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THE PROBLEM OF NOISE REDUCTION WITH REFERENCE TO LIGHT AIRPLANES

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SUMMARY

Experiments by Deming at the Langley Memorial Aeronautical Laboratory confirm completely the formula of Gutin, which permits the convenient calculation of the sound level of any airplane propeller at low forward speeds. A simplification of the Gutin formula has been achieved by the use of a set of functions giving the sound level in the direction of maximum intensity. The sound level can be read from graphs of the functions for various numbers of blades and tip speeds.

Two numerical examples and one experimental example are included; also, a radical fan-type propeller is tentatively treated.

Results of this study show that propeller noise dominates engine exhaust noise even though the exhaust noise has a relatively high intensity. It is concluded, therefore, that in order to reduce the outside sound level of an airplane materially, it will be necessary to modify the propeller to operate at low tip speeds and to have a large number of blades. The practical use of this conclusion is a matter of considerable technical complexity involving many compromises. An effective engine exhaust muffler will also be required.

INTRODUCTION

The problem of designing airplanes deals chiefly with cost, performance, stability, safety, and similar factors; however, questions have occasionally been raised concerning the elimination of airplane noise. This problem must be considered from the standpoint of both the airplane passengers and the people living in the vicinity of airports. The airports located near residential sections are usually small and can accommodate only light airplanes. The present paper emphasizes the study of noise from light airplanes.

In 1936 a paper by Gutin was published (reference 1) which gives the theoretical expression for the sound emission of an airplane propeller as a function of speed, number of blades, thrust and torque, and linear dimensions of the propeller. The formula is, strictly speaking, applicable only to the case of a stationary propeller; in other words, Gutin did not include the effect of the forward or flight speed. It can be shown, however, that the formula is sufficiently accurate for low forward speeds to make it adequate for application to low-powered airplanes. The theoretical results of Gutin were confirmed by extensive measurements by Deming at the Langley Memorial Aeronautical Laboratory, part of which have been reported in reference 2.

The present paper applies the Gutin formula to several cases of light airplanes. The formula has been rewritten in a form convenient for engineering use. The representative sound level is obtained by the use of a single graph.

The human ear is sensitive to sound energies ranging from about 10^{-16} watts per square centimeter to 10^{-4} watts per square centimeter, at which level the sound becomes painful to the listener. Since the power ratio at the two limits corresponds to a million times a million, acoustical workers have adopted a logarithmic scale as a measure of sound energy. The unit of one "decibel" is equivalent to a power ratio of 1.259, which is the antilogarithm of 0.1. The base level adopted by the Acoustical Society of America (reference 3) is 10^{-16} watts per square centimeter. The sound intensity level hence is given by the formula

$$I = 10 \log_{10} \frac{P}{10^{-16}} \quad \text{decibels} \qquad (1)$$

where P is power in watts per square centimeter. Conversely, the rate of energy per square centimeter is given as

$$P = 10^{\left(\frac{I}{10}-16\right)} \quad \text{watts per square centimeter}$$

and, if I is considered as a mean value, the total energy radiated per second is

$$E = 4\pi L^2 \times 10^{\left(\frac{I}{10}-16\right)} \quad \text{watts} \qquad (2)$$

where L is the distance from the source.

The sound intensity level may also be expressed in terms of the root-mean-square pressure of the sound by use of the following formula:

$$P = \frac{p^2}{\rho c} \times 10^{-7} \quad \text{watts per square centimeter}$$

where the root-mean-square pressure p is in dynes per square centimeter, the density ρ is in grams per cubic centimeter, and the velocity of sound c is in centimeters per second. Under standard conditions the energy level of 10^{-16} watts per square centimeter corresponds to a pressure of 0.0002 dyne per square centimeter. Thus the sound intensity level may be expressed as

$$I = 20 \log_{10} \frac{p}{0.0002}$$

$$= 74 + 20 \log_{10} p \quad \text{decibels} \qquad (3)$$

A pressure of one dyne per square centimeter corresponds to 74 decibels.

