REPORT No. 884

EFFECTS OF SWEEPBACK ON BOUNDARY LAYER AND SEPARATION

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SUMMARY

Following the law of stress adopted in the Navier-Stokes equations, the configuration of the viscous flow in planes at right angles to the axis of an infinite cylinder is found to be independent of the axial motion of the cylinder. In the limiting case of a yawed or swept wing of very high aspect ratio, certain boundary-layer and separation phenomena are thus determined independently by the crosswise component of velocity. It follows that the effect of sweepback is to increase the area of stable laminar flow and to decrease the lift coefficient at which flow separation occurs.

INTRODUCTION

Experimental observations of the viscous flow over oblique wings and bodies present such complex phenomena that even the most approximate simplying principle may be of value in interpreting these observations. In the case of compressible flow, such a simplifying principle was found by studying the idealized problem of the perfectly cylindrical oblique flow, corresponding to the infinite aspect-ratio wing, or the infinitely slender body. In that case, the equations of motion together with the boundary conditions applicable to a moving cylinder in a frictionless fluid showed that (1) the flow pattern and pressure disturbances in planes at right angles to the cylindrical axis are determined solely by the component of velocity in these planes, and (2) the axial motion of the cylinder produces no effect on the flow. Now the question is: Can similar generalizations be made concerning the viscous flow produced by an infinite cylinder moving obliquely? For this case, the Navier-Stokes equations, together with the given boundary conditions, supply a definite answer. It can be shown that proposition (1) still applies while proposition (2), of course, does not. Thus it is to be expected that various features of the viscous flow such as boundary-layer thickness and separation point *if observed in planes at right angles to the cylindrical axis* will be determined solely by the component of velocity of the cylinder in these planes.

OBLIQUE VISCOUS FLOW

To verify this feature of the oblique flow, consider first the disturbance produced by a pure crosswise motion of the cylinder. The motion and the state of stress will be the same in all cross sections. Each particle in a given cross section is thus associated with a whole string of particles, or a filament, connecting the various cross sections and moving as a unit. Obviously we may introduce any arbitrary lengthwise translation of these filaments without affecting their progress across the cylinder, since each particle will simply move from one cross section to another similar one. Such a lengthwise motion of the filaments will, of course, introduce shearing stresses in this direction, but these stresses will not affect the rate of shear and hence will not affect the shearing stress in the crosswise direction.

The nondependence of the cross flow and the axial flow is, of course, the result of the law of shearing stress adopted in the Navier-Stokes equations. The shearing stress in one direction is taken to be proportional to the rate of shearing in this direction and is independent of the rate of shearing in other directions. At high rates of shear where an appreciable temperature rise is involved, the independence of the shearing stresses can no longer be assumed. Such conditions occur at very high Mach numbers and are, of course, beyond the range of validity of the Navier-Stokes equations.

THE LAMINAR BOUNDARY LAYER ON AN OBLIQUE FLAT PLATE

The simplest case of viscous, cylindrical flow is the laminar boundary layer over a flat plate (Blasius flow). The well-known result of Blasius shows a parabolic increase of boundary-layer thickness beginning at the leading edge, that is,

$$\delta=k\sqrt{\frac{\nu X}{V}}$$

In accordance with the foregoing statements the boundary-layer thickness at any given point will not be affected by the introduction of an additional velocity parallel to the leading edge of the plate. In fact, Blasius' formula shows no change in δ if we change X so that it is measured along the direction of the new resultant velocity since, in this case, both X and V are changed in the same ratio, figure 1. The resultant drag on the oblique plate lies in the direction of the resultant velocity, but the component force in the direction at right angles to the leading edge can be determined solely from the velocity component in this direction.

An approximate solution for the laminar boundary layer on an oblique plate with a favorable pressure gradient has been given recently by Prandtl in reference 1. On the assumption that the flow in the boundary layer departs only slightly from the direction of the main stream, Prandtl obtains the velocity profile of the boundary layer in two mutually perpendicular directions. It appears that the resultant friction drag is inclined away from the stream direction toward the direction of the favorable pressure gradient.

