TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 875

A ROSETTE STRAIN COMPUTER

By W. B. Klemperer
Douglas Aircraft Company, Inc.

Washington
December 1942

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 875

A ROSETTE STRAIN COMPUTER

By W. B. Klemperer

SUMMARY

An electrical apparatus that serves as a computer for the evaluation of linear strain measurements taken in rosette fashion on the surface of a structural element carrying tension or compression and shear is described herein. The solution of the strain circle is accomplished by coupled rotary transformers, one representing each of the rosette gages. The output of the instrument furnishes directly the maximum shear and the orientation of the principal strain axes with respect to the rosette axes as well as the shear component with respect to any desired axis. The machine affords great time saving over trigonometric or graphical evaluation methods.

INTRODUCTION

Experimental exploration of the stress flow through stress-carrying thin-wall structures is becoming increasingly helpful in developing and proving the stress analysis of metal airplanes for which accurate understanding of stress distribution is a prerequisite for safe and efficient design. On the surface of any monolithic structural element, the strain tensor is fully described by the strain circle (reference 1) or by the magnitude and orientation of the two principal strains. In order to determine the strain state experimentally, it is necessary and sufficient to measure the linear strain in three different directions on the surface element.

The conventional technique consists in the application of three linear strain gages as closely packed as feasible (reference 2), in the use of an equilateral triangular lattice of strain gages, or strain-gage holes (reference 3), or in the use of a single instrument designed to measure the distortion of a base triangle (reference 4). Recent developments of reliable, compact, simple, and inexpensive electrical-resistance strain gages (references 5 to 7) and improvements in indicators and recorders of consistent amplification (reference 8) have extended the application of large numbers of strain gages in inaccessible places on structures subjected to elaborate tests, so that the structural engineer need no longer complain about "too little and too late." Hundreds of delta triplets of "band-aid" type of resistors are now readily cemented along spar webs, gussets, bulkheads, skins, and other sheet-metal plate, shell, or frame structures.

In order to exploit the abundance of experimental data now within easy grasp, need arose for a quick method of evaluating the strain-gage measurements in terms of shear flow, that is, a method of determining the direction and magnitude of maximum and minimum strain or the amount of shear with respect to any particular axis. Various methods have been proposed to facilitate this evaluation for both the 0°-45°-90°-135° type of rosette (references 9 to 11) and the 0°-60°-120° "delta" type of array (references 1, 4, 13, and 14). All these methods, however, which are based on a trigonometric routine (references 4 and 12), on a graphical construction (references 1 and 9), or on the manipulation of a mechanical nomogram device, are tedious and exacting. They require at least about 8 minutes per gage station and load station. When there are several hundred gage stations and half a dozen load stations in any one test, several hundred man-hours of engineering labor are tied up.

In order to simplify the evaluation so that it would no longer absorb the time of mathematically trained personnel, a rosette strain computer has been developed by the Engineering Research Department of the Douglas Aircraft Company, Inc. This machine was designed to solve the strain problem as a computer which derives the orientation of the principal axes and the maximum shear or the shear component in any desired direction from the measured values of strain components. The machine is portable and can be operated by a person with nontechnical training in a fraction of the time required by any longhand method.

The author wishes to acknowledge the valuable contributions of Everett H. Pier for the electrical system, Darrow L. Miller for the mechanical arrangement, Scott

Neuls and Robert D. Gordon for the technique of applying the apparatus to actual problems.

DESCRIPTION OF THE COMPUTER

Several physical principles that have characteristics analogous to those involved in the strain circle might be applied to the construction of such a computer. Because of the nature of the strain cycle, it can be represented by a vector in a polar field in which the phase angle is twice the strain orientation angle measured from a principal axis. An audio-frequency alternating-current magnetic field was chosen as the vector representing the shear component in the double-angle polar field because the apparatus is easy to procure. The apparatus has analogies to the Scott connection of a two-phase-three-phase transformer.

The machine consists of a plurality of rotary transformers, one for each of the strain gages coacting in one rosette. Each rotary transformer is designed to generate an alternating voltage proportional to the input into its stator and to the sine of the angle of rotation of its rotor. Such a rotary transformer has been designated a "sinometer."

When this work was begun, no suitable small-size single-phase rotary transformers of accurately sinusoidal characteristic were on the market. They are now being developed; as soon as they are available, a much more compact machine can be made than the two types which have thus far been applied to practical service. In one model Pioneer Autosyns and in the other General Electric Selsyns model 2JD123A1 were used as sinometers by connecting only one of the two or three rotor coils. In the first model a 400-cycle electronic oscillator supplied power; in the second model, power was taken from the 110-volt 60-cycle utility system.

