinder equal to profile chord) per unit span
N	rotative speed
r	location of center of gravity of profile measured from a in units of half chord
r_g	radius of gyration referred to a in units of half chord
S	static mass unbalance of profile, mrb
t	time
v	critical flutter speed
y	vertical displacement
y_0	maximum bending amplitude
β	phase angle by which torsion vibrational mode lags bending mode during flutter
θ	angle of torsional displacement
θ_0	maximum torsional amplitude
μ	ratio of mass of airfoil per unit span to mass of surrounding air cylinder per unit span, $\frac{m}{m_a} = \frac{m}{\pi\rho b^2}$

ρ	density of surrounding air
ω	frequency of flutter vibration
ω_b	fundamental bending frequency of airfoil
ω_b'	fundamental bending frequency of airfoil corrected for centrifugal-force effect
ω_t	fundamental torsional frequency of airfoil

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Dots over the symbols represent derivatives with respect to time.

ANALYSIS

Flutter equations. - The analysis of the effect of centrifugal force is based on the usual semirigid theory. Reference 8 shows that this theory, which assumes two degrees of freedom, that is, the uncoupled fundamental bending and torsional modes, gives results in close agreement with the exact solution, which assumes an infinite number of degrees of freedom. The assumption is made herein that these results also apply to the case in which centrifugal forces are added. The procedure followed is then to correct the fundamental bending mode for the centrifugal effect and to proceed in the usual manner to obtain the critical speeds and frequencies. The effect of centrifugal force on the fundamental torsional mode is assumed to be small enough to be neglected. The assumption is also made that the oscillatory air forces and moments acting on the blade are the same as those acting on an isolated two-dimensional airfoil in an uniform air stream. Incompressible flow is assumed and structural frictions are ignored.

The equations for dynamic equilibrium (reference 5) are

$$\left.\begin{aligned} m\ddot{y} + S\ddot{\theta} + m(\omega_b')^2\, y - L &= 0 \\ I\ddot{\theta} + S\ddot{y} + I\,\omega_t^2\,\theta - M &= 0 \end{aligned}\right\} \qquad (1)$$

The oscillatory aerodynamic lift L and moment M are given by

$$\left.\begin{aligned} L &= -\,m_a\omega^2\left[(a_y + i\bar{a}_y)\,y + (a_\theta + i\bar{a}_\theta)\,b\theta\right] \\ M &= -\,m_a\omega^2 b\left[(b_y + i\bar{b}_y)\,y + (b_\theta + i\bar{b}_\theta)\,b\theta\right] \end{aligned}\right\} \qquad (2)$$

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where

$$
\left.
\begin{aligned}
a_y &= -\left(1 + \frac{2}{k} G\right) \\
\bar{a}_y &= \frac{2}{k} F \\
a_\theta &= a + \frac{2}{k^2} F - \frac{2}{k}\left(\frac{1}{2} - a\right) G \\
\bar{a}_\theta &= \frac{1}{k} + \frac{2}{k^2} G + \frac{2}{k}\left(\frac{1}{2} - a\right) F \\
b_y &= a + \frac{2}{k}\left(a + \frac{1}{2}\right) G \\
\bar{b}_y &= -\frac{2}{k}\left(a + \frac{1}{2}\right) F \\
b_\theta &= -\left[\frac{1}{8} + a^2 + \frac{2}{k^2}\left(a + \frac{1}{2}\right) F - \frac{2}{k}\left(\frac{1}{4} - a^2\right) G\right] \\
\bar{b}_\theta &= -\left[\frac{2}{k^2}\left(a + \frac{1}{2}\right) G + \frac{2}{k}\left(\frac{1}{4} - a^2\right) F - \frac{1}{k}\left(\frac{1}{2} - a\right)\right]
\end{aligned}
\right\} \qquad (3)
$$

where F and G are functions of reduced frequency k given in reference 5.

Let

$$
\left.
\begin{aligned}
y &= y_0\, e^{i\omega t} \\
\theta &= \theta_0\, e^{i(\omega t - \beta)}
\end{aligned}
\right\} \qquad (4)
$$

then

$$
\left.
\begin{aligned}
\ddot{y} &= -\omega^2 y \\
\ddot{\theta} &= -\omega^2 \theta
\end{aligned}
\right\} \qquad (5)
$$

By combining equations (1), (2), and (5) and dividing through by $m_a \omega^2$

$$\left.\begin{aligned}\left\{a_y + i\bar{a}_y + \mu\left[\frac{(\omega_b')^2}{\omega^2} - 1\right]\right\} y + \left\{a_\theta + i\bar{a}_\theta - c\right\} b\theta &= 0 \\ \left(b_y + i\bar{b}_y - c\right) by + \left[b_\theta + i\bar{b}_\theta + e\left(\frac{\omega_t^2}{\omega^2} - 1\right)\right] b^2\theta &= 0\end{aligned}\right\} \quad (6)$$