The following table conveys a concept of the steps in the sound scale by introducing the effect of distance from a given source and by a comparison with commonly recognized sound levels:

SOUND LEVEL FROM SOURCE OF 4π WATTS AT VARIOUS DISTANCES AND COMPARISON WITH KNOWN NOISES

[Absorption, refraction, and reflection are neglected]

Distance			Sound level	
Kilometers	Miles	Feet	Decibels	Reference standards
1/100		32.81	100	Elevated trains
1/10		328.08	80	Printing press
1	0.6213	3280.8	60	Conversation
10	6.213		40	Dwelling
100	62.13		20	
1000	621.3		0	Threshold

SOUND THEORY

The formula for the sound emission from an airplane propeller is given in an important paper by Gutin, which was published in the Physikalische Zeitschrift der Sowjetunion in 1936 (reference 1), as follows:

$$p = \frac{qn\omega}{2\sqrt{2}\ \pi cL}\left(-T\cos\beta + \frac{cQ}{\omega R^2}\right) J_{qn}\left(qn\sin\beta\,\frac{V}{c}\right) \qquad (4)$$

In this formula the symbols have the following definitions:

p root-mean-square sound pressure, dynes per square centimeter (bars)

n number of blades

q harmonic of sound

ω speed of revolution, radians per second

c velocity of sound, centimeters per second

L distance from propeller, centimeters

T thrust, dynes

Q moment, dyne-centimeters

β angle from propeller axis (zero in front)

R propeller radius (mean value), centimeters

$J_{qn}(x)$ Bessel function of order qn and argument

$$x = qn \frac{V}{c} \sin \beta$$

V velocity of element of propeller at 0.8 radius (mean value), centimeters per second

Figure 1 shows a typical distribution of the pressure for the lowest harmonic of the sound. Note that the peak pressure is near $\beta = 120^\circ$. Experiments by Deming (reference 2) show virtually perfect agreement, particularly when the proper reference conditions are used.

By use of the 0.8 radius as the mean radius and by substitution of the thrust for the torque, the Gutin formula may be rewritten in the simpler form

$$p = \frac{p_o}{2\sqrt{2}} \frac{R_t}{L} M_t \left(1.7 \frac{M}{M_t^2} - \cos \beta\right) B_{qn} \qquad (5)$$

where

$$B_{qn} = qnJ_{qn}\left(qn \frac{V}{c} \sin \beta\right)$$

$$p_o = \frac{T}{\pi R_t^2} \quad \text{(full value of radius used)}$$

R_t radius of propeller (full value)

M_t tip Mach number of blade (rotation only)

M Mach number of advance or of flow velocity through propeller disk (V_o/c)

V_o flow velocity through propeller disk

The conversion factor for p expressed in pounds per square foot and in dynes per square centimeter is

1 pound per square foot = 478.8 dynes per square centimeter (bars)

The formula for p may therefore be written

$$p = 169.3 p_o \frac{R_t}{L} M_t \left(1.7 \frac{M}{{M_t}^2} - \cos \beta\right) B_{qn} \qquad (6)$$

where p_o is given in pounds per square foot.

In regard to the quantity B_{qn}, it may be noted that the subscript qn and the argument $qn \frac{V}{c} \sin \beta$ are related. If fixed values of 1, 0.75, and 0.5 are chosen for V/c and fixed values of 90° and 120° are chosen for the angle β, the entire quantity

$$B_{qn} = qnJ_{qn}\left(qn \frac{V}{c} \sin \beta\right)$$

may be plotted against the argument or frequency qn. By use of the foregoing values, six curves are obtained, each given by a double index V/c and β, where V/c is the mean Mach number of the blade and β is the angle measured from the direction of advance as zero. The six curves, each labeled accordingly, are shown in figure 2. Since the maximum sound pressure is obtained at a value of β of approximately 120°, the curve relating to this angle generally gives sufficient information on the intensity, since the pattern on the whole repeats itself around the origin with zero intensity at 0° and 180° and in the direction for which $\cos \beta \approx 1.7 \frac{M}{{M_t}^2}$.

