REPORT No. 799

CHARTS FOR THE DETERMINATION OF WING TORSIONAL STIFFNESS REQUIRED FOR SPECIFIED ROLLING CHARACTERISTICS OR AILERON REVERSAL SPEED

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SUMMARY

A series of charts are presented by which the wing torsional stiffness required to meet a given standard of rolling effectiveness may be quickly determined. The charts may also be used to obtain quickly the aileron reversal speed and the variation of the loss in rolling effectiveness with airspeed. The charts apply to linearly tapered wings and elliptical wings of tubular-shell construction having various aspect ratios with aileron span and location of ailerons as variables. In the derivation of the charts, induced lift effects have been taken into account and the form of the wing-torsional-stiffness curve has been assumed.

INTRODUCTION

In order to insure adequate rolling control at high speeds, present structural requirements for Army airplanes (reference 1) specify that the computed aileron reversal and divergence speeds be at least 1.15 times the terminal velocity of the airplane. The accuracy of such computations depends on the availability of aerodynamic data, on a knowledge of the wing torsional stiffness, and—to a smaller extent—on the method of computation used.

Since the terminal Mach number for fighter airplanes is approximately 0.85, aerodynamic data should be available at a Mach number of about 1.0 if accurate results are to be obtained in rolling-performance calculations. A Mach number of 1.0 is considerably higher than that at which high-speed wind-tunnel data are available or to which they can be extrapolated.

Data available on the torsional stiffness of wings have indicated that the calculated values of wing torsional stiffness are likely to vary considerably from the test values. Unpublished test data have indicated that, for wings of the same model, significant differences in torsional stiffness can be expected as a result of differences in fabrication.

Many methods are available for computing speeds of aileron reversal and divergence. (See references 2 to 10.) These methods differ mainly in the combination of assumptions used and in the degree of exactness attempted. In most cases, the form of the wing-twist curve has been assumed to be linear or parabolic and the induced lift effects have been either entirely neglected or approximated. In some of the methods (references 4, 6, 8, and 10) the actual wing-torsional-stiffness distribution is required and equilibrium is established between elastic and aerodynamic forces all along the wing span; with the exception of the method of reference 4, however, these methods either omit induced lift effects or approximate them. The application of the exact method of reference 4 requires, according to the author, 100 man hours. Inasmuch as the accuracy to be gained by the use of the most exact method, which includes induced lift effects, is only about 6 percent as compared with methods employing strip theory, the use of the most exact method is considered impractical when the possible inaccuracies in the aerodynamic and structural quantities are considered.

Some improvement in the requirements can be obtained by specifying that the loss in rolling effectiveness due to wing twist at either terminal velocity or high-speed level flight shall not exceed some fixed percentage of that at low speed. Such a specification would then conform better to maneuverability requirements and would not require as much extrapolation of the aerodynamic data. Further improvement could be obtained by requiring that, after fabrication, the torsional stiffness of each wing panel be greater than a specified value.

Charts are given in the present paper from which the required wing torsional stiffness may be readily obtained if a given loss in rolling ability is not to be exceeded. The charts may be used to predict the aileron reversal speed and the loss in rolling effectiveness at any other speed. The charts are based on the application of the usual lifting-line theory to wings of tubular-shell construction and allow variations in wing taper, aileron span, and aileron position to be taken into account. In the derivation of the charts, the form of the wing-torsional-stiffness distribution has been assumed and as a result the twist curves for a given wing vary with the aileron span and position in a manner to be expected.

The advantage of the present method over others is mainly in the speed with which results can be obtained. More nearly accurate results are obtained than with methods that assume the shape of the wing-twist curve to be linear or parabolic.

SYMBOLS

Symbols used in the present paper are defined in the following list (see also fig. 1):

S	wing area, square feet
$\bar{c}$	mean geometric wing chord, feet (S/b)
c	wing chord at any spanwise station, feet
c_a	aileron chord
b	wing span, feet
A	wing aspect ratio (b^2/S)
y	semispan station, feet
k	nondimensional semispan station $\left(\frac{y}{b/2}\right)$

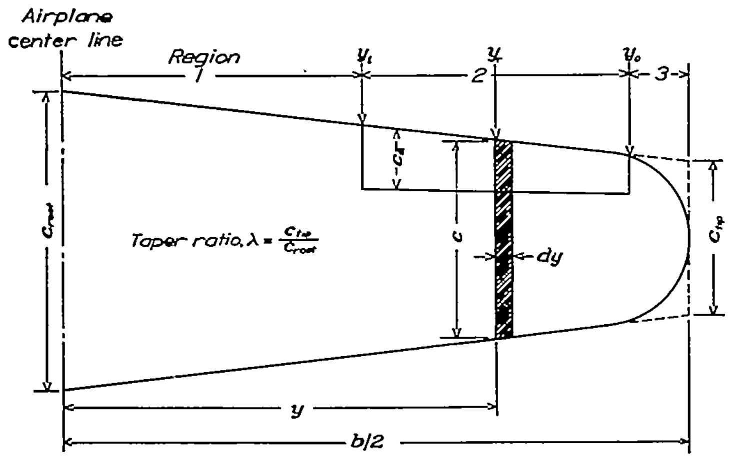


FIGURE 1.—Typical wing-aileron combination illustrating pertinent symbols and dimensions used in analysis.

λ	wing taper ratio; that is, ratio of fictitious tip chord, obtained by extending wing leading and trailing edges to tip, to root chord
m_θ	wing torsional stiffness as obtained by application of concentrated torque near wing tip; foot-pounds per radian
t	torque per unit span, foot-pounds per foot
T	accumulated or total torque, foot-pounds
θ	angle of wing twist, radians
δ	aileron deflection, radians
$dc_m/d\delta$	rate of change of section pitching-moment coefficient with aileron angle as obtained for incompressible flow, per radian
$d\alpha/d\delta$	rate of change of wing angle of attack with aileron angle as obtained for constant lift at section
$pb/2V$	helix angle of roll, radians
L_θ	rolling moment due to antisymmetrical wing twist, foot-pounds
C_l	rolling-moment coefficient
C_{l_δ}	rate of change of rolling-moment coefficient with aileron deflection, per radian
C_{l_p}	rate of change of rolling-moment coefficient with helix angle $pb/2V$, per radian
C_{l_θ}	rate of change of rolling-moment coefficient with wing twist at reference section, per radian
τ	derived constant for airplane (see appendix)
γ	helix-angle parameter
ϕ	fraction of rigid-wing rolling effectiveness to be retained by flexible wing
q	dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$
ρ	density, slugs per cubic foot
V	true airspeed, feet per second
V_e	equivalent airspeed, miles per hour
a	speed of sound, feet per second
M	Mach number (V/a)
$1/\sqrt{1-M^2}$	Glauert compressibility correction factor

Subscripts

y and k denote a particular spanwise station

o	outboard end of aileron
i	inboard end of aileron
r	reference station taken as midaileron span
1	refers to particular spanwise station inboard of aileron span, region 1
2	refers to particular spanwise station in way of aileron, region 2
3	refers to particular spanwise station outboard of aileron span, region 3
L	limit diving speed
R	reversal speed

CHARTS

The design of ailerons that will provide adequate rolling control at high speed requires the determination of (1) the wing torsional stiffness required to meet a given standard of rolling effectiveness, (2) the aileron reversal speed, and (3) the variation with airspeed of helix angle of roll per unit aileron deflection. The charts presented herein have been prepared in order to facilitate the computation of these factors. The results apply to wings of tubular-shell construction, of various aspect ratios and taper ratios (including elliptical wings), and with various lengths and positions of the aileron along the span. The various plan forms considered are shown in figure 2.