Equations (6) have solutions other than $y = \theta = 0$ if and only if the determinant of the coefficients of y and θ vanishes. By setting this determinant equal to zero, separating real and imaginary parts, and rearranging

$$\left.\begin{aligned}\left(\frac{\omega}{\omega_t}\right)^2 &= \frac{b_2}{c_2} \\ \text{and} \quad a_1\left(\frac{\omega}{\omega_t}\right)^4 + b_1\left(\frac{\omega}{\omega_t}\right)^2 + c_1 &= 0\end{aligned}\right\} \quad (7)$$

where

$$b_2 = c\bar{a}_y + \mu\left(\frac{\omega_b'}{\omega_t}\right)^2 \bar{b}_\theta$$

$$c_2 = c\bar{a}_y + \mu\,\bar{b}_\theta + \bar{a}_\theta(b_y - e) + \bar{b}_y(a_\theta - e) - b_\theta\,\bar{a}_y - a_y\,\bar{b}_\theta$$

$$a_1 = (c - b_\theta)(\mu - a_y) + \bar{b}_y\,\bar{a}_\theta - \bar{b}_\theta\,\bar{a}_y - (a_\theta - e)(b_y - e)$$

$$b_1 = ca_y + \mu\left(\frac{\omega_b'}{\omega_t}\right)^2 b_\theta - c\mu\left[\left(\frac{\omega_b'}{\omega_t}\right)^2 + 1\right]$$

$$c_1 = c\mu\left(\frac{\omega_b'}{\omega_t}\right)^2$$

For a given profile, a, b, and c are functions only of the reduced frequency k and the fundamental bending frequency corrected for centrifugal force effects ω_b', which is, in turn, a function of the rotative speed N. For a given rotative speed N, a, b, and c are functions only of the reduced frequency k. Equations (7) can then be solved graphically for the reduced frequency k and the ratio of flutter frequency to fundamental torsional frequency ω/ω_t.

Bending frequency. - The effect of centrifugal force on fundamental bending frequency ω_b was obtained from reference 9.

$$(\omega_b')^2 = \omega_b^2 + KN^2 \quad (8)$$

where ω_b is the fundamental bending frequency without rotation and K is a function of the ratio of blade length to root radius and of the blade taper. By combining equations (7) and (8), the reduced frequency k and flutter frequency ω can be obtained as a function of the rotative speed N. The critical flutter speed v can then be obtained from the relation

$$v = \frac{\omega b}{k} \tag{9}$$

RESULTS AND DISCUSSION

Equations (7) were solved for the reduced frequency k and the frequency ratio ω/ω_t as functions of the ratio of torsional frequency to bending frequency corrected for centrifugal-force effect ω_t/ω_b'; the results are shown in figure 1. A typical profile with the following constants was chosen for the calculations:

$$a = -0.40$$

$$r = 0.10$$

$$r_g = 0.50$$

$$\mu = 100$$

From figure 1 and equation (8) it is possible to obtain the reduced frequency k and the frequency ratio ω/ω_t as a function of the rotative speed N, once a value for the fundamental bending frequency ω_b is chosen. For this purpose, equation (8) can be conveniently rewritten in the form

$$\left(\frac{\omega_t}{\omega_b'}\right)^2 = \frac{\left(\frac{\omega_t}{\omega_b}\right)^2}{1 + \frac{KN^2}{\omega_b^2}} \tag{10}$$

For a given profile, the frequency ratio ω_t/ω_b' will therefore decrease as the rotative speed N increases. From figure 1, for values of frequency ratio ω_t/ω_b' less than approximately 0.90, the value of reduced frequency k will decrease and the value of ω/ω_t will decrease as the rotative speed N increases (or as ω_t/ω_b' decreases). At values of frequency ratio ω_t/ω_b' greater than approximately 0.90, both reduced frequency k and frequency ratio ω/ω_t increase as the rotative speed N increases (or as ω_t/ω_b' decreases). But for values of ω_t/ω_b' close to 0.90, the reduced frequency k increases at a faster rate than ω/ω_t. The critical flutter speed v can be expressed as

$$v = \frac{b\left(\frac{\omega}{\omega_t}\right)\omega_t}{k} \qquad (11)$$

The critical flutter speed v will therefore increase with increasing rotational speed N for values of frequency ratio ω_t/ω_b' less than 0.90 and will decrease with increasing rotative speed N for values of ω_t/ω_b' greater than 0.90. The existence of a minimum value for the critical flutter speed v is therefore indicated.

The foregoing relations can perhaps be more clearly seen if equation (10) is used with figure 1 to plot directly the reduced frequency k and the frequency ratio ω/ω_t as a function of the rotative speed N. This variation is shown in figures 2 and 3 for a fundamental bending frequency ω_b of 100 radians per second and frequency-ratio ω_t/ω_b' values ranging from 1 to 5. The constant K was assumed to be $\left(\frac{2\pi}{60}\right)^2$. This value corresponds to a non-tapered blade with a ratio of root radius to blade length of zero. As shown in figures 2 and 3, the frequency ratio ω/ω_t increases as the rotative speed N increases but the reduced frequency k at first increases and then decreases. If the rate of increase of reduced frequency k in any part of the region where it is increasing is greater than the rate of increase of the frequency ratio ω/ω_t, then from equation (11) a minimum critical flutter speed v can be expected.