Whatever type of electrical device is used as a sinometer in the computer, several such sinometers are stacked on a common shaft or geared together with fixed phase differences between their respective rotors and stators corresponding to twice the angles between the orientation of the individual strain gages which they represent. In the case of a 0°-60°-120° delta array of gages, three sinometers are so coupled that the second and third rotors make angles of 120° and 240° with their respective stator axis when the first one is alined with its own stator. Conversely, in a machine designed to solve the results of a 0°-45°-90°-135° array quadruplet of gages, four sinometers are coupled with their respective rotors phase-shifted to 0°-90°-180°-270°. All rotors are connected in series; whereas all stator terminals are brought out individually. Each sinometer is individually shielded to prevent its field from influencing the others around it.

Each stator is provided with a potentiometer or Variac input control and polarity switch to adjust the input current from a constant-voltage alternating-current source to the strain value measured by the strain gage which it represents. A common alternating-current voltmeter with a circuit selector switch serves to indicate each stator current while it is being adjusted and also to read the synthetic rotor output when the selector switch is turned to the output circuit.

A shaft of the mechanically coupled rotor system is provided with a knob and angular scales or dials graduated in half the actual angle of rotor rotation to correspond to the strain orientation on the test specimen and with a 45° fork to check the angle between maximum and zero strain.

The circuit diagram for a triple-sinometer type of computor for the evaluation of delta strain-gage arrays is shown in figure 1. The three stator-input circuits and the rotor circuit are shown in bold lines, the accessory circuits in finer lines. The indicator is a voltmeter of the rectifier type.

The accompanying photographs show the first two experimental machines assembled: figure 2, the unit containing geared Autosyns; figures 3 and 4, the one containing shaft-coupled Selsyn devices. The second unit, though somewhat bulkier, was arranged for greater convenience of manipulation.

OPERATION OF THE COMPUTER

The procedure for operating the machine for a 60° delta set of three strain-gage readings of, for example, $a = 14.0$, $b = 11.3$, $c = -7.5 \times 10^{-4}$ strain units is as follows:

The selector switch is turned to A, the A potentiometer is so adjusted that the indicator reads 14.0; the selector switch is turned to B, the B potentiometer is so adjusted that the indicator reads 11.3; the selector switch is turned to C, the polarity switch C is snapped to minus, the C potentiometer is so adjusted that the indicator reads 7.5; the selector switch is now turned to S and the rotor knob is rotated until the indicator reads a maximum. This maximum is the shear-strain value, in this example 13.6×10^{-4}; and the index on the dial scale now reads the orientation of the principal axis, that is, that of maximum strain, against the A leg in a conventional sense (in the case of the example, 27°). The entire operation takes less than 30 seconds.

If great accuracy in determining the orientation of the principal axes is required, the dial may be turned to zero output, which should occur at an angle of 90° of the rotors or 45° in the strain field from the maximum. This setting can be readily checked with the aid of the adjustable 45° dial fork. A pair of lamps is provided to indicate polarity of the output and to accentuate the zero position. If all three input values are adjusted to be equal, the lamps will flicker alternately to indicate that there is no solution for the angle. It would be entirely feasible to have the light circuits actuate a pair of relays that would control a servo motor which would automatically find the direction of the principal axis; but thus far the additional time saving of about 5 seconds offered by this automatization has not seemed to justify the extra apparatus.

If the shear component at some definite angle from the A gage line is desired, the knob is turned to the corresponding dial angle and the indicator is read with the selector switch on S. The linear strain in this direction can be obtained by adding this reading, with due regard to polarity, to the average strain $(a + b + c)/3$.

For the convenience of utilizing the indicator at a fair part of its range, its dial is provided with three scales in the ratio $\frac{1}{2}$:1:2. Any one of these scales may be used for each problem but should be kept consistent for all input adjustments and for the output reading. The scale giving the largest meter deflection affords the greatest accuracy for the evaluation.

The maximum and minimum tensile strains are readily found by adding and subtracting the shear amplitude δ to and from the average strain: $\epsilon_p = \epsilon + \delta$ and $\epsilon_q = \epsilon - \delta$. The principal stresses s_p and s_q are determined from the principal strains ϵ_p and ϵ_q by introducing the elastic modulus E and Poisson's ratio μ in the equation

$$s_p = \frac{E}{1 - \mu^2}\left(\epsilon_p + \mu\epsilon_q\right) \quad \text{and} \quad s_q = \frac{E}{1 - \mu^2}\left(\epsilon_q + \mu\epsilon_p\right)^*$$

and the maximum shear stress in the equation

$$\tau_{max} = \frac{1}{2}\left(s_p - s_q\right) = \frac{1}{2}\frac{E}{1 + \mu}\left(\epsilon_p - \epsilon_q\right) = \frac{E\delta}{1 + \mu} = 2G\delta$$

where $G = E/2(1+\mu)$ is the shear modulus, which is a known material constant.