By convention, the root-mean-square pressure of 1 dyne per square centimeter corresponds to a sound level of 74 decibels and the sound level at a pressure P in dynes per square centimeter is then

$$I = 74 + 20 \log_{10} p \quad \text{decibels} \tag{3}$$

In order to obtain the total pressure of several harmonics, it is noted that the energy is proportional to p^2. Since the cross products contribute nothing, the p^2 values of the several harmonics may simply be added and the square root extracted. The total effective pressure is thus

$$p_t = \sqrt{\sum_q p^2}$$

and the sound level is

$$I = 74 + 10 \log_{10} \sum_q p^2 \tag{7}$$

Only the factor B_{qn} changes with the harmonic (see formula (5)); therefore,

$$I = 74 + 20 \log_{10} 169.3 p_o \frac{R_t}{L} M_t \left(1.7 \frac{M}{M_t^2} - \cos \beta\right) + 10 \log_{10} \sum_q B_{qn}^2 \tag{8}$$

This formula may be written

$$I = 118.6 + 20 \log_{10} p_o \frac{R_t}{L} M_t \left(1.7 \frac{M}{M_t^2} - \cos \beta\right) + 10 \log_{10} \sum_q B_{qn}^2 \tag{9}$$

where p_o, which is in pounds per square foot, is the only dimensional term. Note that formula (9) is very convenient to use since the Bessel functions appear only in the last term in the form of the sum of the squares. The last term can be given directly for a given number of blades as a function of V/c and the angle β only. As mentioned, the peak pressure corresponds to a value of β of about 120°. Because only this peak pressure is referred to in the present paper, 120° is the value of β used. This function has been plotted for two-, four-, six-, and eight-blade propellers in figure 3, which gives directly the quantity $10 \log_{10} \sum_q B_{qn}^2$.

Because the Gutin formula was developed for an airplane resting on the ground, strictly speaking it should not be used for the flight or even the take-off condition. Actually the error is very small so long as the forward speed is small compared with the velocity of sound.

EXAMPLES OF CALCULATIONS AND MEASUREMENTS

Calculations are made for the cruising condition of a small airplane A having the following specifications:

Airplane speed, miles per hour	75
Horsepower	46
Propeller speed, rpm	2100
Propeller efficiency, percent	80
Propeller diameter, feet	5.83
Number of propeller blades	2
Propeller disk loading, p_o, pounds per square foot	6.9
Airplane Mach number, M	0.098
Propeller-tip Mach number, M_t	0.57

The values of p_o, M, and M_t were obtained as follows:

$$p_o = \frac{\text{Power}}{\text{Airplane velocity} \times \text{Disk area}}$$

$$= \frac{46 \times 550 \times 0.8}{75 \times \frac{88}{60} \times \frac{\pi}{4} \times (5.83)^2}$$

$$= 6.9 \text{ pounds per square foot}$$

$$M = \frac{\text{Airplane velocity}}{\text{Speed of sound}}$$

$$= \frac{75}{1120} \times \frac{88}{60}$$

$$= 0.098$$

$$M_t = \frac{\text{Propeller tip speed}}{\text{Speed of sound}}$$

$$= \frac{2100 \times 5.83\pi}{1120 \times 60}$$

$$= 0.57$$

From formula (9), for $\beta = 120^\circ$,

$$I = 118.6 + 20 \log_{10} 6.9 \times 0.57 \left[1.7 \frac{0.098}{(0.57)^2} + 0.5 \right]$$

$$- 20 \log_{10} \frac{L}{R_t} + 10 \log_{10} \sum_q B_{qn}^{\;2}$$

$$= 118.6 + 12 - 20 \log_{10} \frac{L}{R_t} + 10 \log_{10} \sum_q B_{qn}^{\;2} \qquad (10)$$

The value of the next to last term in formula (10) is

$$20 \log_{10} \frac{L}{R_t} = 0 \qquad (\text{for } L = R_t)$$

and

$$20 \log_{10} \frac{300 \times 2}{5.83} = 20 \times 2.01 = 40 \qquad (\text{for } L = 300 \text{ ft})$$

This term gives the distance effect. From figure 3 the value of the last term is $10 \log_{10} \sum_{q} B_{qn}{}^2 = -16$ for a two-blade propeller at $\frac{V}{c} = 0.0455$. The appropriate Mach number is obtained by using the 0.8 radius as a reference station and disregarding the forward speed. Thus, $\frac{V}{c} = 0.8M_t = 0.455$.