In order to verify this conclusion, the critical flutter speed v was calculated assuming a blade chord of 2 inches. The results are shown in figure 4 for values of fundamental bending frequency ω_b of 100, 500, 1000, and 2000 radians per second, respectively. The critical flutter speed v decreases to a minimum value as the rotative speed N is increased. This minimum value depends on the bending frequency ω_b, on the ratio of torsional to bending frequencies ω_t/ω_b, and on the rotative speed N. The rotative speed N at which the critical flutter speed v is a minimum increases as the ratio of torsional to bending frequency ω_t/ω_b increases and as the natural bending frequency ω_b increases; also, the higher this ratio, the greater the decrease in critical flutter speed v. This decrease in critical flutter speed v as the rotative speed N is increased is due to the centrifugal force causing only the bending frequency ω_b to increase. Although increasing both the bending frequency ω_b and torsional frequency ω_t will increase the critical flutter speed v, increasing the bending frequency ω_b alone may decrease the critical flutter speed (reference 10). This relation can be seen if the curves of figure 1 are expressed as

$$\frac{\omega}{\omega_t} = f_1\left(\frac{\omega_t}{\omega_b{}'}\right)$$

$$k = f_2\left(\frac{\omega_t}{\omega_b{}'}\right)$$

Then by equation (11)

$$v = \frac{bf_1\left(\frac{\omega_t}{\omega_b{}'}\right)\omega_t}{f_2\left(\frac{\omega_t}{\omega_b{}'}\right)}$$

If the torsional frequency ω_t and bending frequency $\omega_b{}'$ are both increased so that the ratio of torsional to bending frequency $\omega_t/\omega_b{}'$ remains constant, $\dfrac{f_1\left(\frac{\omega_t}{\omega_b{}'}\right)}{f_2\left(\frac{\omega_t}{\omega_b{}'}\right)}$ will remain constant. The value of critical flutter speed v, however, increases because of the increase in torsional frequency ω_t. A change in the value of bending frequency $\omega_b{}'$ alone, however, may either raise or lower the value of $\dfrac{f_1\left(\frac{\omega_t}{\omega_b{}'}\right)}{f_2\left(\frac{\omega_t}{\omega_b{}'}\right)}$ and produce a corresponding change in critical flutter speed v.

At high frequency ratios ω_t/ω_b and at high values of the natural bending frequency ω_b, the critical flutter speeds v obtained are above the lower compressibility limit for which the calculations are valid and the results are therefore meaningless. This limit has been arbitrarily chosen as 800 feet per second and all curves above that speed are shown as dashed lines.

For natural bending frequencies ω_b higher than 1000 radians per second, classical flutter cannot occur in the subsonic range except at low frequency ratios ω_t/ω_b and possibly at high rotative speeds N as shown by figure 4(c) and 4(d). Because current compressor and turbine blades usually have bending frequencies greater than 1000 radians per second, little danger of such blades fluttering classically would exist if the torsional frequency ω_t were several times the bending frequency ω_b. However, at high rotative speeds N, the critical flutter speed v can be greatly reduced and blades that would be safe at zero rpm can become unstable. As an example, figure 4(c) shows that at a frequency ratio ω_t/ω_b of 2 the flutter speed v is reduced from a value that is out of the subsonic range to about 615 feet per second as the rotative speed is increased from 0 to 10,000 rpm.

CONCLUSIONS

From the calculations made to determine the effect of centrifugal force on the flutter of a uniform cantilever beam, the conclusion is drawn that under certain conditions centrifugal force can be detrimental, decreasing the critical flutter speed. A compressor or turbine blade that would be stable at zero rpm might become unstable at high rotative speeds. The flutter characteristics of blades must therefore be investigated at the operating engine speeds. The validity of the semirigid flutter theory with the centrifugal force effect included must still be verified. The effect of cascading must also be investigated.

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REFERENCES

1. Sterne, L. H. G., and Ewing, H. G.: A Comparison of Flutter Tests on Propellers with 2, 3, and 4 Blades. Rep. No. S.M.E. 3287, R.A.E., 1944.

2. Victory, Mary: Flutter at High Incidence. Rep. No. S.M.E. 3240, R.A.E., 1943.

3. Bollay, William, and Brown, Charles D.: Some Experimental Results on Wing Flutter. Jour. Aero. Sci., vol. 8, no. 8, June 1941, pp. 313-318.

4. Mendelson, Alexander: Effect of Aerodynamic Hysteresis on Critical Flutter Speed at Stall. NACA RM No. E8B04, 1948.

5. Theodorsen, Theodore: General Theory of Aerodynamic Instability and the Mechanism of Flutter. NACA Rep. No. 496, 1934.

6. Duncan, W. J., and Collar, A. R.: Calculation of the Resistance Derivatives of Flutter Theory. I. R.&M. No. 1500, A.R.C., 1932.

7. Kassner, R., and Fingado, H.: The Two-Dimensional Problem of Wing Vibration. R.A.S. Jour., vol. XLI, Oct. 1937, pp. 921-944.

8. Goland, Martin: The Flutter of a Uniform Cantilever Wing. Jour. Appl. Mech., vol. 12, no. 4, Dec. 1945, pp. A197-A208.

9. Timoshenko, S.: Vibration Problems in Engineering. D. Van Nostrand Co., Inc., 2d ed., 1937, p. 385.

10. Theodorsen, Th., and Garrich, I. E.: Mechanism of Flutter. A Theoretical and Experimental Investigation of the Flutter Problem. NACA Rep. No. 685, 1940.