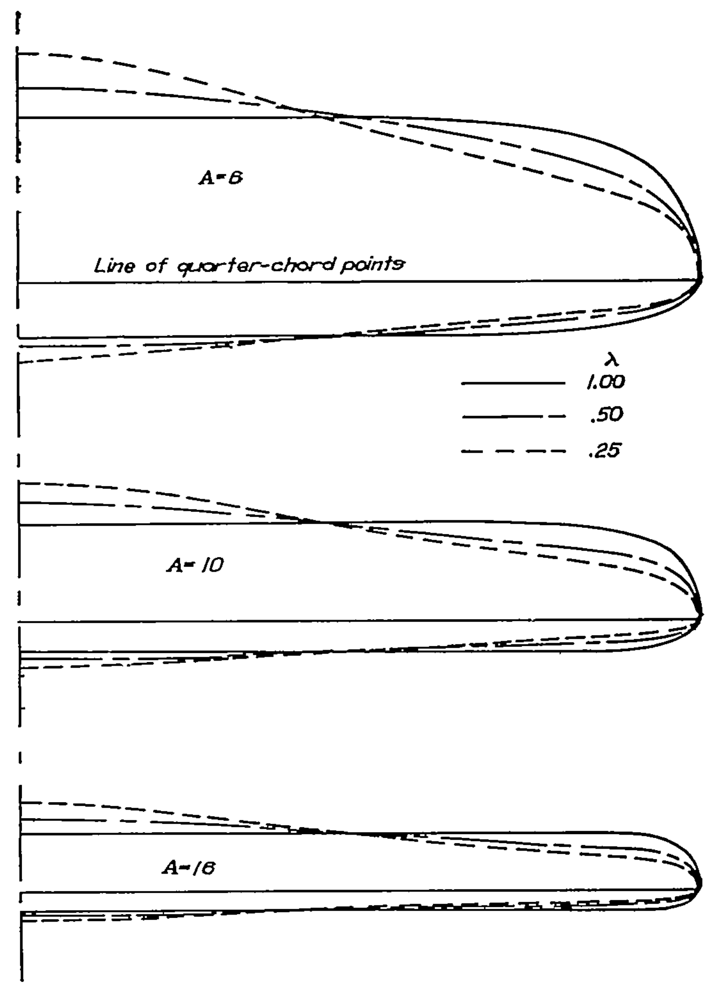
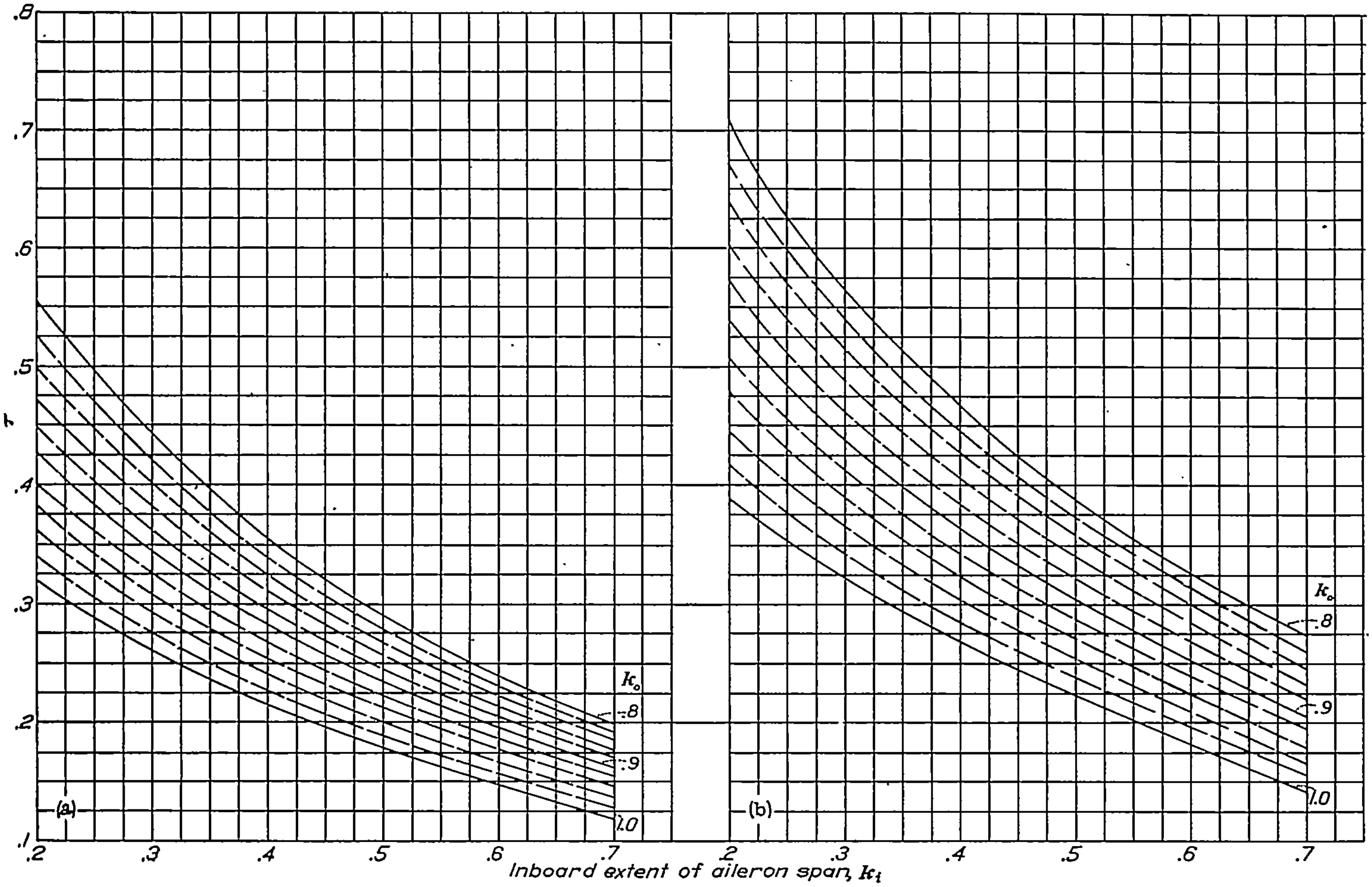


FIGURE 2.—Wing plan forms used in analysis. (From reference 13.)