The sound intensity due to the propeller can now be obtained simply by adding the four terms on the right hand side of equation (10). In the order given, the first of these terms is a constant, the second is due to the disk loading and Mach number of the airplane and the propeller, the third takes into account the distance from the propeller, and the fourth is a function obtained from figure 3 for various values of V/c and various numbers of blades. In the foregoing example, therefore, the sound intensity at a distance of 1 radius from the propeller is

$$I = 118.6 + 12 - 0 - 16 = 114.6 \quad \text{decibels}$$

At a distance of 300 feet the sound intensity of the same propeller is

$$I = 118.6 + 12 - 40 - 16 = 74.6 \quad \text{decibels}$$

The propeller sound intensities have also been calculated for a somewhat larger airplane, which will be called airplane B, having the following specifications:

Airplane speed, miles per hour	165
Horsepower	133
Propeller speed, rpm	2900
Propeller diameter, feet	5.5

The detailed calculations for airplane B are omitted.

For comparison, the calculated propeller sound intensities for airplanes A and B at a distance of 3 feet and 300 feet, respectively, are given as

Airplane	I at 3 feet (db)	I at 300 feet (db)
A	114.6	74.7
B	127	87

The sound energy radiated from the airplane propeller may be obtained by use of formula (2). For simplicity, the intensities in all directions are assumed to be constant and equal to the intensity obtained at $\beta = 120^\circ$, and therefore the total energy radiated through the surface of a sphere of 300-foot radius is

$$E = P \times 4\pi L^2$$

$$= 10^{\left(\frac{I}{10}-16\right)} \times 4\pi(300 \times 30.5)^2$$

$$= 10^{\left(\frac{I}{10}-16\right)} \times 1.05 \times 10^9 \quad \text{watts}$$

For the propeller of airplane A the energy radiated is consequently

$$E = 10^{\left(\frac{74.7}{10}-16\right)} \times 1.05 \times 10^9$$

$$\approx 2 \quad \text{watts}$$

and for airplane B the energy is

$$E = 10^{\left(\frac{87}{10}-16\right)} \times 1.05 \times 10^9$$

$$\approx 52 \quad \text{watts}$$

Strictly speaking, these figures are too high, since the maximum intensity at 120° was inserted in the formulas instead of the mean intensity. On the other hand, the reflection from the ground generally caused a doubling of the sound intensities, particularly in the horizontal plane. The figures given are therefore reasonably representative for the sound energy.

Measurements were made on a certain small airplane, which will be called airplane C, having the following specifications:

Airplane speed, miles per hour	90
Horsepower	49
Propeller speed, rpm	2150
Propeller diameter, feet	6
Number of propeller blades	2

Noise intensities were measured in the cabin of this airplane with a commercial portable meter; the absolute readings are therefore not too accurate. The measurements were made to give an idea of the noise level for different flight conditions and are in fair agreement with calculations made for airplane A, which this airplane resembles. The data obtained for airplane C are as follows:

Sound intensity (db)	Airplane speed (mph)	Propeller speed (rpm)	Remarks
90 to 92	5	1000	Taxiing
94	0	1500	Magneto check
106	40	2300	Take-off
98 to 101	60	2300	Climb
93 to 95	85	2150	Cruising
84	65	800	Normal glide
92			Landing approach

Finally, a radical modification of airplane A is considered. This airplane, which will be called airplane D, is supposed to employ a fan-type propeller. The value of the propeller advance ratio is increased from 0.54 for airplane A to 1.62 for airplane D by reducing the tip speed of the propeller in the ratio of 3 to 1. An eight-blade fan-type propeller is chosen for airplane D to reduce the noise level. In order to keep the induced losses of the propeller at a constant value, it is necessary to increase the disk area in the ratio of the mass coefficients (reference 4). The mass coefficient for airplane A at cruising speed is 0.68. For the projected eight-blade propeller the mass coefficient is 0.48. The disk area must be thus increased in the ratio of 0.68/0.48 or 1.41 and the propeller diameter for airplane D becomes $5.83\sqrt{\frac{0.68}{0.48}} = 6.95$ feet. The disk loading p_o for airplane D is $6.9 \times \frac{0.48}{0.68} = 4.9$ pounds per square foot, and the tip Mach number M_t is $\frac{1}{3} \times 0.57 = 0.19$.