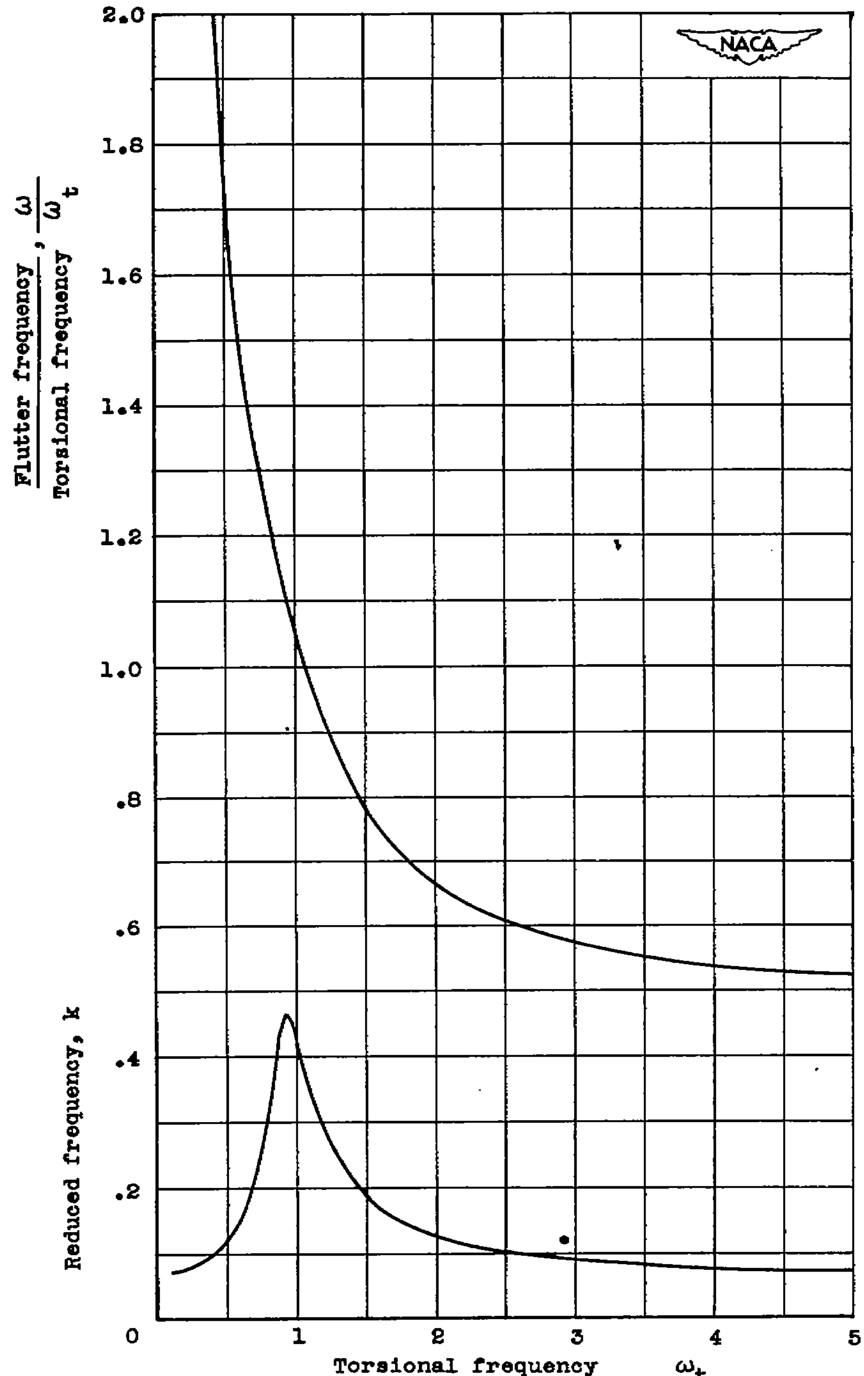

Figure 1. - Variation of ratio of flutter frequency to torsional frequency and of reduced frequency with ratio of torsional frequency to corrected bending frequency.

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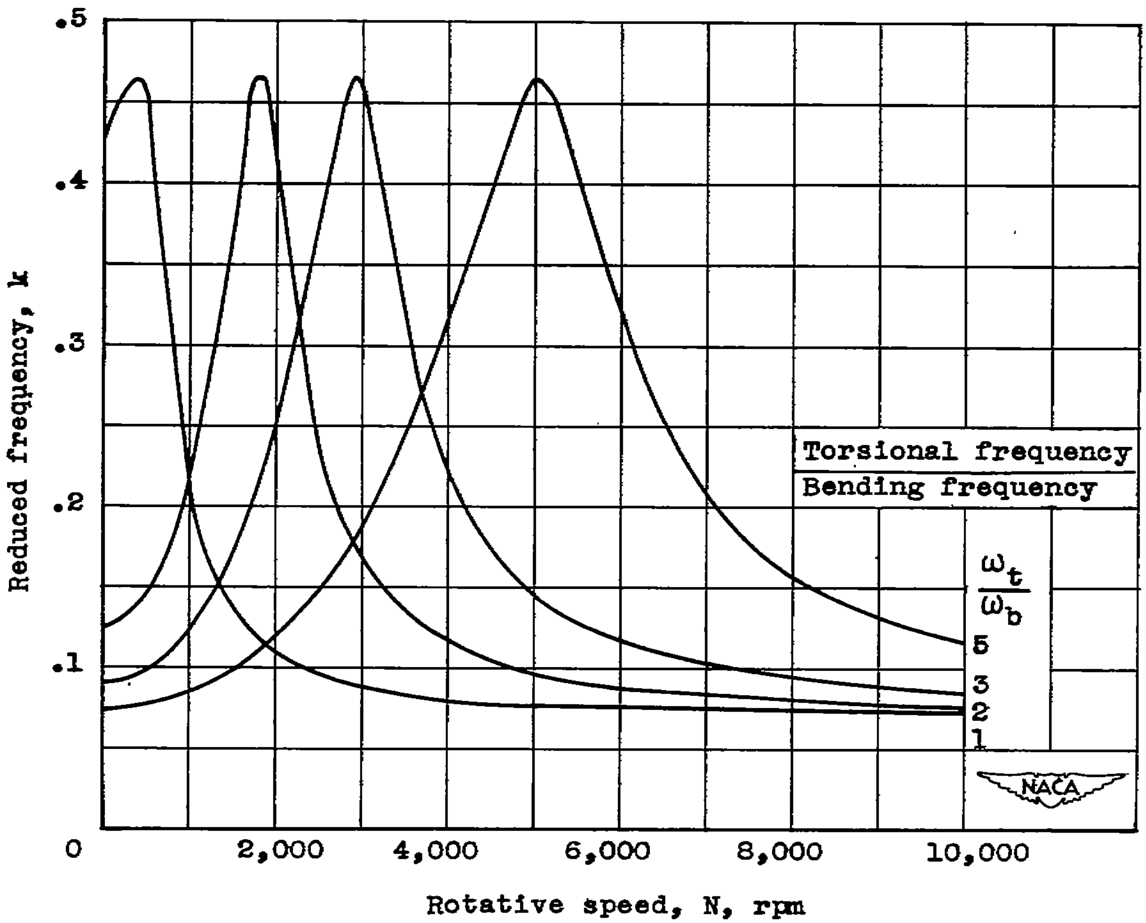

