TECHNICAL NOTES

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A SEMI-GRAPHICAL METHOD FOR ANALYZING STRAINS MEASURED ON THREE OR FOUR GAGE LINES INTERSECTING AT 45°

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I. INTRODUCTION

The determination of the state of stress at a point in a plane, from strain measurements made on intersecting gage lines, is not an exceptionally difficult problem, the solution being based on the generally well-known biaxial relationships between stress and strain. In the treatment of this problem, as in many others, there will always be those who will employ the mathematical manipulations in the reduction of the data, while others will prefer a graphical method for accomplishing the same purpose. The necessary mathematical manipulations and a simplified tabular form for making the calculations have been discussed by Professors Beggs and Timby (reference 1). Osgood and Sturm (reference 2) have described the so-called "dyadic circle" graphical method which they attribute to Professor H. M. Westergaard. Both of these methods have been in use at our laboratories for the past eight years.

The method herein described is essentially graphical, but does require a small amount of arithmetical manipulation. It is not, however, a combination of the two methods mentioned above. To the best of the writer's knowledge, this treatment constitutes an entirely new procedure for determining stresses from strains measured on gage lines intersecting at 45°. The method is thought to have some advantages in simplicity and directness over both of those previously mentioned.

The manner in which the method is to be used will first be described, to be followed by a proof of its validity.

II. DESCRIPTION OF THE METHOD FOR THREE GAGE LINES

The method will first be described as applicable to the case of three gage lines intersecting at 45°. For the purpose of this description, a rosette will be considered with gage lines 1 and 3 mutually perpendicular, and gage line 2 bisecting the first and third quadrants, measuring clockwise from gage line 1. The procedure to be followed in applying the method is as follows:

1. Construct, on transparent paper or cloth, a master curve of the function $f = \cos 2\theta$, plotting values of f as ordinates and θ as abscissas. It is convenient to have this curve plotted for the region $-45^\circ < \theta < 135^\circ$. This master curve becomes a piece of more or less permanent equipment, to be used each time the method is applied to a set of strain data.

2. Divide the unit strains measured on the three gage lines (e_1, e_2, and e_3) by an adjustment factor K, to be determined from the equation

$$K = 0.707 \sqrt{(e_1 - e_2)^2 + (e_2 - e_3)^2} \qquad (1)$$

3. Plot the adjusted strain readings, as ordinates, preferably on cross-section paper, to the same scale as used for f values on the master curve. The θ values (angles between gage lines) are the abscissas. The choice of origin for this plot is immaterial, the important thing being the relative positions for the three gage lines.

4. Draw an auxiliary horizontal line representing the average of the adjusted strain values for gage lines 1 and 3. The ordinate, y, for this line will be

$$y = \frac{\frac{e_1}{K} + \frac{e_3}{K}}{2}$$

5. Place the master curve over the plot, with its horizontal axis coinciding with the auxiliary line of the plot, and in such a location that the plotted points fall on the master curve.

6. The maximum and minimum points of the master

curve, referred to the origin of the plot and its original axis, indicate the directions of the principal strains, and the magnitude of their adjusted values. To obtain the true magnitudes of the principal strains, these values must be multiplied by the adjustment factor, K.

7. Having the magnitudes of the principal strains, e_p and e_q, the principal stresses p and q can readily be determined from the biaxial stress-strain relationships,

$$p = \frac{E}{1 - \mu^2} (e_p + \mu e_q) \qquad (2)$$

and

$$q = \frac{E}{1 - \mu^2} (e_q + \mu e_p) \qquad (3)$$

where

E is modulus of elasticity, lb. per sq. in.

and

μ, Poisson's ratio.

8. The maximum shearing stress (τ), which occurs on planes inclined at angles of 45° to the direction of principal stress, can readily be evaluated from the equation,

$$\tau = \frac{p - q}{2} \qquad (4)$$

The various steps in the method can be followed in figure 1. In this figure the master curve has been superimposed on the plot with the proper axes coinciding and in such a manner that the master curve passes through the plotted points.

It may be found more convenient to plot the strain data on transparent paper and superimpose this plot on the master curve. Obviously, the method would remain unchanged.

