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RESEARCH MEMORANDUM

LOCATION OF DETACHED SHOCK WAVE IN FRONT OF A BODY MOVING AT SUPERSONIC SPEEDS

By

Edmund V. Laitone and Otway O'M. Pardee

Ames Aeronautical Laboratory
Moffett Field, Calif.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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RESEARCH MEMORANDUM

LOCATION OF DETACHED SHOCK WAVE IN FRONT OF A BODY MOVING AT SUPERSONIC SPEEDS

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SUMMARY

It is shown that for velocities slightly in excess of sonic, the position of the detached shock wave located in front of a given body at zero angle of attack may be estimated theoretically to a reasonable degree of accuracy. The theory developed compares favorably with the available experimental data.

INTRODUCTION

The solvable fluid-flow problems are in general divided into two distinct classes: those in which the field of flow is completely subsonic and those in which the flow is supersonic, each regime having its special methods of solution and approximation. As yet, very little has been accomplished toward the solution of any fluid-flow problems in that region between the critical Mach number and the shock detachment Mach number, the latter being defined later. This region is sometimes termed the transonic regime. The difficulty of solution is due in large measure to the combination of mixed subsonic and supersonic flows together with pronounced viscosity or boundary-layer effects. There is, however, one problem capable of solution which should prove very useful both in flight and wind-tunnel work – that of estimating for Mach numbers slightly in excess of 1 the position of the detached shock wave preceding a body.

It is characteristic of supersonic flight that preceding every body or attached to its nose is a shock wave. Here a differentiation should be made between pointed and blunt-nosed bodies. In the case of blunt-nose bodies, the bow wave always remains detached similar to that shown in figure 1. However, for any given sharply pointed body, there is a Mach number below which the shock wave is detached but above which it is attached in the characteristic fashion of a Mach wave, as shown in reference 1. For pointed bodies this Mach number is the detachment Mach number, and, as noted before, represents the upper limit of the transonic region. For blunt-nose bodies, on the other hand, there is no upper limit defined.

The solution of the present problem in transonic flow is somewhat simplified since there is no interaction between the shock and boundary layer. The viscosity effects are almost all relegated to the region of the wake and for the present problem are relatively unimportant. Moreover, certain of the results from linear perturbation theory may be used which at first glance might not seem applicable.

Linear perturbation theory has in the past found wide uses in the study of subsonic and supersonic flow fields. It is based upon the assumption that the disturbance created by the presence of the body is small; that is, the perturbation velocities due to the body are small compared to the free-stream velocity. With these approximations for subsonic flows, perturbation theory shows that, for very slender bodies of revolution, the pressure coefficients along the body are independent of Mach number; whereas for two-dimensional flow they are not. The development and discussion of these points are given in references 2, 3, 4, and 5.

SYMBOLS

The following is a list of the more important symbols used in this report, given in order of their introduction:

M Mach number

M_0 free-stream Mach number

M_1 Mach number on downstream face of shock wave

δ angle shock wave makes with normal to free-stream direction

$\Delta\theta$ deviation angle of flow at shock wave

ϵ excess of free-stream Mach number over 1, small compared to 1

Δs change in entropy

p pressure

ρ density

γ - ratio of specific heat at constant pressure to specific heat at constant volume (c_p/c_v)

V velocity

V_0 free-stream velocity

V_1 velocity on downstream face of shock wave

a speed of sound

x distance from nose of body

T maximum thickness of body

S distance along stagnation line from the shock wave to the nose of the body

L length of body

THEORY

The flow field to be considered is shown in figure 1. The body of maximum thickness T is symmetrical about the XX'-axis and at zero incidence to the free stream. It has a stagnation point at O and the stream line XO leading up to this point is called the stagnation line.

This body is moving at a supersonic speed such as to produce the detached shock wave AA' which extends to infinity in both directions. The shock wave intersects the stagnation line at the point S. At this point, the shock wave is normal to the stream lines. At other points in the field, such as P, the shock is not normal to the free-stream direction but makes an angle δ with this normal. The angle δ varies from zero at S to the complement of the Mach angle at infinity, for the shock wave has an asymptote whose angle with the horizontal is the Mach angle.