Figure 2. - Variation of reduced frequency with rotative speed for various ratios of torsional to bending frequencies. Fundamental bending frequency, 100 radians per second.

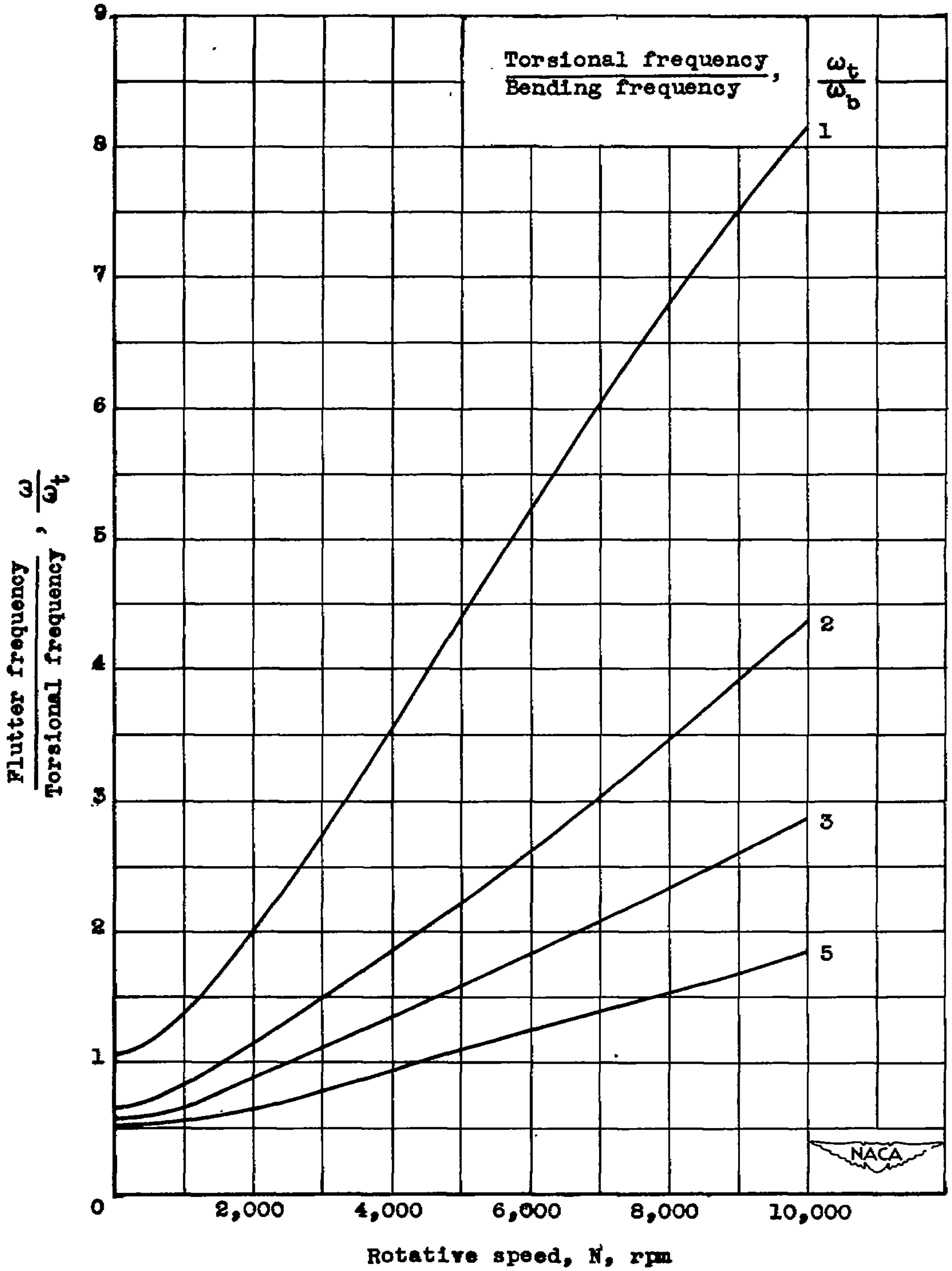

Figure 3. - Variation of ratio of flutter frequency to torsional frequency with rotative speed for various ratios of torsional to bending frequencies. Fundamental bending frequency, 100 radians per second.