III. PROOF OF VALIDITY OF THE METHOD

Consider an element in a plane subjected to the principal stresses p and q. Then the mutually normal stresses corresponding to the position represented by the angle θ, measured clockwise from the direction of p, will be

$$\sigma_\theta = p \cos^2 \theta + q \sin^2 \theta \qquad (5)$$

and

$$\sigma_\theta' = p \sin^2 \theta + q \cos^2 \theta \qquad (6)$$

Making the following substitutions,

$$p = \frac{E}{1 - \mu^2} (e_p + \mu e_q) \qquad (2)$$

$$q = \frac{E}{1 - \mu^2} (e_q + \mu e_p) \qquad (3)$$

and

$$\sigma_\theta = \frac{E}{1 - \mu^2} (e_\theta + \mu e'_\theta) \qquad (7)$$

$$\sigma_\theta' = \frac{E}{1 - \mu^2} (e'_\theta + \mu e_\theta) \qquad (8)$$

Equations (5) and (6) become

$$e_\theta + \mu e'_\theta = (e_p+\mu e_q) \cos^2 \theta + (e_q+\mu e_p) \sin^2 \theta \qquad (5a)$$

and

$$e'_\theta + \mu e_\theta = (e_p+\mu e_q) \sin^2 \theta + (e_q+\mu e_p) \cos^2 \theta \qquad (6a)$$

Solving these equations simultaneously for e_θ yields

$$e_\theta = e_p \cos^2 \theta + e_q \sin^2 \theta \qquad (9)$$

which by trigonometric substitution can be reduced to

$$e_\theta = \frac{1}{2} \left[(e_p+e_q) + (e_p-e_q) \cos 2\theta \right] \qquad (9a)$$

This equation is of the form

$$e_\theta = A + B \cos 2\theta \tag{9b}$$

where A and B are constants for any particular state of stress.

$$A = \frac{e_p + e_q}{2} \quad \text{and} \quad B = \frac{e_p - e_q}{2}$$

The variation in e_θ can therefore be represented by a curve of the form of the function $f = \cos 2\theta$, the scale depending on the constants A and B which are functions of the measured strains.

The scale to which the $\cos 2\theta$ curve represents the strains is involved in the adjustment factor K of equation (1). To derive this equation, consider the ordinate of the $\cos 2\theta$ curve at an angle α to be a and at an angle $\alpha + 45^\circ$ to be b (see fig. 1), then

$$\cos 2\alpha = a \tag{10}$$

and

$$\cos (2\alpha + 90) = - \sin 2\alpha = b \tag{11}$$

but

$$\sin^2 2\alpha = 1 - \cos^2 2\alpha, \quad \text{also} \quad \cos^2 + \sin^2 = 1$$

therefore

$$b^2 = 1 - a^2 \tag{12}$$

but, from figure 1,

$$a = \frac{e_1}{K} - \frac{e_1 + e_3}{2K} \tag{13}$$

and

$$b = \frac{e_1 + e_3}{2K} - \frac{e_2}{K} \tag{14}$$

Simplifying and substituting in equation (12), yields the equation

$$(e_1 - 2e_2 + e_3)^2 = 4K^2 - (e_1 - e_3)^2$$

which, when solved for K, reduces to equation (1):

$$K = \frac{\sqrt{2}}{2} \sqrt{(e_1 - e_2)^2 + (e_2 - e_3)^2} \qquad (1)$$

IV. THE METHOD APPLIED TO FOUR GAGE LINES

The advantage of the four gage line rosette (reference 2) lies largely in the automatic check provided on the reliability of the strain measurements. Consider a fourth gage line (4) added to the three gage line rosette, bisecting the second and fourth quadrants. The "balance" of the rosette can be checked by the relation

$$e_1 + e_3 = e_2 + e_4$$

If the rosette is in perfect balance, the state of stress may be obtained from any three consecutive gage lines using the method as previously described. It is seldom, however, that the readings on a four gage line rosette balance exactly. Failure to balance by a considerable amount may mean that the measurement on one of the gage lines is unreliable. Further study of the data may indicate which of the gage lines is at fault, and this reading can then be eliminated, leaving a three gage line rosette.

In general, however, failure of the readings on the four gage lines to balance is by an amount consistent with the limits of accuracy of the strain gage. In such cases, all four readings may be used in determining the state of stress, by applying the method as outlined above, but with the following modifications.