The shock wave divides the field into two parts. Everywhere upstream of the shock wave, the flow is uniform and the total-head or stagnation pressure is constant. Downstream of the shock wave, the flow varies throughout the field and each streamline has a different stagnation pressure. This variation in stagnation pressure or total head is due to the variation in entropy change through the shock wave. The entropy change is primarily a function of the free-stream Mach number and the angle δ, being greatest when $\delta = 0$; that is, on the stagnation line.

The deviation in direction of the flow upon passing through the shock wave is $\Delta\theta$ as shown in figure 1. This deviation varies from zero at the stagnation line to a maximum angle $\Delta\theta_{max}$ some point a finite distance out on the shock wave and approaches zero again as the shock wave approaches its asymptote.

The Mach number on the downstream side of the shock wave varies also with the angle δ as well as the free-stream Mach number. The lowest Mach number is less than 1 and occurs where the shock is normal at the point S. Going out along the shock wave, the Mach number increases with increasing angle δ, approaching the free-stream Mach number as δ approaches the complement of the Mach angle. The variation then is from subsonic in the vicinity of S to supersonic far out on the shock wave.

The method of solution developed in this report covers only free-stream velocities slightly greater than sonic, being at all times at a free-stream Mach number of $1 + \epsilon$, where ϵ is small compared to 1. It then will be shown that the maximum entropy change and maximum deviation angle $\Delta\theta_{max}$ are of higher order than ϵ and consequently negligible. To further simplify the problem, only the distance OS is determined, which is sufficient to determine the shock wave in the vicinity of the body since, for free-stream Mach numbers of the order of $1 + \epsilon$, the shock wave is nearly plane. It will then only be necessary to consider a normal shock and the variation of velocity along the stagnation line, since it will be shown that these are sufficient to determine the distance OS. The entropy change through a shock wave is given by

$$\Delta s = c_v \ln \frac{p/\rho^\gamma}{p_o/\rho_o{}^\gamma} \tag{1}$$

where the subscript o refers to the free-stream conditions. Expanding this in powers of $M_o{}^2 - 1$, where M_o is the free-stream Mach number, by means of the relations of conservation of mass, momentum, and energy given in reference 1, the following expression is obtained for normal shock waves:

$$\Delta s = c_v \frac{2\gamma(\gamma-1)}{3(\gamma+1)^2} \left(M_o{}^2 - 1 \right)^3 + \ldots \tag{2}$$

Then since $M_0 = 1 + \epsilon$, the maximum possible increase in entropy is given by

$$\Delta s = c_v \frac{16}{3} \gamma \frac{\gamma-1}{(\gamma+1)^2} \epsilon^3 + \ldots \tag{3}$$

Therefore, the entropy is approximately constant and consequently so is the total head or stagnation pressure.

It can be shown also that the maximum flow deflection is of higher order than ϵ. Von Kármán has shown (equation (8.9) of reference 6) that the minimum Mach number for a given flow deflection is given by

$$M = 1 + \frac{3}{2} \left(\frac{\gamma+1}{4}\right)^{2/3} \cdot (\Delta\theta)^{2/3} \tag{4}$$

and since M is a monotonically increasing function of $\Delta\theta$ the same equation defines the maximum flow deviation for an arbitrary Mach number. Then replacing M by $1 + \epsilon$ the maximum flow deviation is given by

$$\Delta\theta_{max} = \frac{8}{3(\gamma+1)} \sqrt{\frac{2}{3}} \epsilon^{3/2} \tag{5}$$

and it can be seen that the maximum deviation of flow is of higher order than ϵ.

From the conservation laws referred to previously, the Mach number M_1 after (downstream) a normal shock is given in terms of the Mach number M_0 before (upstream) the shock by

$$M_1^2 = \frac{2+(\gamma-1)M_0^2}{2\gamma M_0^2-(\gamma-1)} \tag{6}$$

The ratio of the velocity V_1 after a shock to the velocity V_0 before the shock (the free-stream velocity) is given by

$$\frac{V_1}{V_0} = \frac{1}{\gamma+1}\left(\frac{2}{M_0^2} + \gamma-1\right) \qquad (7)$$

Setting $M_0 = 1 + \epsilon$ and $\gamma = 7/5$ in equations (6) and (7) and neglecting ϵ^2 and higher power, the equations become

$$M_1 = 1 - \epsilon \qquad (8)$$

and

$$\frac{V_1}{V_0} = 1 - \frac{5}{3}\epsilon \qquad (9)$$

Then M_1 and V_1/V_0 differ from free-stream conditions by the order ϵ.