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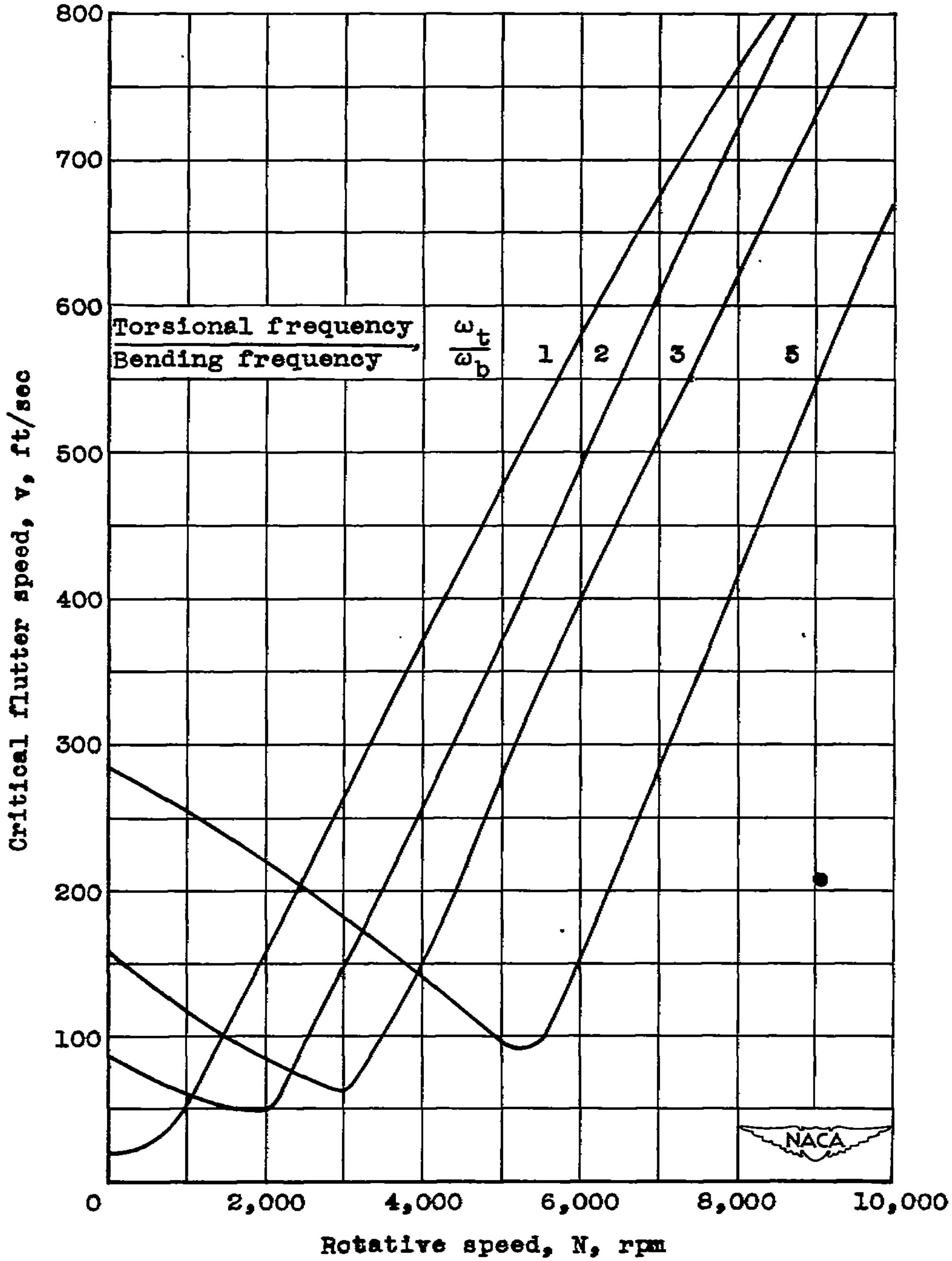
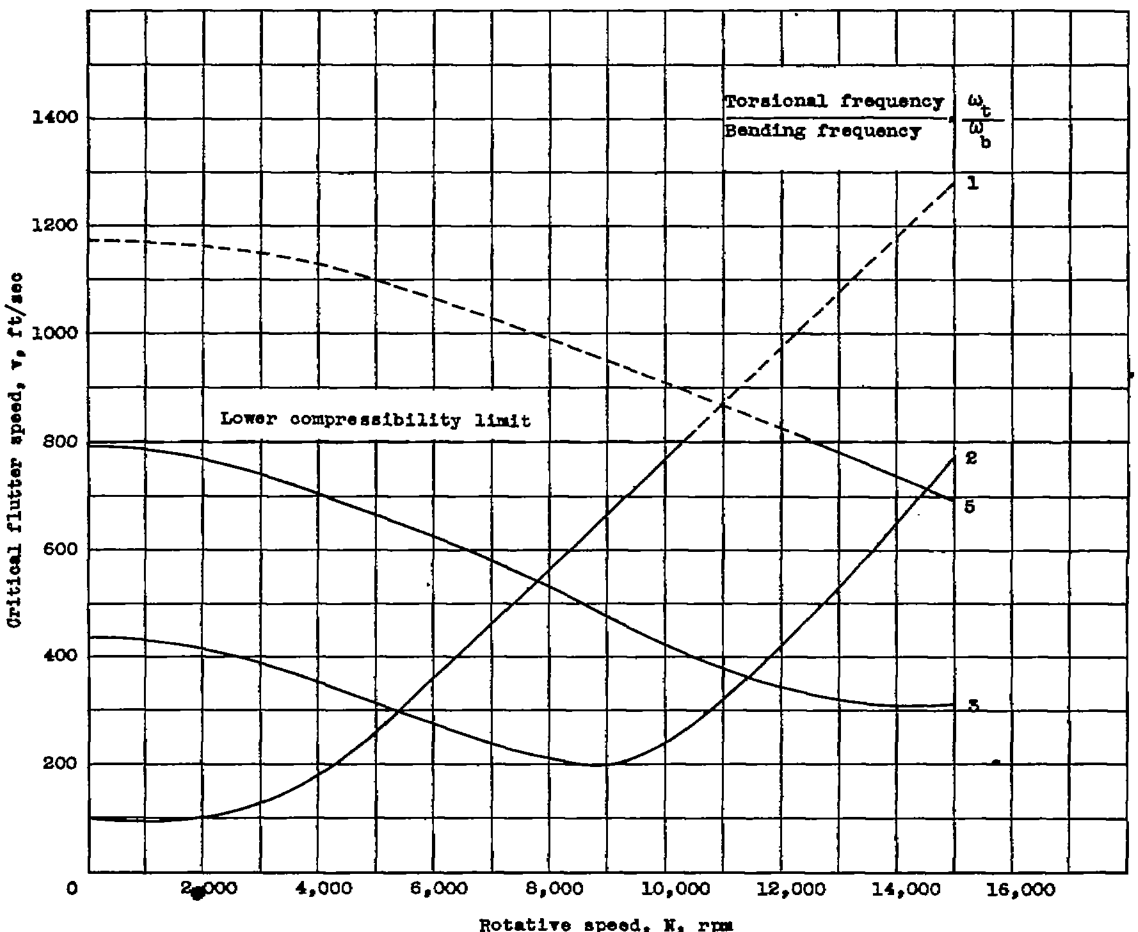