The propeller sound pressure for the case of airplane D is calculated to be about 25 decibels at 1 radius and about -13 decibels at 300 feet. The value of -13 decibels means that the sound from the fan-type propeller would be below the threshold of human hearing, since the threshold under ideal conditions is by definition at 0 decibel. The sound of the propeller for airplane D would be inaudible at about 50 feet. Such a propeller would be very heavy, would have to be geared, and, since it operates at a high advance ratio, would require a variable-pitch mechanism. Whether such changes can be incorporated will be left unanswered, as the problem involves several fields of engineering other than that of sound and must be arrived at by extensive compromises or regulations imposed by law.

Recently a series of tests has been made on two-, four-, and seven-blade propellers driven by an electric motor. The results of these tests show good agreement with the Gutin formula, particularly at tip Mach numbers from 0.5 to 0.9. The agreement between theory and experiment is good over a sound energy range of as much as 10,000 to 1. For conventional propellers, therefore, the Gutin formula gives the sound output correctly. For a fan-type propeller as suggested for airplane D, the possibility exists, however, that the sound as calculated by the Gutin formula at a sufficiently low level may become masked by vortex noises.

The foregoing formulas give physical noise levels as measured by instruments. The sensitivity of the human ear is dependent on the frequency, particularly at low noise levels. A correction factor must therefore be applied in order to obtain the audibility of a particular sound. Thus, an indicated physical reduction is not necessarily accompanied by a corresponding reduction in audibility. It should be remembered that the greatest sensitivity of the ear is in the range of approximately 1000 to 4000 cycles per second. The fundamental of the propeller noise is therefore rarely audible.

The effect of exhaust noise was studied in connection with the light airplanes A and B. It is contended that an index of the relative importance of the exhaust noise may be obtained by the use of the "masking" effect of the propeller noise. By masking is meant the property of a certain loud noise to render the ear unable to perceive a simultaneous weaker noise. If the average observer is uncertain as to whether he can hear the weaker noise, this noise is said to be masked by the louder one, which in the present case is the propeller noise. In such a case the elimination of the weaker noise is technically without merit.

By means of aural listening tests it was determined that the exhaust noise on airplane A was drowned out by the propeller at a speed of about 2100 rpm. Since this speed is about the cruising speed, the effect of an exhaust muffler might just be discernible but the exhaust muffler would not reduce the sound output appreciably except when the airplane was idling on the ground. On a larger airplane, airplane B for example, the exhaust noise was masked at about 1500 rpm. This speed is very far from the cruising speed of the airplane, which is at about 2900 rpm. Airplane B would therefore definitely not gain from an improvement in the muffler.

In order to check these conclusions further, exhaust-noise measurements were made at a distance of 3 feet from an unmuffled gasoline engine having about the same exhaust frequency and power as a light-airplane engine. The measured values were 82 decibels for idling and 92 decibels for full power. Since the airplane engines usually have shorter exhaust stacks than the engine tested, it may be assumed that the exhaust noise of a light-airplane engine is 95 to 100 decibels at a distance of 3 feet from the exhaust opening. By use of these values for the exhaust intensity, the combined exhaust and propeller noise is computed by means of formulas (3) and (7). Thus, the following table is obtained:

Airplane	Assumed exhaust noise at 3 ft (db)	Calculated propeller noise at 3 ft (db)	Combined propeller and exhaust noise at 3 ft (db)
A	95	114.6	114.68
B	100	127.0	127.01

The foregoing table shows that the combined engine and exhaust noise is absolutely indistinguishable from the propeller noise alone even when the relatively high sound intensity level of 95 to 100 decibels is used for the exhaust noise. Conversely, it is to be noted that if or when the propeller is silenced a "perfect" muffler will be required on the exhaust, since the exhaust noise must be brought down to approximately the same level.

CONCLUSIONS

1. Extensive measurements on many propellers at the Langley Memorial Aeronautical Laboratory show that the Gutin formula gives the sound level for propellers at low forward speeds with adequate accuracy; therefore the necessity for measurements of the propeller noise no longer exists.

2. A type of measurement of the relative level of the exhaust noise is indicated. A masking of the exhaust noise by the propeller noise at a certain low speed and fractional power is a condition necessary to insure adequate muffling. The exhaust noise should not be audible through the propeller noise at some given low propeller speed. The sound is dominated by the propeller to such an extent that excessive muffling is useless in the average case.