Since the entropy and total head are constant throughout the field, to the order of approximation used, the flow downstream of the shock wave is derivable from a velocity potential. The boundary conditions necessary to specify this velocity potential are the shape of the body and the velocity vector distribution over any surface which encloses the body. It is now necessary to consider variations in the flow field with change in boundary conditions.

For subsonic flow, the shape of the body and the flow at infinity are sufficient boundary conditions; while for supersonic flow, the condition upstream at infinity is replaced by one on the downstream face of the shock wave.

Since the boundary conditions in either subsonic or supersonic flow are a continuous function of Mach number, the velocity potential is a continuous function of Mach number inside the boundary limits. Now if the two potential fields (subsonic and supersonic) are to approach a common limit at a Mach number of 1 and consequently be continuous from subsonic through to supersonic free-stream velocities, it is only necessary that the boundary conditions approach a common limit.

As the free-stream Mach number approaches 1 from above, the shock wave recedes upstream to infinity and the flow deviation vanishes since it is of higher order than ϵ, the Mach number increment; and furthermore the Mach number on the downstream face of the shock wave approaches 1. The limit then is a uniform, parallel free stream at infinity the same as for the more obvious subsonic case.

Hence, the velocity potential, velocity, and pressure coefficient for any given point in the subsonic region between S and O is a continuous function of the Mach number. If in this region the variation of pressure coefficient with Mach number can be determined for subsonic free streams, then, a good approximation to the pressure coefficients for free-stream Mach numbers slightly greater than 1, of the order of $1 + \epsilon$, is obtained by a mathematical continuation of the subsonic variation of pressure coefficient with Mach number, this by virtue of the velocity potential being a continuous function of Mach number.

The velocity variation along the stagnation line can be given by an equation of the form

$$\frac{V}{V_o} = f\ (x/T,\ M) \tag{10}$$

where nondimensional form is used for convenience, V being the velocity at any point x along the OX-axis and M the local Mach number at the point x. At the point S where the shock is located $V = V_1$, $x = S$, and $M = M_1$ the equation then becomes

$$\frac{V_1}{V_o} = f\ (S/T,\ M_1) \tag{11}$$

and S/T is determined since M_1 and V_1/V_o are given by equations (6) and (7) or approximately from equations (8) and (9) by

$$M_1 = 1 - \epsilon = 2 - M_o \tag{12}$$

and

$$\frac{V_1}{V_o} = 1 - \frac{5}{3}\epsilon = \frac{8}{3} - \frac{5}{3}M_o \tag{13}$$

With these approximations equation (11) becomes

$$\frac{8}{3} - \frac{5}{3}M_o = f\left(\frac{S}{T},\ 2 - M_o\right) \tag{14}$$

which defines S/T as a function of M_o.

APPLICATIONS

In order to evaluate the functions defined by equation (14) it is necessary to resort to linear perturbation theory. With this purpose in mind the variation in the velocity ratio V/V_o

along the stagnation line has been obtained for a number of standard two- and three-dimensional shapes in incompressible flow, the results of which are presented in table I. The case of three dimensional bodies of revolution will be considered first.

It can be shown from the methods of reference 2 that for slender bodies of revolution the velocity ratio V/V_o along the stagnation line is independent of Mach number. That is,

$$\frac{V}{V_o} = \left(\frac{V}{V_o}\right)_{M=0}$$

Using this in equations (11) and (14) gives at S

$$\left(\frac{V_1}{V_o}\right)_{M=0} = \frac{8}{3} - \frac{5}{3} M_o \qquad (15)$$

For illustrative purposes, the method is applied to the three-dimensional source and an experimental comparison made. Referring to table I,

$$\left(\frac{V}{V_o}\right)_{M=0} = 1 - \frac{\frac{1}{16}}{\left(\frac{x}{T} + \frac{1}{4}\right)^2}$$

Then using this in equation (15) the point S is determined from

$$M_o = 1 + \frac{0.0375}{\left(\frac{S}{T} + \frac{1}{4}\right)^2} \qquad (16)$$

The curves presented in figure 2 were obtained in this manner under the assumption that even in the case of a sphere the pressure coefficients on the stagnation line are independent of Mach number. It is worthy of note that the curve for the source approximates very closely the values for any Rankine Ovoid of thickness ratio less than 0.10.