(a) Taper ratio, 0.25. (b) Elliptical wing.

FIGURE 3.—Variation of rolling-moment-loss parameter with extent of aileron span. Aspect ratios ranging from 5 to 16.

The derivation, which is given in detail in the appendix, is based on an application of the lifting-line theory to the wing plan forms chosen. (See figs. 1 and 2.) In the derivation, the wing section aerodynamic coefficients were assumed to be unaffected by torsional deformation and the shape of the torsional-stiffness curve was assumed to be inversely proportional to the cube of the distance from the wing center line. It was also assumed that the ratio of the wing chord to the aileron chord was constant and that no twist occurred about the aileron hinge axis.

The results, which apply to wings having aspect ratios ranging from 5 to 16, are presented in the charts of figures 3 and 4. In figure 3 the nondimensional coefficient τ is given as a function of k_i and k_o. This coefficient τ is directly proportional to the rolling-moment loss due to the torsional deformation of the wing and is inversely proportional to the dynamic pressure.

If it is desired to determine the wing stiffness required at the reference section (midaileron) to retain a specified rolling effectiveness, the following equation may be applied:

$$m_{\theta_r}=\tau \frac{dc_m/d\delta}{d\alpha/d\delta} \frac{b^3}{2A^2(1-\phi)} \frac{q}{\sqrt{1-M^2}} \tag{1}$$

The dynamic pressure at reversal speed may be determined from the equation

$$\frac{q_R}{\sqrt{1-M^2}}=\frac{2m_{\theta_r}}{\tau\left(\frac{dc_m/d\delta}{d\alpha/d\delta}\right)\left(\frac{b^3}{A^2}\right)} \tag{2}$$

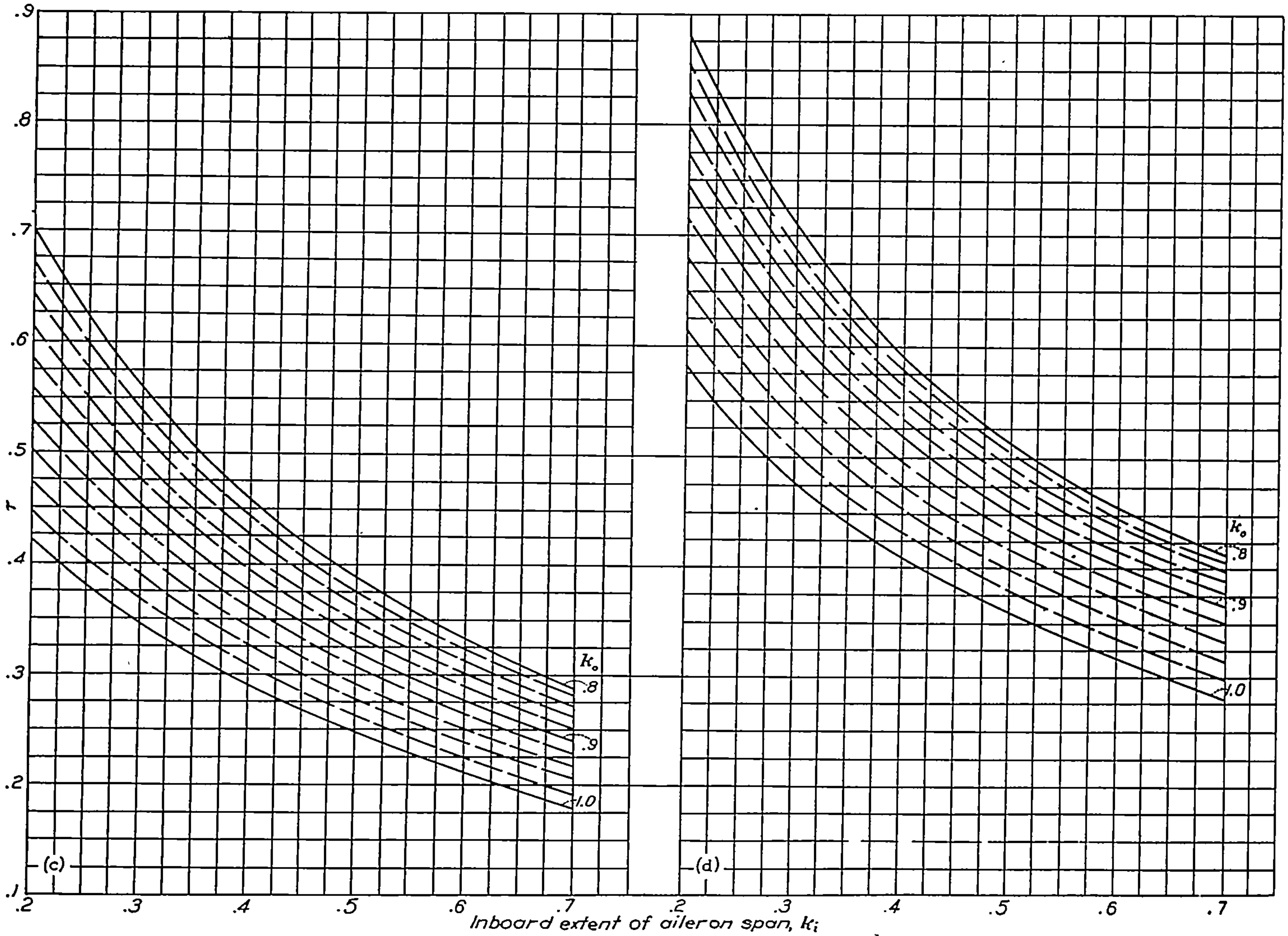
If the actual variation of $dc_m/d\delta$ with Mach number is known this variation may be substituted in equations (1) and (2) in place of the Glauert compressibility correction factor $1/\sqrt{1-M^2}$.

The coefficient γ given in figure 4 is a nondimensional quantity representing a helix-angle parameter for a rigid wing. This parameter may be used in the equation

$$\frac{pb/2V}{\delta}=\gamma\phi\frac{d\alpha}{d\delta} \tag{3}$$

to obtain the helix angle $pb/2V$ of a flexible wing for a unit aileron deflection.

The results given in equations (2) and (3) may be used to determine the loss in aileron effectiveness with airspeed due to wing flexibility. Because the relation between $\frac{pb/2V}{\delta}$ and $q/\sqrt{1-M^2}$ is linear, a straight line drawn between the point representing the value for $\frac{pb/2V}{\delta}$ at $\frac{q}{\sqrt{1-M^2}}=0$ and the point for zero helix angle will yield the value of $\frac{pb/2V}{\delta}$ at all intermediate values of $q/\sqrt{1-M^2}$.