BASIS OF THEORY

The theory of the instrument, as previously suggested, is based on the fact that the linear strain in any direction defined by an angle θ from the axis of maximum strain can be expressed by the relation

$$\epsilon_\theta = \epsilon + \delta \cos 2\theta \tag{1}$$

where ϵ is the average strain and δ the amplitude, which is half the difference between maximum and minimum tensile strain and numerically equal to the maximum shear strain. The average strain ϵ is immediately determined as the arithmetical average of the three linear-strain components ϵ_a, ϵ_b, and ϵ_c, measured at three directions 60° from each other (or, for that matter, of any number of strain-gage readings taken at regular subdivisions of 180°). The values of the shear-strain amplitude δ and of the orientation angle θ are determined by the machine.

From the description of the construction, circuit,

---

*The principal stresses themselves can also be indicated directly by the machine. For this purpose, a separate circuit and switch comprising a resistor adjusted to the value of Poisson's ratio can be provided.

and operation of the delta triplet type of instrument, it will be evident that the voltage generated in each of the three sinomotor rotors is expressed by a time-phase vector proportional to $\epsilon_a \cos \varphi$, $\epsilon_b \cos (\varphi + 120)$, $\epsilon_c \cos (\varphi - 120)$, respectively, if φ is the incidental angle at which the magnetic axis of the A rotor is set with respect to its stator-coil axis. If the various measured linear-strain values are regularly spaced components built of the terms defined by equation (1), then

$$\left.\begin{aligned} \epsilon_a &= \epsilon + \delta \cos 2\theta \\ \epsilon_b &= \epsilon + \delta \cos (2\theta + 120) \\ \epsilon_c &= \epsilon + \delta \cos (2\theta - 120) \end{aligned}\right\} \qquad (2)$$

and the sum S of the voltages generated by all rotors in series, inasmuch as they are all in phase because they are derived from the same primary alternating-current source, is the sum of the three lines

$$\left.\begin{aligned} \epsilon_a \cos \varphi &= \epsilon \cos \varphi + \delta \cos \varphi \cos 2\theta \\ \epsilon_b \cos(\varphi+120) &= \epsilon \cos(\varphi+120) + \delta \cos(\varphi+120) \cos(2\theta+120) \\ \epsilon_c \cos(\varphi-120) &= \epsilon \cos(\varphi-120) + \delta \cos(\varphi-120) \cos(2\theta-120) \end{aligned}\right\} \qquad (3)$$

and may be expressed as

$$\begin{aligned} S &= \epsilon \cos \varphi \left(1 - \frac{1}{2} - \frac{1}{2}\right) + \delta \cos \varphi \cos 2\theta (1 + 2 \cos^2 120) \\ &\quad - \epsilon \sin \varphi \left(\frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3}\right) + \delta \sin \varphi \sin 2\theta (2 \sin^2 120) \\ &= 0 + \frac{3}{2} \delta (\cos \varphi \cos 2\theta + \sin \varphi \sin 2\theta) \\ &= \frac{3}{2} \delta \cos (\varphi - 2\theta) \end{aligned} \qquad (4)$$

This equation proves that, for $\varphi = 2\theta$, the value of S becomes a maximum and that it varies as a cosine function of the departure of the phase angle from this principal axis. The quantity S is 3/2 that of the unknown strain amplitude. This factor 3/2 is taken care of in the machine by a shunt connected when the selector switch is in the S position, so that the true value of δ can be read directly on the same scale of the indicator as the values ϵ_a, ϵ_b, and ϵ_c to which the inputs had been adjusted.