In figure 3 are shown some experimental data for a 20-millimeter shell which were obtained from the U.S. Army firing range at the Aberdeen Proving Grounds. The theoretical curve shown for comparison is for a source, since the shell which was used had a fairly large nose radius making it approximately a three-dimensional source shape. The agreement with the theory is rather good even though the shell was continually decreasing its speed; for due to the deceleration of the shell, the shock wave is likely to be at a different location than would be found at a steady velocity.

In the case of bodies of revolution the result was simple. For two-dimensional bodies, however, the pressure coefficient varies with Mach number, and here a slight difficulty appears. It is necessary to realize that in calculating the velocity field, linear perturbation theory in and of itself makes no distinction between local and free-stream Mach number. In fact, they are assumed to differ by a negligible quantity. That this assumption is not valid in the present case is self-evident; however, it can be shown in unidimensional flow, where an exact solution is possible, that there is less error in calculating the velocities in a decreasing velocity field in using the local Mach number rather than the free-stream Mach number. Using this criterion and equation (13), the velocity ratio at the point S is

$$\frac{V_1}{V_o} = 1 - \frac{1-\left(\frac{V_1}{V_o}\right)_{M=0}}{\sqrt{1 - M_1^2}} = \frac{8}{3} - \frac{5}{3} M_o$$

Now $M_1 = 1 - \epsilon$ and neglecting ϵ^2 and higher powers

$$\sqrt{1 - M_1^2} = \sqrt{2\epsilon} = \sqrt{2(M_o - 1)}$$

therefore

$$\left(\frac{V}{V_o}\right)_{M=0} = 1 - \frac{5\sqrt{2}}{3}(M_o - 1)^{3/2} \tag{17}$$

where $(V/V_o)_{M=0}$ can be obtained from table I. The curves shown in figure 4 were obtained in this manner, where it has been assumed that even in the case of the circular cylinder the Prandtl-Glauert rule holds on the stagnation line.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif.

Edmund V Laitone,
Aeronautical Engineer.

Otway O'M Pardee
Otway O'M. Pardee,
Electrical Engineer.

Approved:

Donald H. Wood
Donald H. Wood,
Aeronautical Engineer.

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TABLE I.- THE VELOCITY PARAMETER V/V_0 ALONG THE STAGNATION LINE FOR VARIOUS BODIES IN INCOMPRESSIBLE FLOW

Bodies of revolution
Three-dimensional source, $1 - \dfrac{1/16}{\left(\frac{x}{T} + \frac{1}{4}\right)^2}$
Sphere, $1 - \dfrac{1/8}{\left(\frac{x}{T} + \frac{1}{2}\right)^2}$
Rankine Ovoid, T/L = 0.10, $1 - 0.6284 \left[\dfrac{1}{\left(\frac{x}{T} + 0.2506\right)^2} - \dfrac{1}{\left(\frac{x}{T} + 9.7494\right)^2}\right]$
Prolate Spheroid, T/L = 0.10, $1 - 0.005181 \left[\dfrac{9.9499\left(\frac{x}{T} + 5\right)}{\left(\frac{x}{T} + 5\right)^2 - 24.750} - \log\left(\dfrac{\frac{x}{T} + 9.9749}{\frac{x}{T} + 0.0251}\right)\right]$
Two-dimensional symmetrical bodies
Two-dimensional source, $1 - \dfrac{0.1592}{\left(\frac{x}{T} + 0.1592\right)}$
Circular cylinder, $1 - \dfrac{1/4}{\left(\frac{x}{T} + \frac{1}{2}\right)^2}$
Rankine Oval, T/L = 0.05, $1 - \dfrac{3.23}{\left(\frac{x}{T} + 10\right)^2 - 96.77}$
Rankine Oval, T/L = 0.10, $1 - \dfrac{1.65}{\left(\frac{x}{T} + 5\right)^2 - 23.35}$
Rankine Oval, T/L = 0.16, $1 - \dfrac{1.052}{\left(\frac{x}{T} + 3.125\right)^2 - 8.714}$
Elliptic cylinder, T/L = 0.05, $1 - \left[\dfrac{21}{4\left(\frac{x}{T} + 10\right)^2 - 399 + 2\left(\frac{x}{T} + 10\right)\sqrt{4\left(\frac{x}{T} + 10\right)^2 - 399}}\right]$
Elliptic cylinder, T/L = 0.10, $1 - \left[\dfrac{11}{4\left(\frac{x}{T} + 5\right)^2 - 99 + 2\left(\frac{x}{T} + 5\right)\sqrt{4\left(\frac{x}{T} + 5\right)^2 - 99}}\right]$