(c) Taper ratio, 0.50. (d) Taper ratio 1.00.

FIGURE 3.—Concluded

APPLICATION OF CHARTS

Determination of torsional stiffness required at the reference section to fulfill a specified rolling requirement.—It is desired to determine the torsional stiffness required of the wing at the reference section (midaileron station) in order that at limit diving speed at sea level the airplane will retain 0.25 of the rolling effectiveness of the rigid wing. The following values are given:

Fraction of rolling effectiveness to be retained, ϕ (by stipulation)	0. 25
Limit diving speed, V_L, miles per hour	553
Mach number, M	0. 728
Wing span, b, feet	41
Aspect ratio, A	5. 6
Plan form	Elliptical
Distance to inner end of aileron, k_i, fraction of semispan	0. 538
Distance to outer end of aileron, k_o, fraction of semispan	0. 945
Dynamic pressure at limit diving speed, q_L, pounds per square foot	782. 8
$d\alpha/d\delta$	0. 36
$dc_m/d\delta$	0. 42

From figure 3, for an elliptical wing and with the given values of k_i and k_o, τ is determined as 0.249. When these values are substituted in equation (1),

$$m_{\theta_r}=0.249\times\frac{0.42}{0.36}\times\frac{41^3}{2\times5.6^2\times0.75}\times\frac{782.8}{0.685}$$

$$=486{,}000 \text{ foot-pounds per radian}$$
$$=102{,}000 \text{ inch-pounds per degree}$$

Thus, if a concentrated torque of 10,200 inch-pounds applied outboard of the midaileron section produced less than 0.1° wing twist of the reference section relative to the wing root, the wing would exceed the required stiffness.

Determination of aileron reversal speed.—The same quantities used in the previous example, together with an experimentally determined value of the wing stiffness m_{θ_r} (equal to 527,000 ft-lb per radian, $\phi=0$) may be used to obtain from equation (2)

$$\frac{q_R}{\sqrt{1-M^2}}=\frac{2\times527{,}000}{0.249\times\frac{0.42}{0.36}\times\frac{41^3}{5.6^2}}$$

$$=1652 \text{ pounds per square foot}$$

From figure 5, which gives the relations between $q/\sqrt{1-M^2}$ and V_e, the reversal speed at sea level is determined to be 619 miles per hour.

Determination of variation of helix angle of roll with speed.—The helix angle $pb/2V$ per unit aileron angle for the rigid wing ($\phi=1.0$) is found from figure 4 and equation (3). Figure 4 applies to all normal taper ratios and aspect ratios. For the airplane of the example $\gamma=0.91$, and since $d\alpha/d\delta$ was assumed to be 0.36, $\frac{pb/2V}{\delta}$ for the rigid wing is 0.328 radians per radian deflection or 0.00573 radians per degree of aileron deflection. At this value of $\frac{pb/2V}{\delta}$, $\frac{q}{\sqrt{1-M^2}}=0$. If $\frac{q_R}{\sqrt{1-M^2}}=1652$ pounds per square foot, the corresponding helix angle is zero. The helix angle for any intermediate value of $q/\sqrt{1-M^2}$ or airspeed may be determined by drawing a straight line through these two points.

Discussion of examples.—In the application of the charts to determine the wing stiffness, reversal speed, or helix-angle variation, certain quantities will be obtained from the geometry of the wings; other quantities will be determined either by performance specification or by the aerodynamic characteristics of the reference airfoil section, which is located at the midspan of the aileron.

The values of b, A, k_i, and k_o can easily be determined from the geometry of the wing. In equation (1), the value of ϕ must be specified and $q/\sqrt{1-M^2}$ must be known. An alternative method for determining the aileron reversal speed may be used instead of equation (2) in cases in which the variation of $dc_m/d\delta$ is not a function of $1/\sqrt{1-M^2}$. This procedure involves the calculation of ϕ at speeds greater than the limit diving speed by the use of equation (1). The values of $dc_m/d\delta$ and $d\alpha/d\delta$ corresponding to the Mach numbers of the airspeeds chosen are used and a plot of ϕ against Mach number is made. As the Mach number is increased, ϕ approaches zero and when ϕ reaches zero, the aileron reversal speed is determined. These computations may be made for various altitudes to determine the variation of aileron reversal speed with altitude.

In equation (2) the altitude must be specified and the value of the torsional stiffness m_{θ_r} must be known either from tests or calculation. At present, experimental values of m_{θ_r} are greatly preferred since the calculated values may be considerably in error. The experimental value of the wing stiffness may be easily obtained by applying a unit torque of about 20,000 inch-pounds to a section near the wing tip and determining the angle of twist at the reference section relative to the wing root. It is recommended in reference 9 that in order to obtain best results an antisymmetrical torque be applied to the other wing tip.

The quantities $d\alpha/d\delta$ and $dc_m/d\delta$ are aerodynamic quantities that apply at the reference section. In general, they will vary primarily with flap-chord ratio and Mach number and secondarily with other variables such as nose shape, gap,