The proof that a similar machine having four sinometers at 90° phase difference to represent a type rosette quadruplet of gages placed at 0°-45°-90°-135° from a reference line can be adduced in a similar manner. Here it is presumed that the four strain values really are consistent with

$$\left.\begin{aligned} \epsilon_a &= \epsilon + \delta \cos 2\theta \\ \epsilon_b &= \epsilon - \delta \sin 2\theta \\ \epsilon_c &= \epsilon - \delta \cos 2\theta \\ \epsilon_d &= \epsilon + \delta \sin 2\theta \end{aligned}\right\} \qquad (5)$$

When these values are set into the computer, the sum of the voltages generated in the rotor circuit is

$$\left.\begin{aligned} \epsilon_a \cos\varphi &= \epsilon \cos\varphi + \delta \cos\varphi \cos 2\theta \\ -\epsilon_b \sin\varphi &= -\epsilon \sin\varphi + \delta \sin\varphi \sin 2\theta \\ -\epsilon_c \cos\varphi &= -\epsilon \cos\varphi + \delta \cos\varphi \cos 2\theta \\ \epsilon_d \sin\varphi &= \epsilon \sin\varphi + \delta \sin\varphi \sin 2\theta \end{aligned}\right\} \qquad (6)$$

and may be expressed as

$$\begin{aligned} S &= 0 + 2\delta\,(\cos\varphi \cos 2\theta + \sin\varphi \sin 2\theta) \\ &= 2\delta \cos(\varphi - 2\theta) \qquad (7) \end{aligned}$$

This equation also proves that, if $\varphi = 2\theta$, S becomes a maximum and for this angle indicates the principal axes. The factor here is 2 rather than 3/2.

It may be noted that four strain gages actually furnish a redundant set of data. If these data contain small errors, they would not be consistent with the four equations (5), which have three unknown quantities. In order to be consistent with all four equations, alternating pairs of mutually perpendicular strains would have to add up to identical sums, namely,

$$\epsilon_a + \epsilon_c = \epsilon_b + \epsilon_d \tag{8}$$

If they do not add up to identical sums, their cross sums may differ by a discrepancy $e = \epsilon_a + \epsilon_c - \epsilon_b - \epsilon_d$. The machine, if operated without regard for this discrepancy, will yield the same solution as if each of the individual strain values had been corrected by one-quarter of the discrepancy subtracted from the values for which the cross sum was long and added to the values for which the cross sum was short. This result is readily seen when $\frac{1}{4}e \cos \varphi$, $\frac{1}{4}e \sin \varphi$, $-\frac{1}{4}e \cos \varphi$, and $-\frac{1}{4}e \sin \varphi$, respectively, are added to the four equations (6). The sum of the corrections equals zero and the error is thus canceled, if it is assumed that all measurements are of equal accuracy. It may be noted that this adjustment is the same as would result from a least-square adjustment of the four readings. Some experimenters sometimes prefer the redundant quadruplet method to the determinate delta method because of the check of the accuracy of the measurements afforded by the discrepancy e for individual gage stations of particular interest.

UTILITY OF THE COMPUTERS

The machines previously described have been extensively used in the evaluation of many hundreds of rosette strain-gage measurements of new airplanes, both in static proof tests and in flight (reference 12). The time saved in comparison with earlier methods of evaluation not only has been much appreciated in the regular routine of this work but also can be of decisive value when evaluations of critical stresses are to be made on the spot during expensive structural tests and in work in which instantaneous knowledge of the results may prompt the test engineer to alter the test program.

The usefulness of the computer is not limited to the problem of compound axial and shear stresses. There are many other problems of a vectorial nature which can be solved by this machine. One such problem which has been proposed by William Van Dyke is that of determining the magnitude and orientation of the plane of bending in a tubular or strut member of cross section of circular symmetry from three strain gages mounted at 120° spacing around its circumference, each with its gage length parallel to the axis. The average strain here is the average axial strain; the strain amplitude is the additional bending strain; the angular spacing relation between gage location and phase value is here 1:1, and not 1:2; Poisson's ratio does not enter into this bending problem.

The accuracy of the machine results has been frequently verified by control calculations. The results thus far encourage further attempts at electrification of stress-analysis problems in which the desired degree of accuracy is readily attainable with electrical and electromagnetic circuits.

Engineering Research Department,
Douglas Aircraft Company, Inc.,
Santa Monica, Calif.,

REFERENCES

1. Wise, Joseph A.: Circles of Strain. Jour. Aero. Sci., vol. 7, no. 10, Aug. 1940, pp. 438-440.

2. Karpov, A. V., and Templin, R. L.: Building and Testing an Arch Dam Model. Civil Engineering, vol. 2, no. 1, Jan. 1932, pp. 11-16.

3. Stang, Ambrose H., and Greenspan, Martin: Perforated Cover Plates for Steel Columns: Program and Test Methods. Res. Paper 1473, Nat. Bur. of Standards Jour. Res., vol. 28, no. 6, June 1942, pp. 669-686.