3. A general large reduction in the sound level of an airplane can be achieved only by extensive and radical changes in the design of the propeller. The noise from a fan-type propeller is shown to be practically inaudible. In such a case perfect muffling is necessary and permissible. The imaginary airplane considered, with a low-tip-speed fan-type propeller and presumably a perfect muffler, is virtually inaudible at less than 300 feet (except for possible vortex noises).

4. It is evident from the theoretical formulas presented that the main and essential factor in propeller noise reduction is the propeller tip speed and the second factor is the number of propeller blades. Whether any practical application can be made by incorporating features of the fan-type propeller will depend on

conditions beyond the scope of this paper. No other solution is available for a propeller-driven airplane.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
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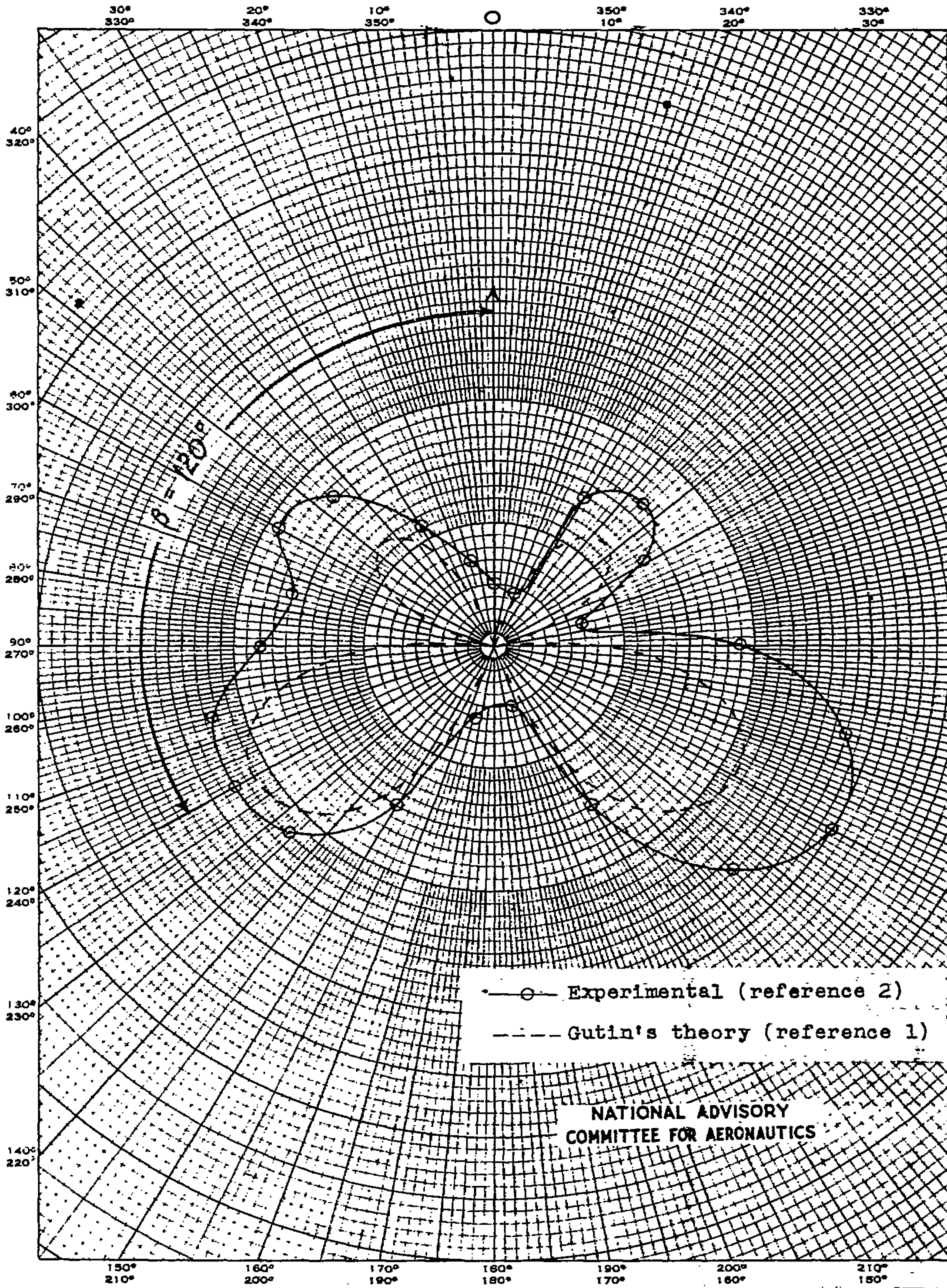

Figure 1.- Experimental sound measurements of first harmonic from two-blade propeller compared with Gutin's formula. (Fig. 1(a), reference 2.)