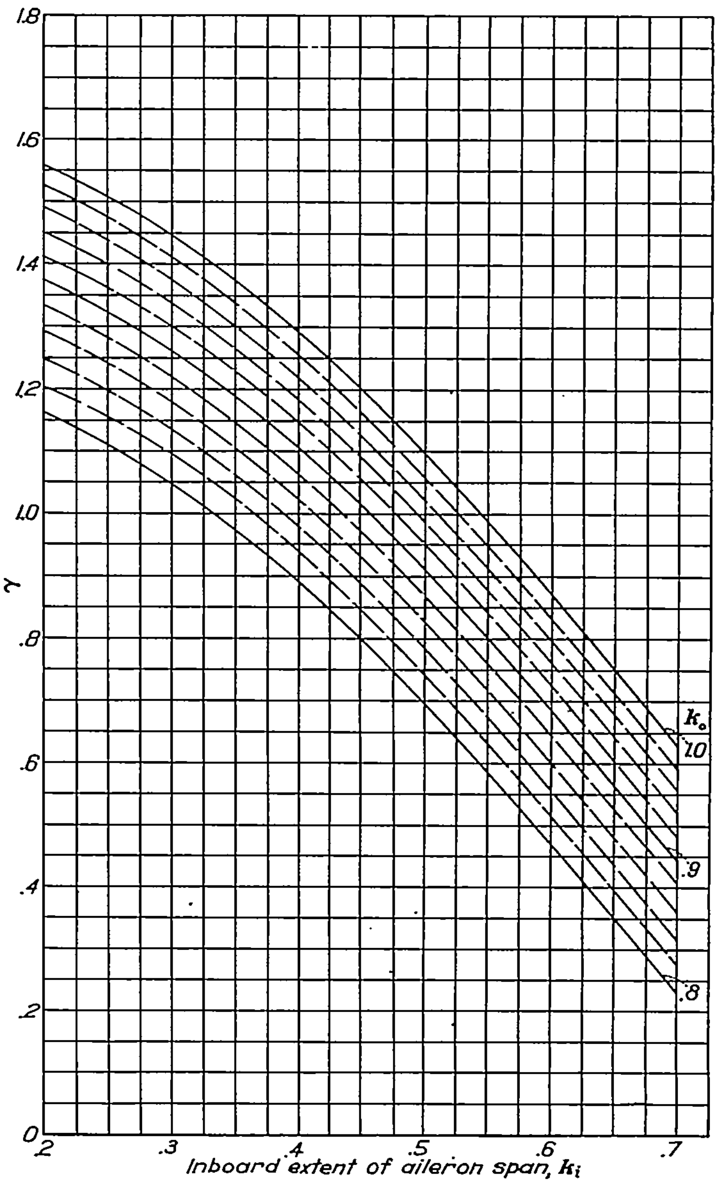


FIGURE 4.—Variation of helix-angle parameter for a rigid wing with extent of aileron span for all aspect ratios and taper ratios. Aspect ratios ranging from 5 to 16.

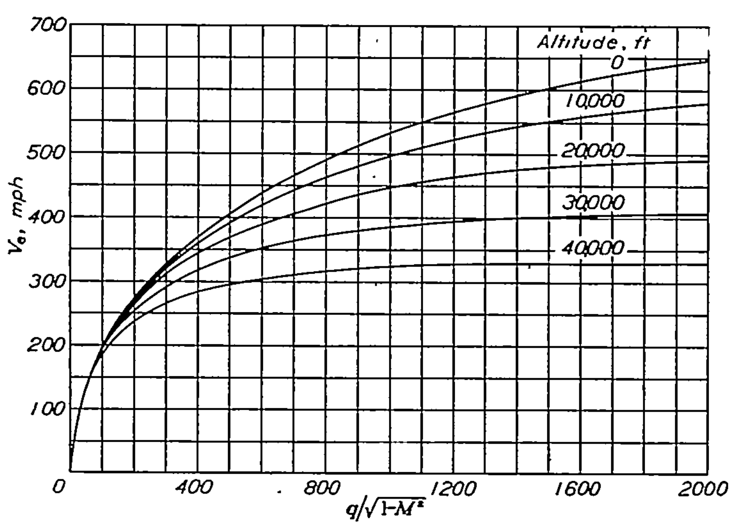


FIGURE 5.—Variation of equivalent airspeed with the factor $q/\sqrt{1-M^2}$.

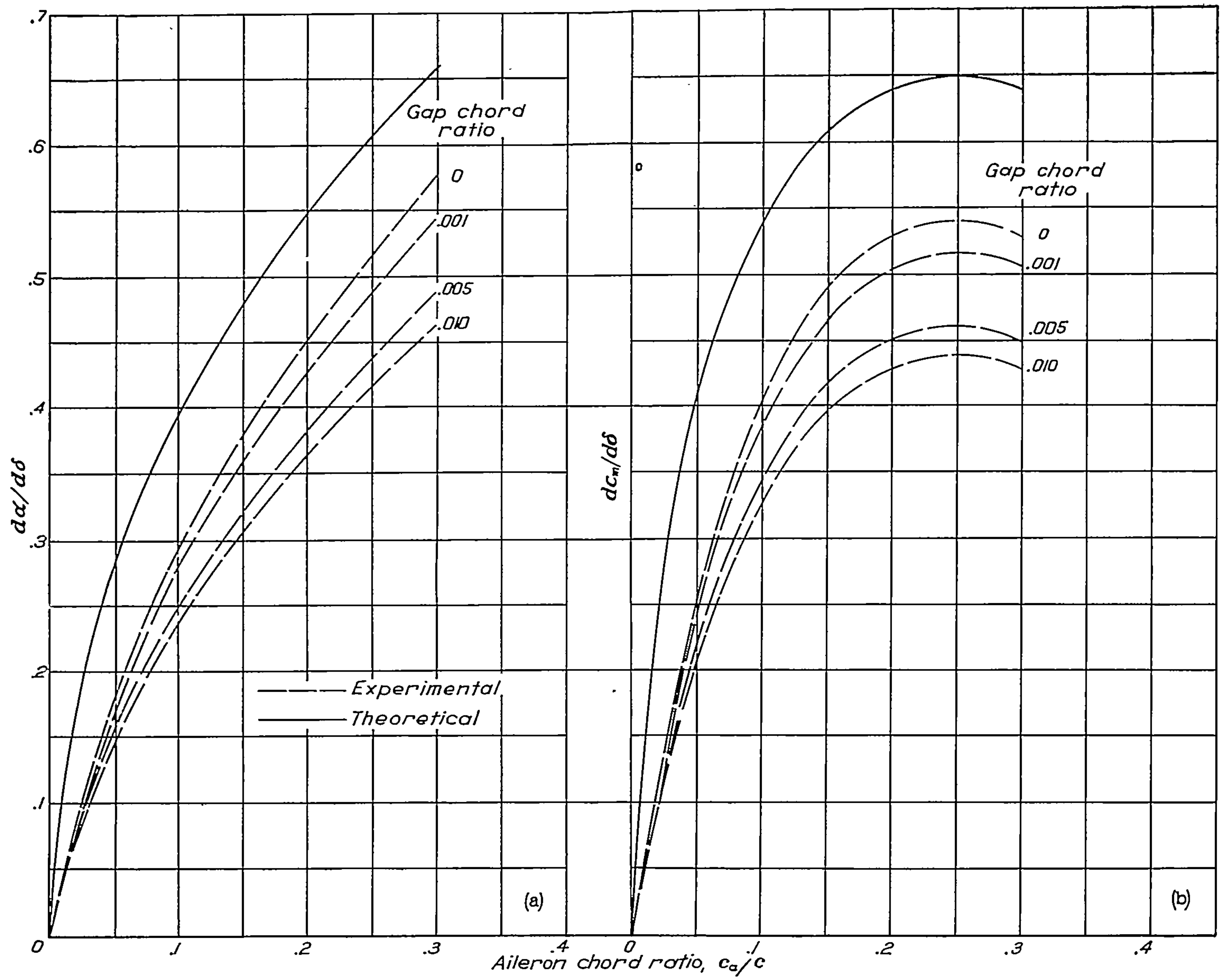


FIGURE 6.—Variation of section coefficients with aileron-chord ratio for ailerons with a balance factor of zero.