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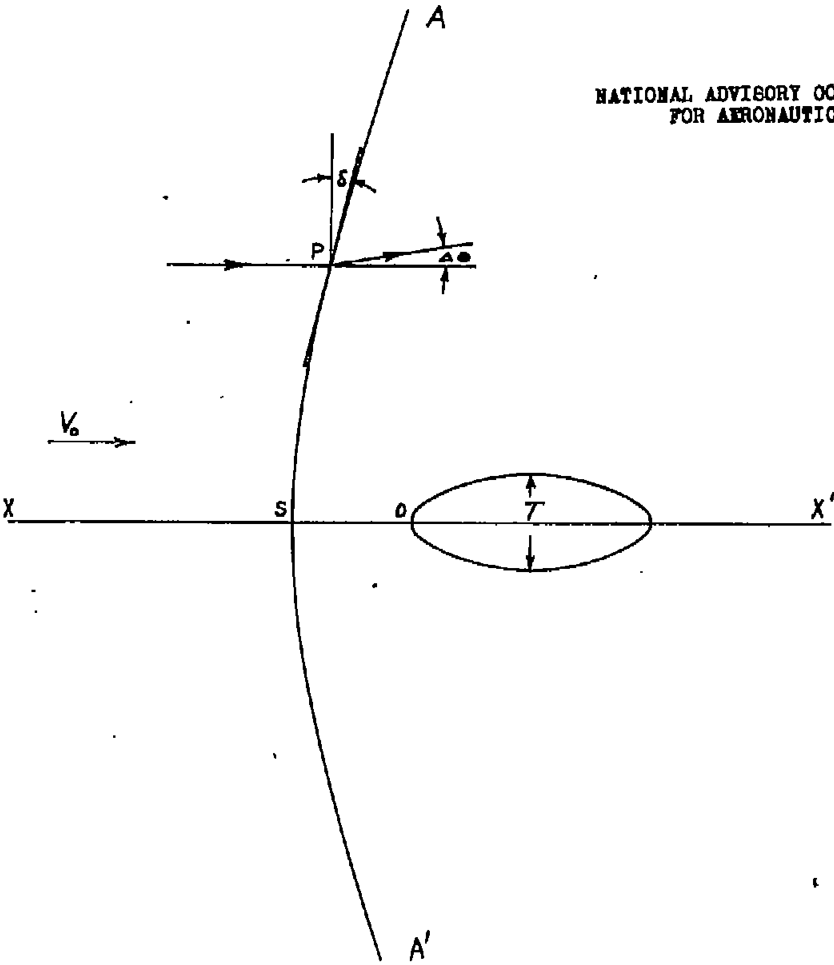

Figure 1.— Body with detached shock wave in supersonic flow.