4. Klemperer, W. B.: The Tensor Gage. Jour. Aero. Sci., vol. 7, no. 9, July 1940, pp. 403-404.

5. deForest, A. V., and Leaderman, H.: The Development of Electrical Strain Gages. T.N. No. 744, NACA, 1940.

6. Howland, W. L., and Beed, C. F.: Tests of Pressurized Cabin Structures. Jour. Aero. Sci., vol. 8, no. 1, Nov. 1940, pp. 17-23.

7. Jackman, Kenneth R.: Modern Aircraft Materials and Their Testing. SAE Jour., vol. 47, no. 5, Nov. 1940, pp. 461-473.

8. deForest, A. V.: Characteristics and Aircraft Applications of Wire Resistance Strain Gages. Instr., vol. 15, no. 4, April 1942, p. 112.

9. Osgood, Wm. R.: Determination of Principal Stresses from Strains on Four Intersecting Gage Lines 45° Apart. Res. Paper 851, Nat. Bur. of Standards Jour. Res., vol. 15, no. 6, Dec. 1935, pp. 579-581.

10. Sibert, H. W.: Principal Stresses from Three Strains at 45°. Jour. Aerc. Sci., vol. 7, no. 1, Nov. 1939, pp. 26-27.

11. Hill, H. N.: A Semi-Graphical Method for Analyzing Strains Measured on Three or Four Gage Lines Intersecting at 45°. T.N. No. 709, NACA, 1939.

12. Strang, C. R.: Progress in Structural Design through Strain-Gage Technique. SAE Jour., vol. 50, no. 8, Aug. 1942, pp. 346-357.

13. Beggs, G., and Tremby, E.: Interpreting Data from Strain Rosettes. Engineering News Record, vol. 120, no. 10, March 10, 1938, pp. 366-370.

14. Osgood, Wm. R., and Sturm, Rolland G.: The Determination of Stresses from Strains on Three Intersecting Gage Lines and Its Application to Actual Tests. Res. Paper 559, Nat. Bur. of Standards Jour. Res., vol. 10, no. 5, May 1933, pp. 685-692.

A,B,C	Sinometer circuits	L	Polarity lamps	R	Voltage regulator
D	Angle dial	M	Rectifier	S	Sum
E	Indicating voltmeter	N	Auxiliary transformer	T	Transformer
J	Amplifier	P	Polarity switches	U	Current source
K	Thyratron	Q	Selector switch	V	Variac