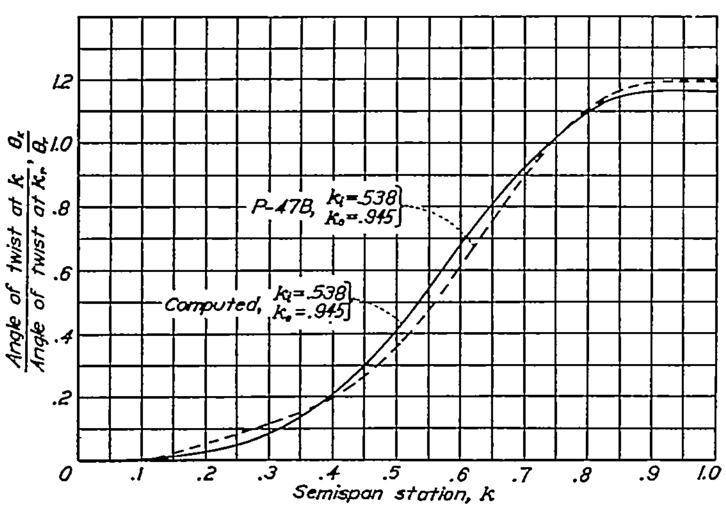


FIGURE 7.—Comparison of the wing-twist curve for the P-47B airplane with a twist curve computed for an elliptical wing with a cubic stiffness distribution.

aileron section, and angle of attack. Figure 6 has been prepared to show average variations of these quantities with flap chord and gap ratio. More accurate values can, however, be obtained from specific tests of the aileron section used or from reference 11, which analyzes the data obtained from a large number of wind-tunnel tests.

DISCUSSION

The agreement that may be obtained between the calculated values of wing twist and actual values of wing twist when the form of the wing-torsional-stiffness distribution is assumed is illustrated in figure 7. Computations were made for the P-47B wing by use of stiffness data furnished by the Army Air Forces, Air Technical Service Command, Wright Field, Ohio. The twist curve resulting from these computations is compared in figure 7 with a twist curve computed by assuming that wing torsional stiffness varied inversely with the cube of the distance from the airplane center line.

In calculations involving the use of a wing-twist curve, more consistent results will be obtained by assuming a cubic stiffness distribution than by assuming the wing-twist curve to be either linear or parabolic. In a practical case the trailing-edge portion of the wing usually contributes a negligible amount to the wing torsional stiffness, but the twist curves for a given wing will differ considerably with aileron span and position since the twist is dependent upon the magnitude and position of the applied torques. Such a variation is included in the results given by the charts.

Equation (2) shows that, other things being equal, any increase in τ lowers the aileron reversal speed. From figure 3, an increase in τ is seen to occur when the aileron span is decreased about a given reference position. For an elliptical wing with ailerons extending from $k_i=0.4$ to $k_o=0.8$, τ is therefore 0.467; whereas, for the same wing with ailerons extending from $k_i=0.2$ to $k_o=1.0$, τ is 0.388. In both cases the reference section is located at 0.6 semispan.

In order to determine the wing stiffness required to insure a specified rolling effectiveness, the largest value of $q/\sqrt{1-M^2}$ obtainable should be used.

If the present Army Air Force specification is used as a guide—namely, that the reversal speed should be 1.15 times the terminal velocity of the airplane—calculations show that the value of ϕ to be used in equation (1) would in general yield overly conservative results for wing torsional stiffness. Another procedure for determining the wing stiffness would be to specify a value of ϕ at high-speed level flight. Either specification is believed to be more useful than the current one ($V_R > 1.15V_L$) that yields results at Mach numbers beyond which data would be completely lacking and that would introduce complications in the equations. As an illustration, if an airplane were capable of reaching a Mach number of 0.87, the design Mach number would be 1.0 With Glauert's approximation, which is used with the present method, the required wing stiffness would then be infinite. In recognition of this difficulty, Victory has introduced in reference 9 the concept of an equivalent Mach number while still retaining the requirement that the reversal speed be 1.15 times the terminal velocity of the airplane. Reference 9, in introducing this concept, interprets the present requirement that $V_R > 1.15V_L$ as referring to an equivalent speed in an incompressible flow.

Grinsted, in reference 12, has suggested an alternative procedure—namely, that the wing stiffness be determined so that aileron reversal would occur at limit diving speed and that this value of wing stiffness then be increased by the factor 1.15^2.

From a consideration of references 9 and 12, together with current Army Air Force requirements, the following values of ϕ and $q/\sqrt{1-M^2}$ are recommended for use with the charts presented herein:

Method I.—With limit diving speed as a basis, use $\phi=\frac{1}{4}$ and $q/\sqrt{1-M^2}$ at limit diving speed at sea level.

Method II.—With high-speed level flight as a basis, use $\phi=\frac{4}{5}$ and the largest level-flight value of $q/\sqrt{1-M^2}$, regardless of the altitude at which it occurs.

By employing the detailed results and the equations given in the present report together with the various specifications that have been advanced, the wing stiffness at the reference section has been computed for the airplane used in the example. The following table shows the stiffness as obtained by the use of the various requirements:

Requirement	Sea-level basic speed (mph)	Stiffness m_{θ_r} (ft-lb per radian)
Method I		
$\phi=\frac{1}{4}$ at V_L	553	486,000
Method II		
$\phi=\frac{4}{5}$ at maximum level-flight speed	325	475,000
Army Air Forces, $V_R=1.15\,V_L$	1.15×553	601,000
Victory (reference 9), $M_R=1.086\,M_L$	1.086×553	478,000
Grinsted (reference 12), $m_{\theta_r}=1.32\,m_{\theta_L}$ where m_{θ_L} is the stiffness assuming $V_R=V_L$	553	481,000

Also, for comparison, the following numerical values of m_{θ_r} are listed for the airplane used in the example:

Source [1]	Condition	m_{θ_r} (ft-lb/radian)
Experimental	Ammunition doors closed	527,000
Experimental	Ammunition doors open	335,000
Calculated	Ammunition doors open	209,000

[1] Experimental data furnished by Army Air Forces, Air Technical Service Command, Wright Field, Ohio. Calculated data from Republic Aviation Corporation.

CONCLUDING REMARKS

Charts have been prepared for use in determining the wing torsional stiffness for wings of tubular-shell construction with aspect ratios ranging from 5 to 16 and taper ratios ranging from 0 to 1 including the elliptical. The loss in rolling effectiveness and the aileron reversal speed may also be calculated. The chief advantage of the present method over previous methods is the speed with which the results may be obtained. More accurate results may be obtained by the use of this method than by the use of methods that assume the shape of the wing-twist curve to be linear or parabolic.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *November 9, 1944.*

APPENDIX

DERIVATION OF CHARTS

Although there are a number of types of air loading and inertia loading that contribute to the wing twist about the elastic axis of wings in flight, so far as the problem of rolling effectiveness is concerned, only the twist due to aileron deflection need be considered. In fact, since the distribution due to damping in roll is likely to be almost the same as the spanwise air load distribution resulting from aileron deflection, only the increase in section pitching moment in way of the aileron need be taken into account in determining the wing twist.