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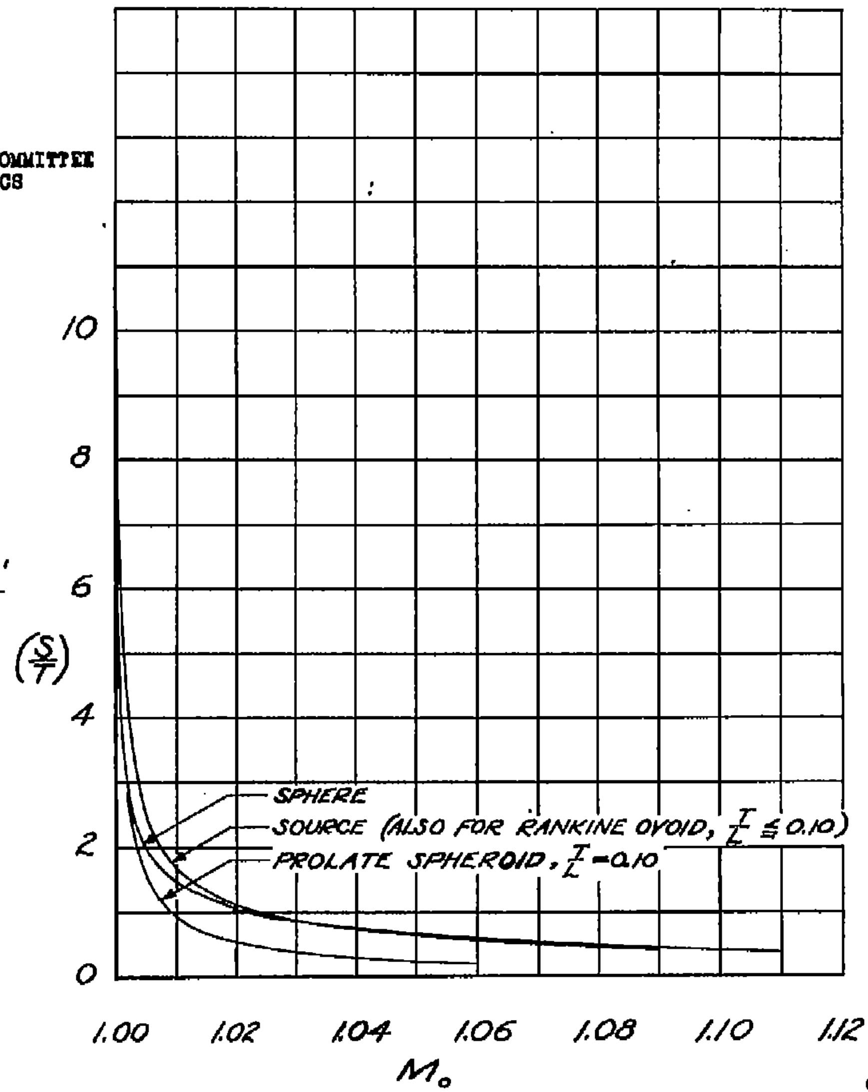

Figure 2.— Location of shock wave in front of three-dimensional bodies of revolution.