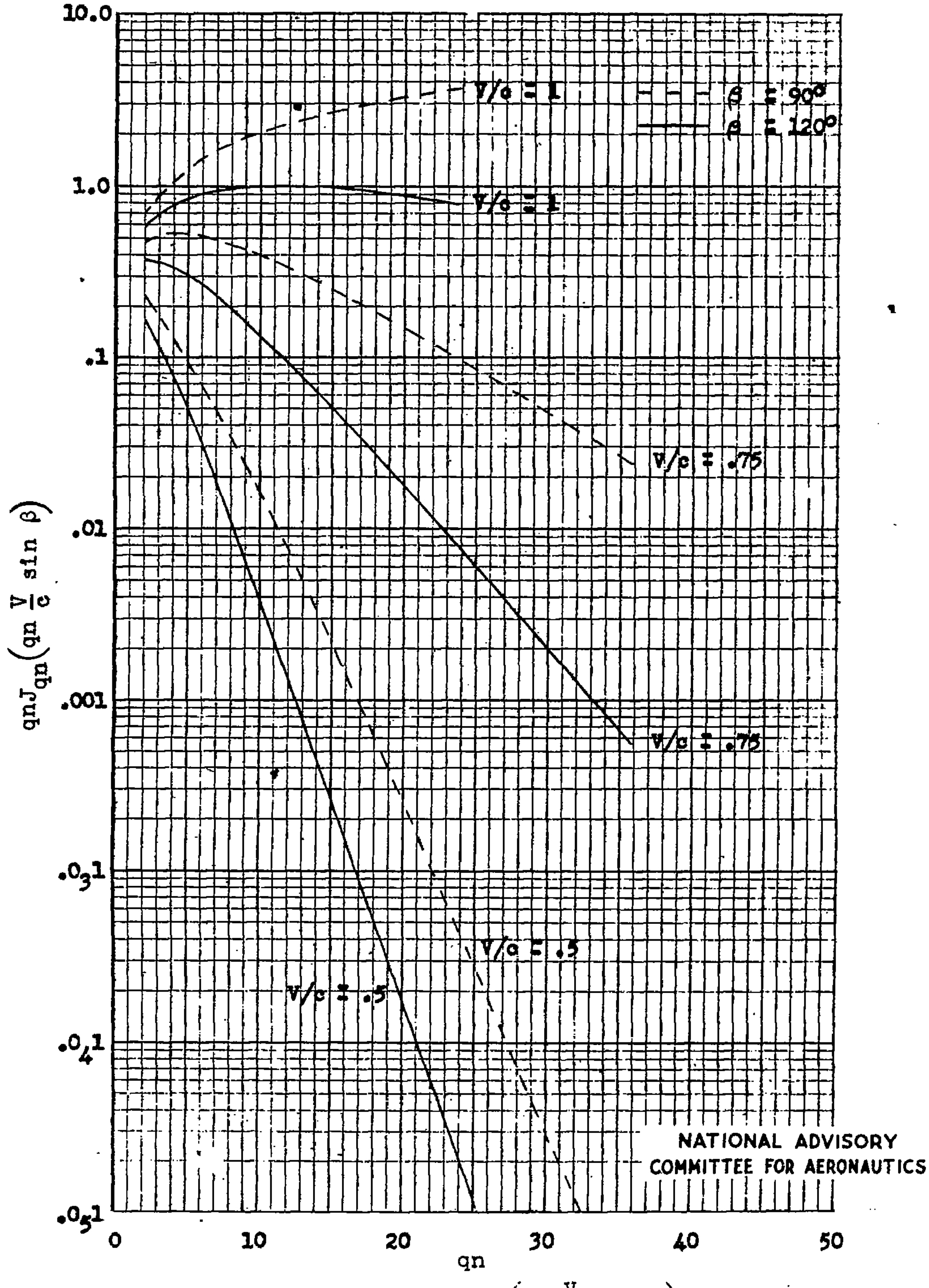

Figure 2.- Function $qnJ_{qn}\left(qn \frac{V}{c} \sin \beta\right)$ for various values of V/c and β.