(a) Fundamental bending frequency, 100 radians per second.

Figure 4. - Variation of critical flutter speed with rotative speed for various ratios of torsional to bending frequencies.



(b) Fundamental bending frequency, 500 radians per second.

Figure 4. - Continued. Variation of critical flutter speed with rotative speed for various ratios of torsional to bending frequencies.

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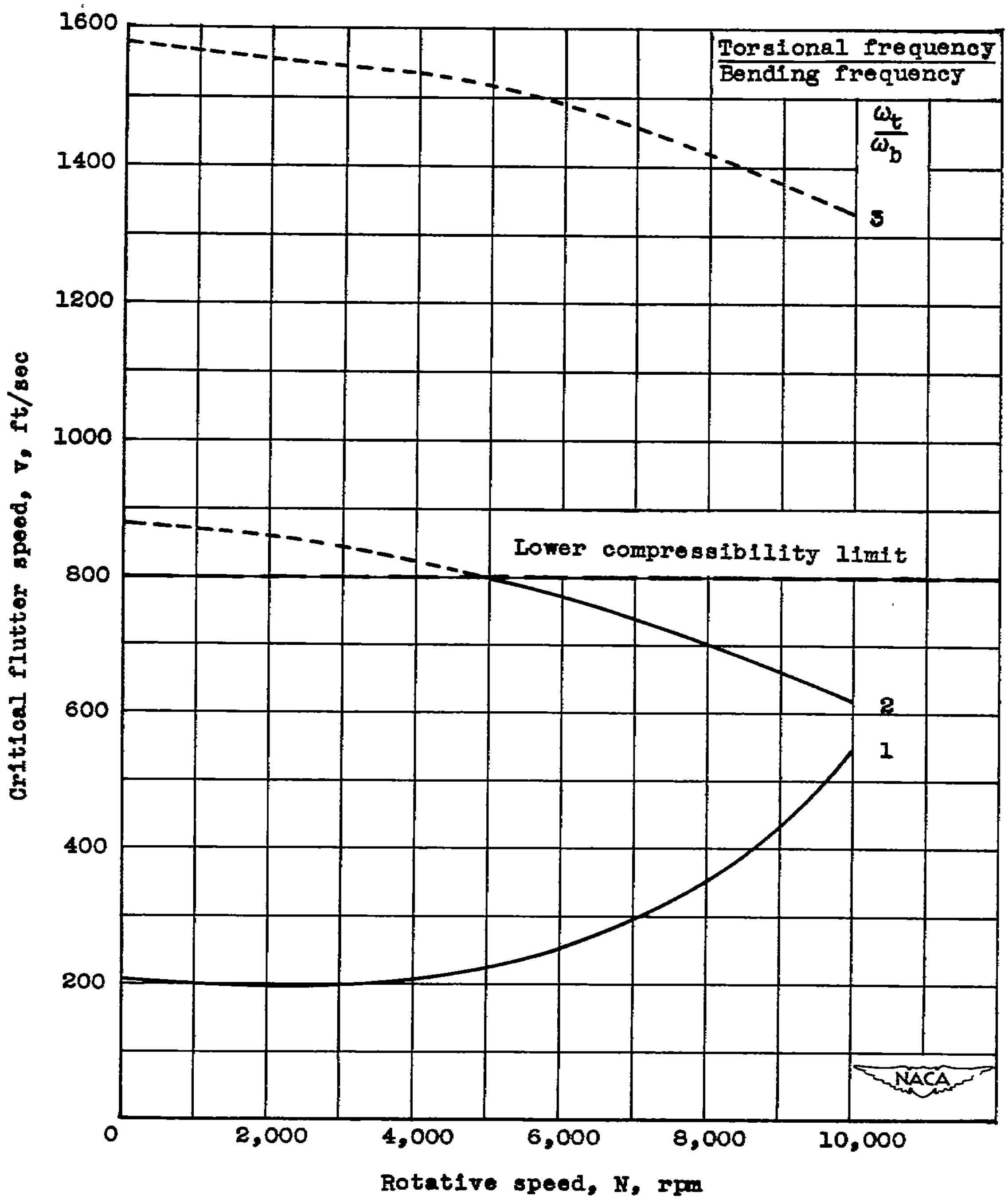