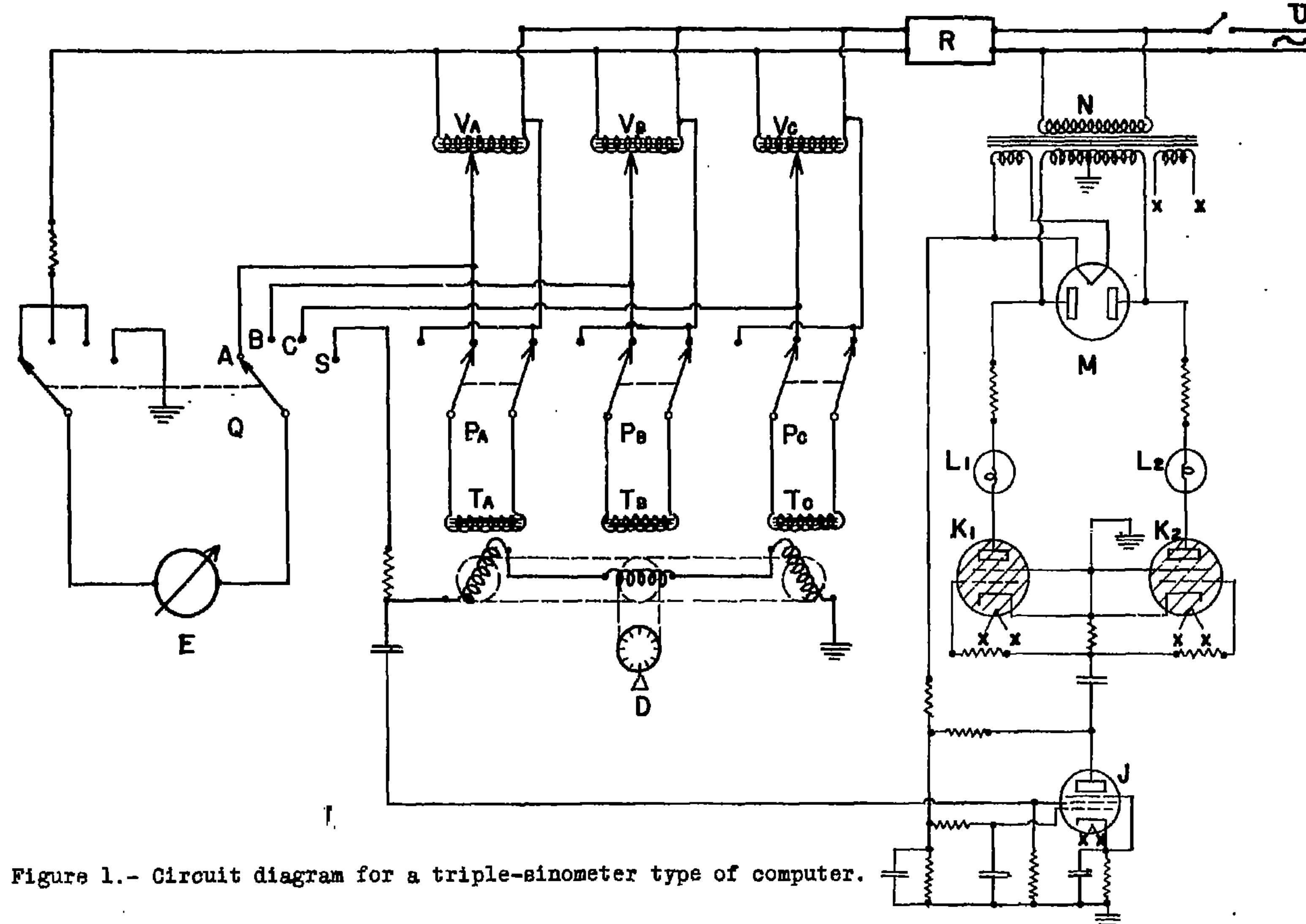

Figure 1.- Circuit diagram for a triple-sinometer type of computer.