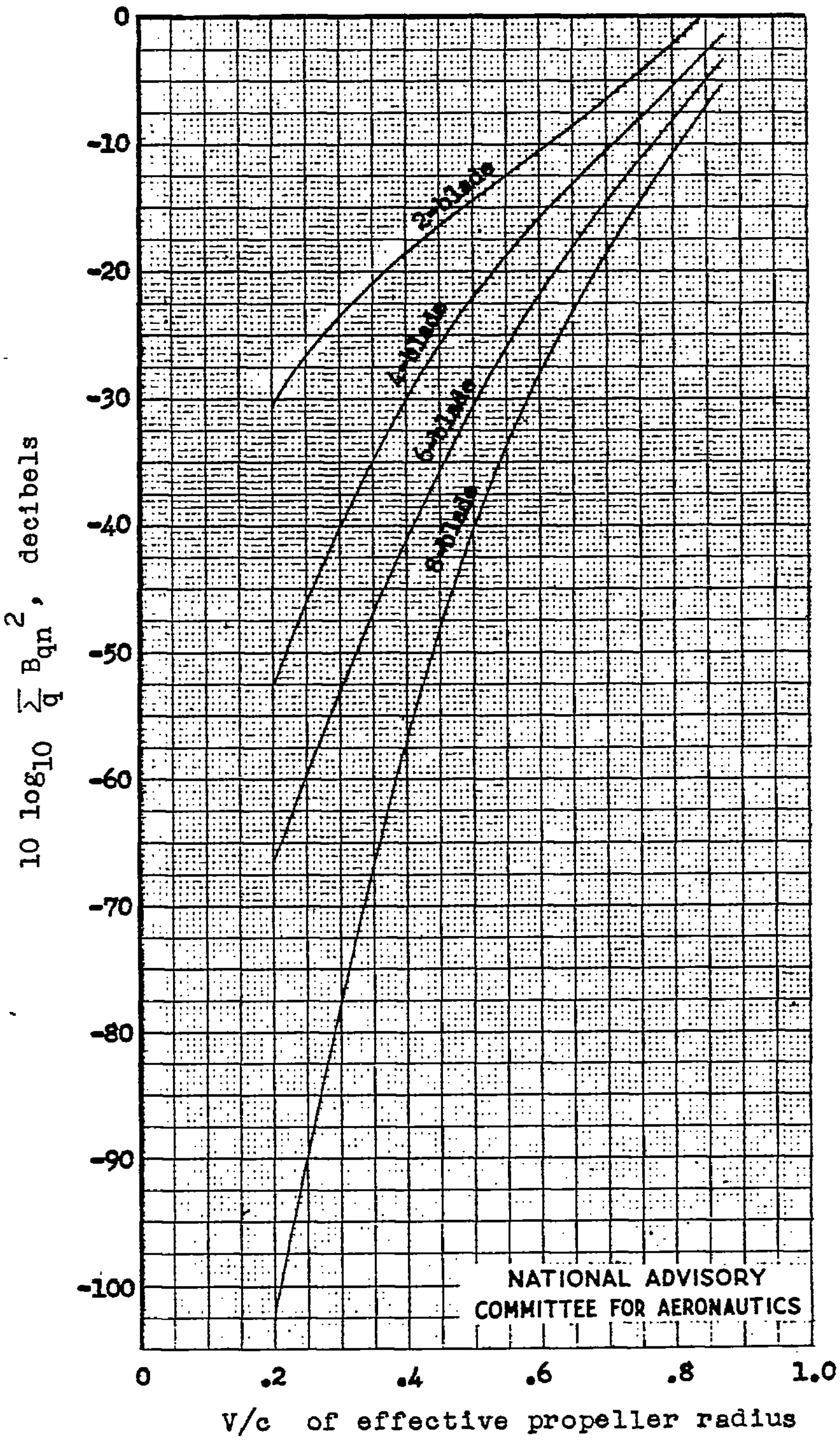

Figure 3.- Function $10 \log_{10} \sum_q B_{qn}^2$ for various numbers of propeller blades.