WIND-TUNNEL TESTS OF CIRCULAR WIRES

With more complex flows involving pressure stresses as well as viscous stresses, complete solutions have been obtained only for very low Reynolds numbers. At high Reynolds numbers the pressure stresses predominate but viscosity

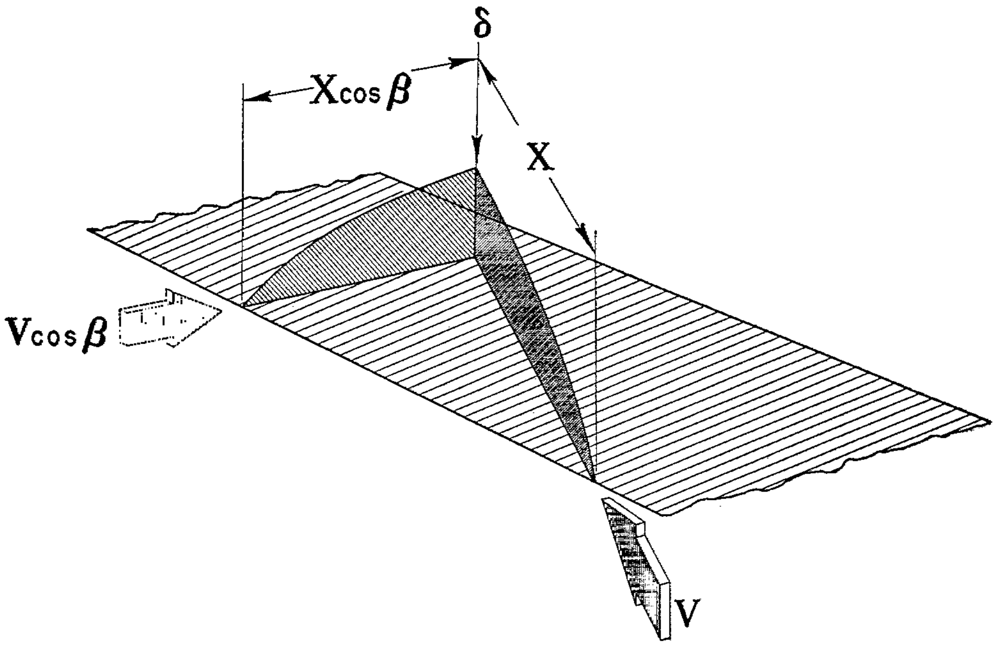

FIGURE 1.—Boundary layer on an oblique flat plate.

plays an important part through its effect on separation. Thus in the case of a circular cylinder the flow separates approximately half-way around the surface and a Kármán vortex street is formed in the wake. One would expect that the geometry of this pattern, the vortex frequency, etc., would depend only on the crosswise component of velocity.

Figure 2 shows the result of a wind-tunnel test on a circular wire at different angles of yaw (reference 2). In these tests the Reynolds number based on the crosswise component varied from 10^2 to 10^3—a range in which the drag coefficient is nearly constant. Over this range the crosswise component of the drag force on the wire should be proportional to $(V\cos\beta)^2$ and the tests do indicate this variation.

In the case of the circular cylinder there is a certain critical Reynolds number at which the line of separation shifts rapidly to a more rearward position. This shift is accompanied by a marked reduction in the drag coefficient. If the cylinder is oblique, the critical Reynolds number should be delayed to a higher speed such that