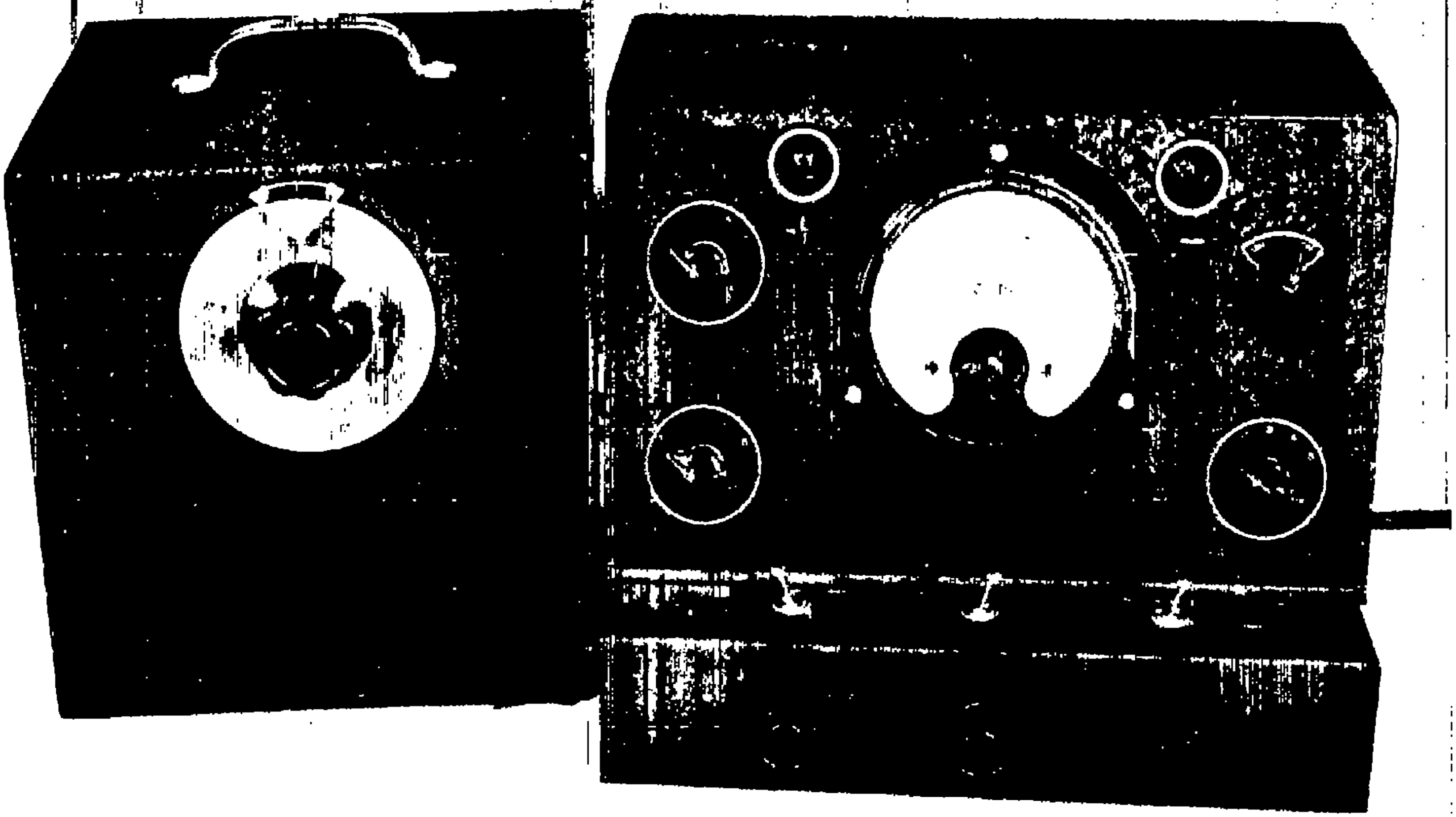

Figure 2. - Delta strain computer containing geared Autosyns.

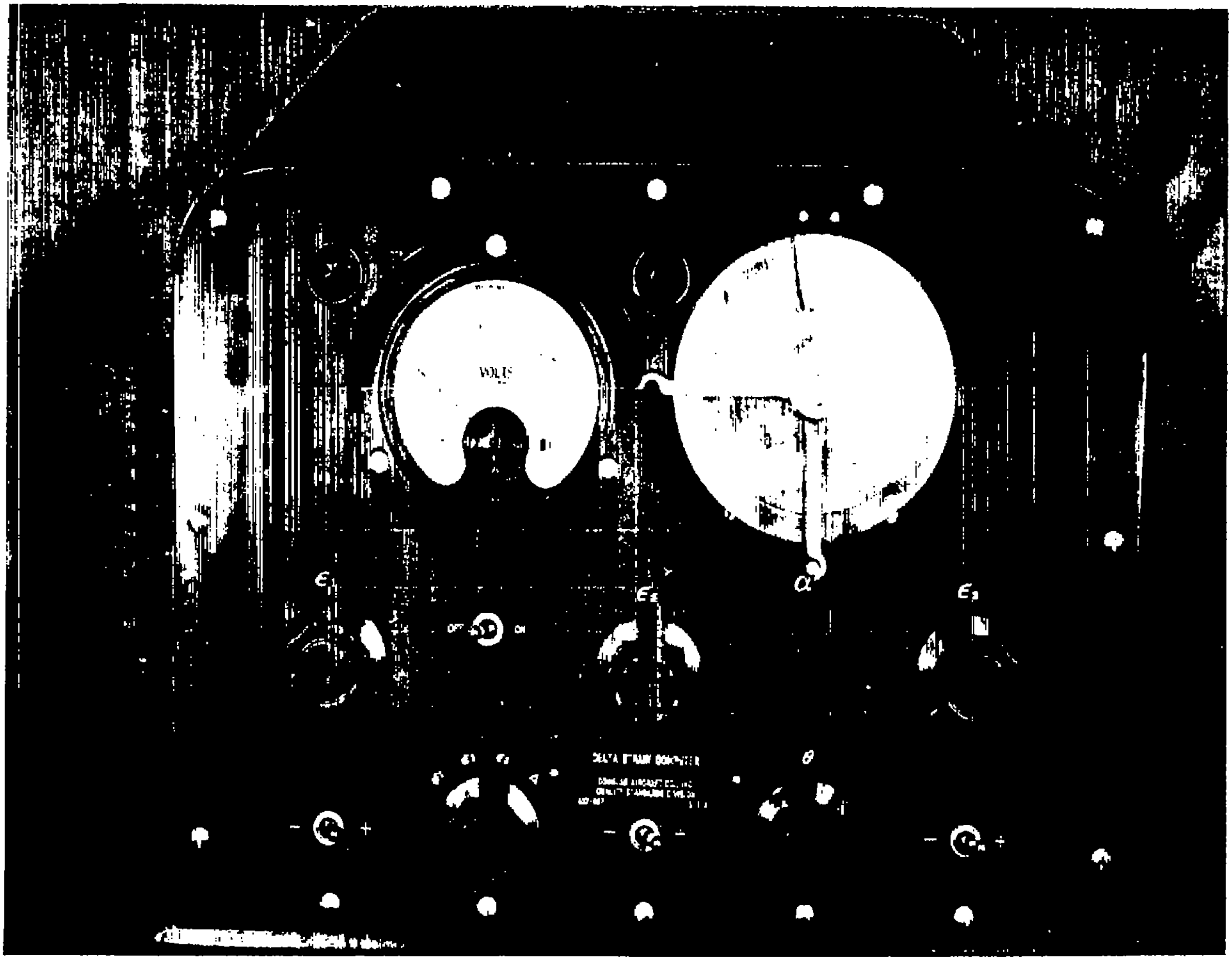

Figure 3. - Delta strain computer containing shaft-coupled Selsyn devices.