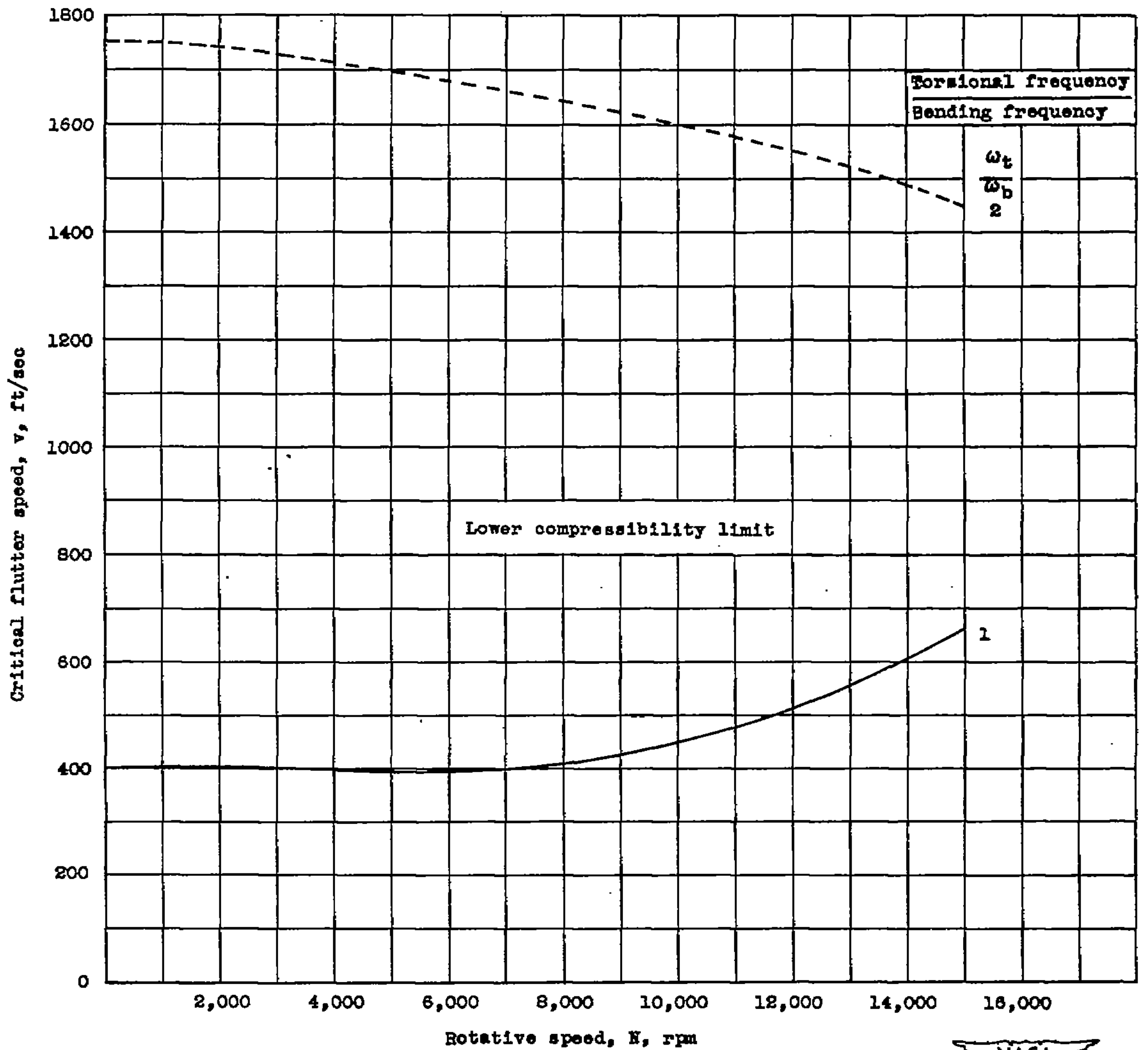


(c) Fundamental bending frequency, 1000 radians per second.

Figure 4. - Continued. Variation of critical flutter speed with rotative speed for various ratios of torsional to bending frequencies.



(d) Fundamental bending frequency, 2000 radians per second.

Figure 4. - Concluded. Variation of critical flutter speed with rotative speed for various ratios of torsional to bending frequencies.

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