$$\frac{V\cos\beta\, d}{\nu}=R_{critical}$$

STABILITY OF LAMINAR FLOW

It is well known that a laminar boundary layer or surface of discontinuity becomes unstable at certain Reynolds numbers. Calculations of the stability of the laminar boundary layer in two-dimensional motion have been made by Tollmein, Schlichting and, more recently, by Lees and Lin. The theoretical predictions have been verified experimentally in important respects by Dryden, Schubauer, and Skramstad (references 3 and 4), who find that transition to turbulent flow can result from fluctuations in the laminar layer. In the case of a yawed plate or airfoil the calculated boundary-layer waves would, of course, have their crests and troughs aligned with the leading edge and their stability would be determined by the Reynolds number of the crosswise component of the stream velocity. In case transition is caused by such oscillations one would expect the transition line to recede from the leading edge with increasing angles of yaw in a stream of constant velocity. On the other hand, if transition is caused by roughness of the surface, a cylindrical flow cannot be assumed and the transition may, of course, be affected differently.

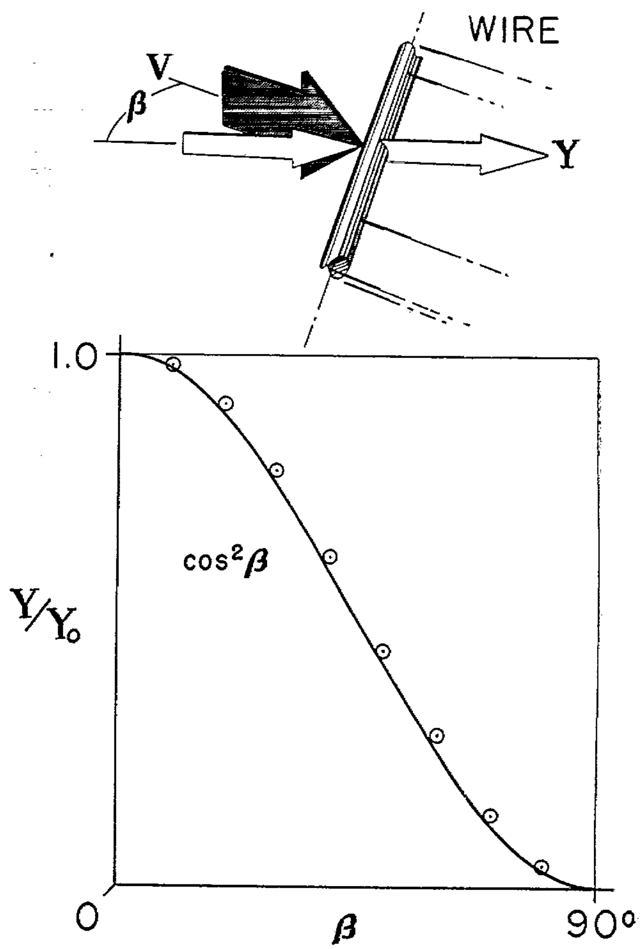

FIGURE 2.—Experimental variation of cross-wise force on an oblique wire.

An interesting case which departs radically from the usual assumptions made for a cylindrical boundary has been investigated by G. I. Taylor (see reference 5). Taylor investigates the stability of viscous flow in the annular space between two concentric circular cylinders in relative rotation. After a certain Reynolds number is exceeded the cylindrical form of the flow disappears and a regular vortex formation appears with the vortex rotations at 90° to the rotation of the cylinders, and alternating periodically along their length. Gortler and Liepmann find similar three-dimensional disturbances in other cases of boundary-layer flow along convex walls. According to Liepmann (reference 6) transition from laminar to turbulent flow results from this three-dimensional type of instability if the surface is concave, but transition on a plane or convex surface results from the two-dimensional type of wave motion mentioned earlier.

FLOW SEPARATION AND MAXIMUM LIFT

The separation of flow over a straight wing occurs when the adverse pressure increase opposing the motion is sufficient to reverse the momentum of the fluid in the boundary layer. (See fig. 3.) In the case of the oblique wing the resultant pressure gradient is, of course, at right angles to the long axis of the wing and both this pressure and the components of the viscous stress distribution lying in this direction will be determined by the crosswise component of velocity. Hence the circumstances leading to separation of flow over the straight wing will be reproduced if the crosswise component of motion of the oblique wing is the same as that of the straight wing. In both cases separation will be taken to mean that the fluid in the boundary layer has lost the component of momentum that carries it across the wing. In the oblique case the boundary layer will then flow in a direction parallel to the long axis.