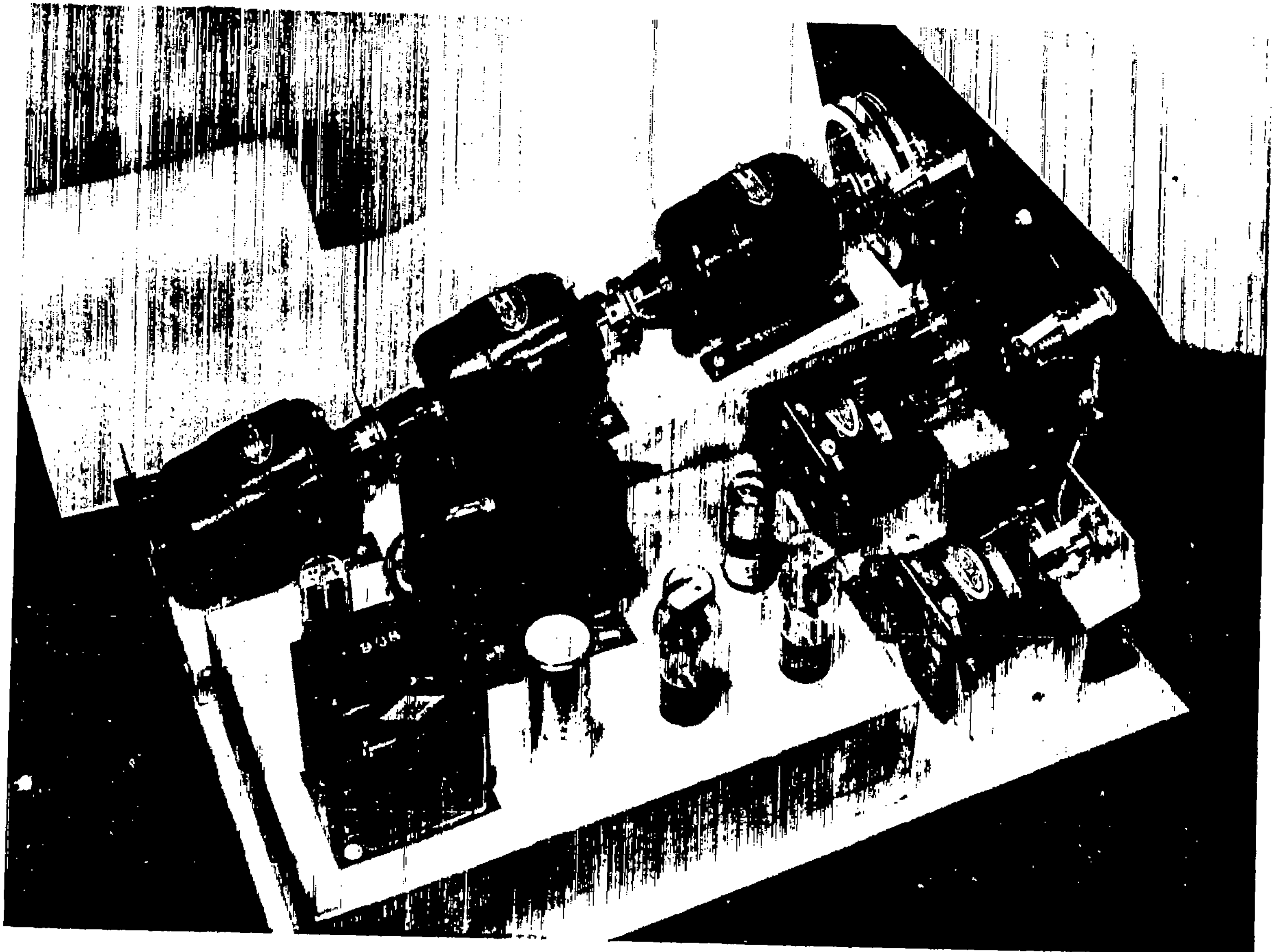

Figure 4. - View of delta strain computer showing shaft-coupled Selsyn devices.