A strip of the wing dy in way of the aileron (see fig. 1) will have acting on it an increment in torque as follows:

$$\Delta t\, dy=\frac{dc_m}{d\delta}\delta\frac{qc^2}{\sqrt{1-M^2}}dy \tag{A1}$$

The factor $1/\sqrt{1-M^2}$ is introduced in equation (A1) in order to increase low-speed values of $dc_m/d\delta$ for Mach number effects. If the correct variation of $dc_m/d\delta$ is available, the quantity $(dc_m/d\delta)\ (1/\sqrt{1-M^2})$ may be replaced by the actual variation with Mach number. The accumulated increment in torque at a particular station y_2 in way of the aileron is

$$\Delta T_{y_2}=\int_{y_2}^{y_o}\Delta t\, dy \tag{A2}$$

and, similarly, the accumulated increment in torque at any station y_1 inboard of the aileron is

$$\Delta T_{y_1}=\int_{y_i}^{y_o}\Delta t\, dy \tag{A3}$$

In the derivation of the charts for the determination of wing torsional stiffness required for specified rolling characteristics or aileron reversal speed, it is desirable to use the wing center line as the reference and to define the torsional stiffness for a wing of tubular-shell construction as the concentrated torque which, when applied outboard of a given station, would produce a unit deflection with respect to the reference section. Although this definition of the torsional stiffness makes the analytical development somewhat longer, it is better suited to the test procedures that are now in use when the torsional-stiffness variation along the span is to be determined. The angle of twist due to aileron deflection θ_{y_2} at any station y_2 in way of the aileron (region 2, fig. 1) is thus given by

$$\theta_{y_2}=\frac{1}{m_{\theta_{y_2}}}\int_{y_2}^{y_o}\Delta t\, dy+\int_{y_i}^{y_2}\frac{\Delta t}{m_\theta}dy \tag{A4}$$

The twist at any station y_1 in region 1 inboard of the aileron is

$$\theta_{y_1}=\frac{1}{m_{\theta_{y_1}}}\int_{y_i}^{y_o}\Delta t\, dy \tag{A5}$$

In region 3 outboard of the aileron end the twist is

$$\theta_{y_3}=\int_{y_i}^{y_o}\frac{\Delta t}{m_\theta}dy \tag{A6}$$

Since no antisymmetrical torque is acting outboard of the aileron tip, θ_{y_3} is constant to the wing tip. By substituting equation (A1) in equation (A4) and introducing $k=\frac{y}{b/2}$ a new equation may be obtained. This equation may then be put into more convenient form by multiplying each term by the ratio of the stiffness $m_{\theta_{k_2}}$ to the square of the mean geometric chord $\bar{c}$. The resulting equation may be rearranged to give the following equation applying to region 2 (for convenience, the factor $1/\sqrt{1-M^2}$ will be grouped with q instead of with $dc_m/d\delta$):

$$\frac{\theta_{k_2}}{\delta q/\sqrt{1-M^2}}\frac{2m_{\theta_{k_2}}}{\frac{dc_m}{d\delta}\bar{c}^2b}=\int_{k_2}^{k_o}\left(\frac{c}{\bar{c}}\right)^2dk+\int_{k_i}^{k_2}\left(\frac{c}{\bar{c}}\right)^2\frac{m_{\theta_{k_2}}}{m_\theta}dk \tag{A7}$$

The equations for the wing twist at stations inboard and outboard of the aileron (equations (A5) and (A6)) similarly become

$$\frac{\theta_{k_1}}{\delta q/\sqrt{1-M^2}}\frac{2m_{\theta_{k_1}}}{\frac{dc_m}{d\delta}\bar{c}^2b}=\int_{k_i}^{k_o}\left(\frac{c}{\bar{c}}\right)^2dk \tag{A8}$$

$$\frac{\theta_{k_3}}{\delta q/\sqrt{1-M^2}}\frac{2m_{\theta_{k_o}}}{\frac{dc_m}{d\delta}\bar{c}^2b}=\int_{k_i}^{k_o}\left(\frac{c}{\bar{c}}\right)^2\frac{m_{\theta_{k_o}}}{m_\theta}dk \tag{A9}$$

Equations (A7), (A8), and (A9) define the angle of twist in the three regions in terms of the chord and stiffness distribution, subject to the assumptions that $dc_m/d\delta$ is a constant along the aileron span and that the aileron does not twist about its hinge axis. Inspection of figure 6(b) indicates that the factor $dc_m/d\delta$ is essentially constant for flap-chord ratios from 0.2 to 0.3 and, since the variation of aileron-chord ratio along the span will normally fall within this range, the assumption is justified. In order to evaluate equations (A7) to (A9), the twist curves will be obtained in terms of the twist θ_r at a reference section, which will be taken at the midspan of the aileron. From equation (A7) the twist at the reference section becomes

$$\frac{\theta_r}{\delta q/\sqrt{1-M^2}}\frac{2m_{\theta_r}}{\frac{dc_m}{d\delta}\bar{c}^2b}=\int_{k_r}^{k_o}\left(\frac{c}{\bar{c}}\right)^2dk+\int_{k_i}^{k_r}\left(\frac{c}{\bar{c}}\right)^2\frac{m_{\theta_r}}{m_\theta}dk \tag{A10}$$

With the stiffness as defined in the present report, the torsional stiffness is infinite at the wing center line and decreases with distance to some finite value at the tip. Analysis of data for typical fighter airplanes indicates that negligible errors will result in twist computations if the torsional stiffness distribution along the span is assumed to be

$$m_\theta=\frac{\text{Constant}}{k^3} \tag{A11}$$

When equation (A11) is substituted into equations (A7) to (A10), the following ratios of θ/θ_r will be obtained in the various regions:

$$\left.\begin{aligned}
\frac{\theta_1}{\theta_r}&=\left(\frac{k_1}{k_r}\right)^3\left[\frac{\int_{k_i}^{k_o}\left(\frac{c}{\bar{c}}\right)^2 dk}{\int_{k_r}^{k_o}\left(\frac{c}{\bar{c}}\right)^2 dk+\frac{1}{k_r^3}\int_{k_i}^{k_r}\left(\frac{c}{\bar{c}}\right)^2 k^3\, dk}\right]\\
\frac{\theta_2}{\theta_r}&=\left(\frac{k_2}{k_r}\right)^3\left[\frac{\int_{k_2}^{k_o}\left(\frac{c}{\bar{c}}\right)^2 dk+\frac{1}{k_2^3}\int_{k_i}^{k_2}\left(\frac{c}{\bar{c}}\right)^2 k^3\, dk}{\int_{k_r}^{k_o}\left(\frac{c}{\bar{c}}\right)^2 dk+\frac{1}{k_r^3}\int_{k_i}^{k_r}\left(\frac{c}{\bar{c}}\right)^2 k^3\, dk}\right]\\
\frac{\theta_3}{\theta_r}&=\left(\frac{k_o}{k_r}\right)^3\left[\frac{\frac{1}{k_o^3}\int_{k_i}^{k_o}\left(\frac{c}{\bar{c}}\right)^2 k^3\, dk}{\int_{k_r}^{k_o}\left(\frac{c}{\bar{c}}\right)^2 dk+\frac{1}{k_r^3}\int_{k_i}^{k_r}\left(\frac{c}{\bar{c}}\right)^2 k^3\, dk}\right]
\end{aligned}\right\} \tag{A12}$$

It will be noted in equations (A12) that only geometrical terms such as spanwise extent of aileron span and chord ratios $c/\bar{c}$ occur and that, in order to determine the resultant twist distribution, only these values need be specified.