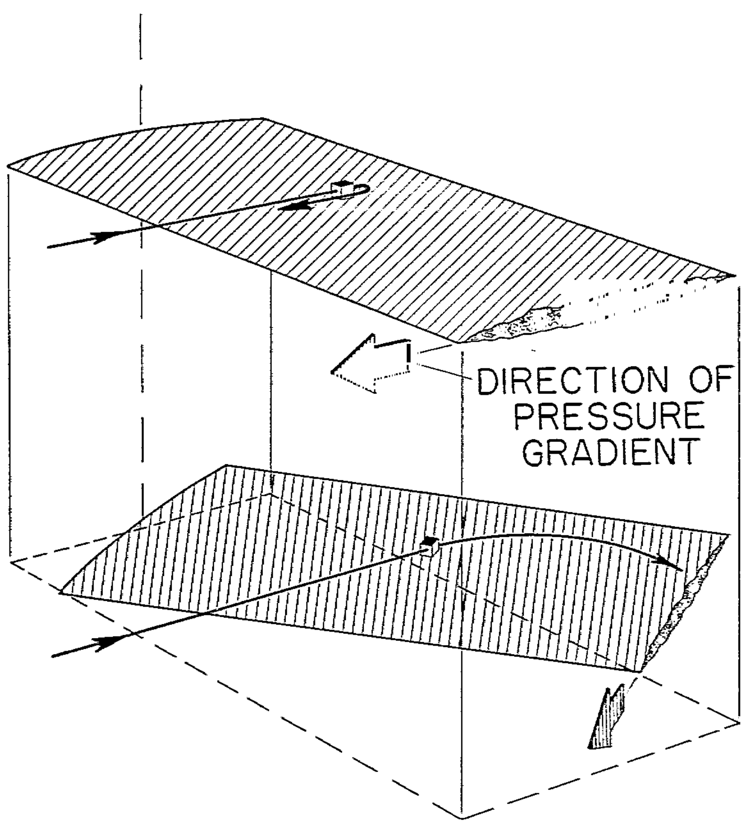

FIGURE 3.—Effect of yaw on paths of particles in separated boundary layer.

The known adverse effects of sweepback on the lift and drag of a wing can be at least partially explained by this analysis. According to the two-dimensional theory a wing which shows boundary-layer reversal and maximum lift at $C_L=1.4$, if yawed 45°, would show separation accompanied by a fully developed lateral motion of the boundary layer at $C_L=0.7$. In each case the lift would drop and the resultant force would fall back to a position nearly at right angles to the chord because of the loss of the suction force on the leading edge. At 60° yaw the predicted maximum lift coefficient would be only 0.35. Wind-tunnel observations of the boundary-layer flow over swept-back wings agree qualitatively with these predictions in regard to flow separation but do not show the expected loss in maximum lift. Instead of a drop in lift after boundary-layer separation, some experiments indicate an increase in lift-curve slope at this point. Ordinarily the forces and moments on the swept wing do begin to show nonlinear variations at the separation point, however, and there is a sharp rise in drag at this point indicating that the loss of suction at the leading edge does occur. The tendency to follow the two-dimensional theory up to the point of separation and the partial failure of the theory after separation indicates that the "end effects" are much greater in the separated flow than in the unseparated flow. This situation is not surprising from the physical standpoint, since the influence of the tips would obviously be more extensive in the case of a thick separated region than in the case of a relatively thin boundary layer. It must be supposed, however, that as the aspect ratio of the swept-back wing is increased its maximum lift coefficient will show an increasing tendency to follow the $\cos^2$ law. The experiments on oblique circular wires, which involve large regions of separated flow and yet follow the two-dimensional theory, lend support to this hypothesis.

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MOFFETT FIELD, CALIF., *July, 1947*.

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