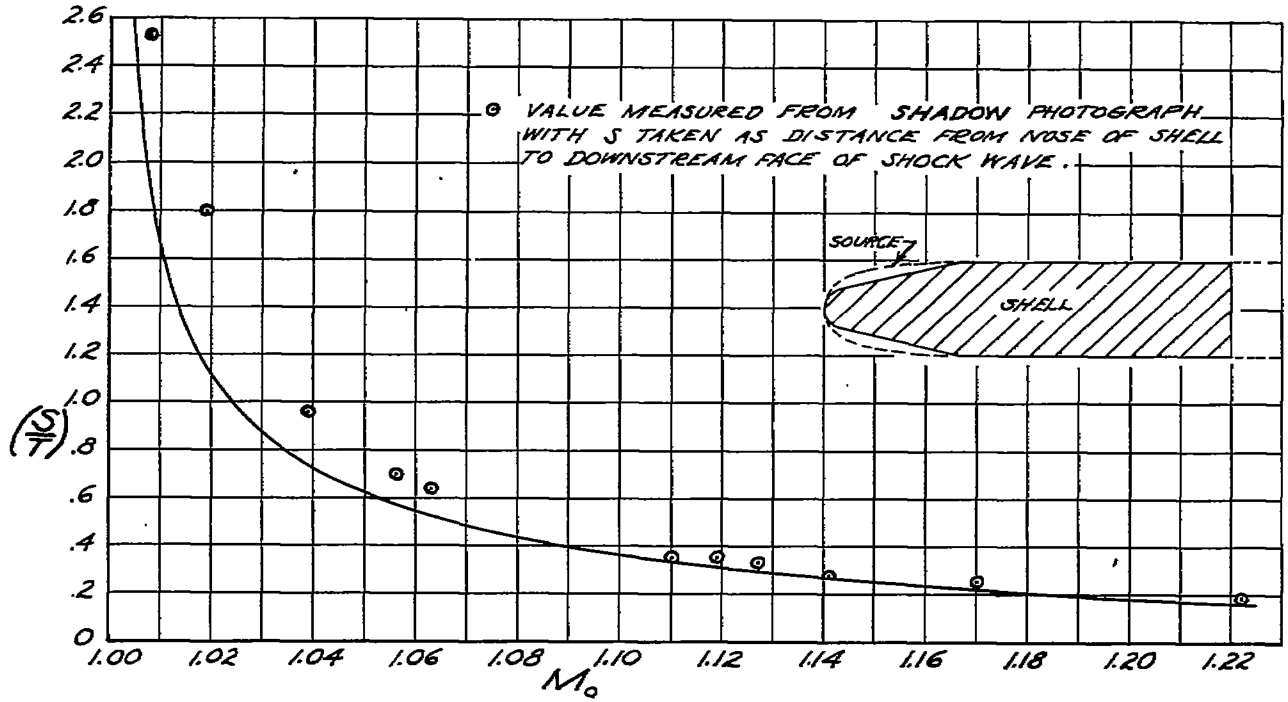

FIGURE 3.– COMPARISON OF MEASURED LOCATION OF SHOCK WAVE IN FRONT OF A 20 MM SHELL WITH THEORETICAL LOCATION FOR A THREE-DIMENSIONAL SOURCE.

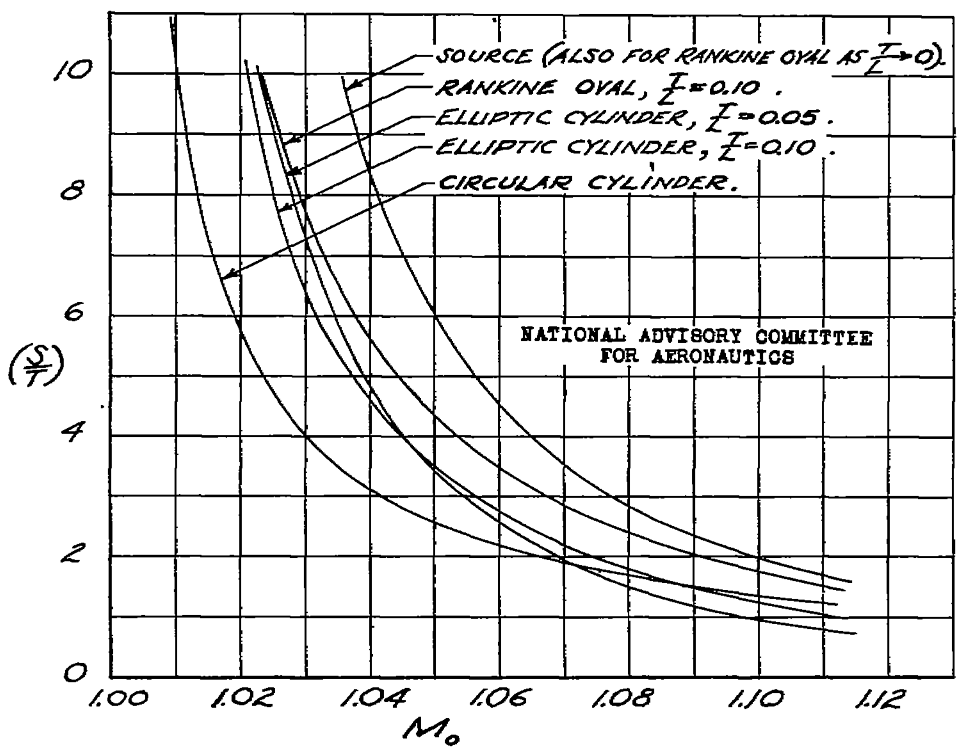

FIGURE 4.– LOCATION OF SHOCK WAVE IN FRONT OF TWO-DIMENSIONAL BODIES.