1. The adjustment factor, K, will be determined from the equation

$$K = 0.707 \sqrt{(e_1 - e_2)(e_4 - e_3) + (e_2 - e_3)(e_1 - e_4)}$$

2. The ordinate, y, for locating the auxiliary line will be

$$y = \frac{\frac{e_1}{K} + \frac{e_2}{K} + \frac{e_3}{K} + \frac{e_4}{K}}{4}$$

3. The master curve will be adjusted so as to best fit the four points representing the plotted strains.

The merits of this method of reducing strain data can best be judged after the method has been applied to several cases. It will be found that the form in which the magnitudes and directions of the principal strains are obtained permits of direct interpretation. After several applications, the method can easily be followed from memory.

Aluminum Company of America,
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New Kensington, Penna., February 15, 1939.

REFERENCES

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2. Osgood, W. R., and Sturm, R. G.: The Determination of Stresses from Strains on Three Intersecting Gage Lines and Its Application to Actual Tests. National Bureau of Standards Research Paper No. 558, May 1933.

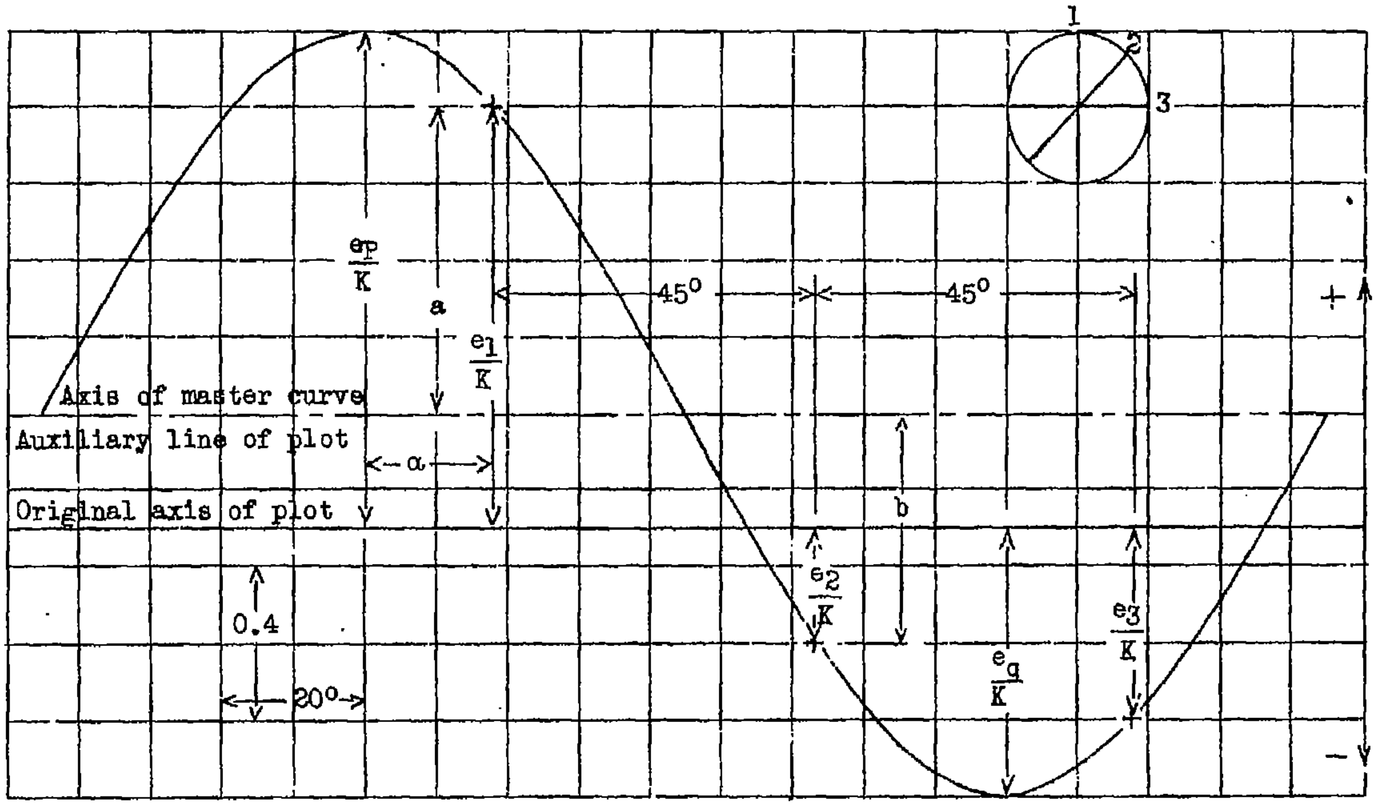

Figure 1.- Semi-graphical method for analyzing strains on 3 gage lines intersecting at 45°