In reference 13 influence lines are presented for a series of tapered wings (see fig. 2) of several aspect ratios, which make possible the computation of a coefficient of rolling-moment loss C_{l_θ} due to any sort of twist distribution. As a first step in the evaluation of a loss coefficient C_{l_θ}, the ratios of θ/θ_r were evaluated for the series of wings shown in figure 2 with ailerons of various span.

The loss in rolling moment due to a twist θ_r, at the reference section, was then defined by the equation

$$\begin{aligned}\text{Rolling moment loss}&=L_\theta\\&=C_{l_\theta}\theta_r qSb\end{aligned} \tag{A13}$$

where

$$C_{l_\theta}=\frac{dC_l}{d\theta_r}$$

$$=\frac{dC_l/d\theta_r}{\theta/\theta_r}\frac{\theta}{\theta_r} \tag{A14}$$

The results shown in figure 16 of reference 13 were used to determine C_{l_θ} for the twist variations computed from equations (A12). The coefficient C_{l_θ} was also determined for elliptical wings of aspect ratios 6, 10, and 16, with values of k_i of 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7 and for values of k_o of 0.8, 0.9, and 1.0 as well as for the wing plan forms shown in figure 2. The numerical results of these steps are not given herein because they are only intermediate steps in the procedure.

In the steady rolling condition, the damping moment equals the moment impressed by the ailerons minus the loss in moment due to twist. In coefficient form, this relation may be expressed as

$$C_{l_p}\frac{pb}{2V}qSb=C_{l_\delta}\delta qSb-C_{l_\theta}\theta_r qSb \tag{A15}$$

from which the helix angle per unit aileron deflection $\frac{pb/2V}{\delta}$ is obtained as

$$\frac{pb/2V}{\delta}=\frac{C_{l_\delta}-C_{l_\theta}\frac{\theta_r}{\delta}}{C_{l_p}} \tag{A16}$$

The coefficients C_{l_δ}, C_{l_θ}, and C_{l_p} will vary with Mach number but of these coefficients only the variation of C_{l_δ} with Mach number can readily be determined from wind-tunnel tests. At present C_{l_p} must be obtained either from results of low-speed tests or from results of computations and C_{l_θ} must always be obtained by computation. For this reason it would appear reasonable to use consistent values and to assume that each varies with Mach number according to $1/\sqrt{1-M^2}$. From equation (A10), for a particular wing aileron combination,

$$\theta_r=\frac{\frac{dc_m}{d\delta}\bar{c}^2b}{2m_{\theta_r}}\frac{\delta q}{\sqrt{1-M^2}}B_1 \tag{A17}$$

where the constant B_1 equals the right-hand side of equation (A10). Also, from equations (A12) and (A14), C_{l_θ} is seen to be a constant for a particular wing-aileron combination. When these values of C_{l_θ} and θ_r are substituted in equation (A16), the following equation results:

$$\frac{pb/2V}{\delta}=\frac{C_{l_\delta}-\frac{\frac{dc_m}{d\delta}\bar{c}^2b}{2m_{\theta_r}}\frac{q}{\sqrt{1-M^2}}B_2}{C_{l_p}} \tag{A18}$$

where the constant

$$B_2=C_{l_\theta}B_1$$

At the aileron reversal speed,

$$C_{l_\delta}=\frac{\frac{dc_m}{d\delta}\bar{c}^2b}{2m_{\theta_r}}\frac{q}{\sqrt{1-M_2}}B_2$$

and the value of the dynamic pressure is

$$\frac{q}{\sqrt{1-M^2}}=\frac{2m_{\theta_r}}{\frac{dc_m}{d\delta}\bar{c}^2b}\frac{C_{l_\delta}}{B_2}=\frac{2m_{\theta_r}}{\frac{dc_m}{d\delta}\frac{b^3}{A^2}}\frac{C_{l_\delta}}{d\alpha/d\delta}\frac{d\alpha/d\delta}{B_2} \tag{A19}$$

By setting

$$\frac{B_2}{C_{l_\delta}\Big/\frac{d\alpha}{d\delta}}=\tau \tag{A20}$$

the dynamic pressure at aileron reversal speed is

$$\frac{q_R}{\sqrt{1-M^2}}=\frac{2m_{\theta_r}}{\tau\frac{dc_m/d\delta}{d\alpha/d\delta}\frac{b^3}{A^2}} \tag{A21}$$

In the determination of the values of τ the necessary numerical values of B_2 were obtained by the procedure outlined and the necessary values of $\frac{C_{l_\delta}}{d\alpha/d\delta}$ were taken directly from figure 16 of reference 13. When the values of τ were plotted, the results were found to be essentially the same for aspect ratios of 6, 10, and 16; the average deviation was less than 1 percent. The value of τ did, however, vary with aileron position and wing taper as shown in figure 3.

For an infinitely rigid wing, the helix angle per degree aileron deflection can be obtained from equation (A18) as

$$\left(\frac{pb/2V}{\delta}\right)_{rigid}=\frac{\frac{C_{l_\delta}}{d\alpha/d\delta}\frac{d\alpha}{d\delta}}{C_{l_p}}$$

$$=\gamma\frac{d\alpha}{d\delta} \tag{A22}$$

where C_{l_p} was obtained from figure 8 of reference 13. Figure 4 gives the values of γ in terms of k_i and k_o. By specifying that the flexible wing retain some fraction ϕ of the rigid wing rolling effectiveness at a specified dynamic pressure (say, terminal velocity), the following equation results

$$\left(\frac{pb/2V}{\delta}\right)_{flexible}=\phi\gamma\frac{d\alpha}{d\delta} \tag{A23}$$

By substituting results from equations (A20) and (A23) into equation (A18), the wing stiffness m_{θ_r} required at the reference section to retain a specified value of rolling ability at a given value of $q/\sqrt{1-M^2}$ is given by

$$m_{\theta_r}=\tau\frac{dc_m/d\delta}{d\alpha/d\delta}\frac{b^3}{2A^2(1-\phi)}\frac{q}{\sqrt{1-M^2}} \tag{A24}$$

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