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CHARTS FOR CALCULATING THE PERFORMANCE OF AIRPLANES HAVING CONSTANT-SPEED PROPELLERS

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SUMMARY

Charts are presented for determining the performance of airplanes having variable-pitch propellers, the pitch of which is assumed to be adjusted to maintain constant speed for all rates of flight.

The charts are based on the general performance equations developed by Oswald in reference 1, and are used in a similar manner.

Examples applying the charts to airplanes having both supercharged and unsupercharged engines are included.

INTRODUCTION

Within the past several years the two-pitch controllable propeller has been developed to a reliable form and at present the multiposition, or variable-pitch, propeller is being perfected. Because of these recent advances, Dr. C. B. Millikan suggested the problem of calculating a set of performance charts for airplanes having constant-speed propellers. The presentation of these charts is the subject of this paper. The authors wish to extend their appreciation to Dr. Millikan for invaluable assistance rendered throughout the preparation of the paper.

As the full-throttle brake horsepower of an airplane is approximately proportional to the engine speed, it is evident that the maximum brake horsepower for a given altitude will be realized only when the engine is operating at rated revolution speed; therefore, aside from special settings, the most efficient use of the variable-pitch propeller is obtained by adjusting its pitch so as to maintain the rated revolution speed for all speeds of flight.

Propellers operated in this manner have been termed "constant-speed propellers."

No attempt will be made to develop the basic performance equations or to describe in detail the method of calculating the present charts, as the process is similar to that of reference 1. The difference between the present charts and those developed for fixed-pitch propellers lies in the T_a and T_v functions, which give the variation of thrust horsepower available with altitude and velocity of flight.

GENERAL PERFORMANCE EQUATIONS

For the purpose of performance calculation, the characteristics of the airplane are represented by three design parameters:

$l_p = W/f$, parasite loading,

$l_s = W/e(kb^2)$, effective span loading,

$l_t = W/P_o \eta_o$, thrust horsepower loading,

where

W is gross weight of the airplane,

f, equivalent parasite area (sq. ft.) defined by the equation $f = C_{D_p} S$,

kb, Munk's equivalent monoplane span,

e, airplane efficiency factor,

η_o, design propulsive efficiency,

P_o, design brake horsepower.

These parameters are combined into a single parameter, Λ, which is plotted as the abscissa in the various performance charts,

$$\Lambda = \frac{l_s \, l_t^{4/3}}{l_p^{1/3}}$$

The maximum speed at sea level, V_m, is expressed in terms of the three design parameters and speeds for other conditions of flight, which are given in terms of R_v, a dimensionless speed ratio,

$$R_v = \frac{V}{V_m}$$

The fundamental performance equation

$$\frac{dh}{dt} = \frac{(t.hp._a - t.hp._r)\ 550}{W} \tag{1}$$

is expressed in engineering units in the form

$$\frac{dh}{dt} = \frac{33000}{\sigma R_v\ l_t}\left[(T_a T_v \sigma R_v - \sigma^2 R_v^{\ 4}) - \frac{l_s - l_t}{3.014\ V_m}(1 - \sigma^2 R_v^{\ 4})\right] \tag{2}$$

where

$\frac{dh}{dt}$ is rate of climb at altitude σ and velocity R_v,

$t.hp._a$, thrust horsepower available,

$t.hp._r$, thrust horsepower required,

$\sigma = \rho/\rho_o$ ($\rho_o = 0.002378$),

$$T_v = \frac{t.hp._a \text{ at velocity } V}{t.hp._a \text{ at } V_m} \qquad \text{(At sea level)}$$

$$T_a = \frac{t.hp._a \text{ at altitude}}{t.hp._a \text{ at sea level}} \qquad \text{(At constant velocity)}$$

By the imposing of the various flight conditions on equation (2), expressions are obtained enabling charts for the major performance characteristics to be calculated, when suitable expressions for T_a and T_v are introduced.

EXPRESSION FOR T_v

The T_v function gives the effect of variation of $t.hp._a$ at sea level due to different speeds of flight. In general,

$$t.hp._a = b.hp.\ \eta \qquad (3)$$

and, in the case of the constant-speed propeller, $b.hp. = b.hp._o$ at sea level for all speeds of flight; therefore

$$T_v = b.hp.\ \eta/b.hp._o\ \eta_o = \eta/\eta_o \qquad (4)$$

By reference to Weick's propeller charts (reference 2), the value of η_o is determined in the usual manner from the design values of the propeller parameters C_{s_o} and J_o, where J_o is V/nD.

$$C_{s_o} = \frac{0.638\ V_m\ \sigma^{1/5}}{P_o^{\ 1/5}\ N_o^{\ 2/5}}$$

$$J_o = \frac{88\ V_m}{N_o\ D}$$

As the b.hp. and N (r.p.m.) are invariant with speed, the values of C_s and J are both directly proportional to the speed of flight; hence,

$$C_s = R_v\ C_{s_o} \qquad (5)$$

$$J = R_v\ J_o \qquad (6)$$

For a given C_{s_o} and J_o it is possible, by using equations (5) and (6), to determine the propulsive efficiency η and the blade angle β as a function of R_v, which has been done for various values of C_{s_o} for both the BEST PERFORMANCE PROPELLER AND PEAK EFFICIENCY PROPELLER* on the propeller charts presented in figures 1 and 2, respectively.

As a definite J_o is associated with each C_{s_o} for both types of propeller, it is necessary only to specify the value of C_{s_o}. Since the value of η for different values of R_v is known, equation (4) enables a curve of T_v to be plotted as a function of R_v, which is the desired relation.

This T_v function will depend upon C_{s_o}, J_o, and the type of engine installation. Curves of T_v have been plotted in figure 3 based on the propeller chart for the case of fuselage 6 (reference 2), which is most representative for modern airplanes.

In order that the charts may be used for propeller designs other than those resulting in best performance or peak efficiency propellers, three representative curves were drawn to represent this family of T_v functions and have been used in calculating the performance charts. These representative curves were chosen to pass through T_{v_c} = 0.86, 0.90, and 0.94, where T_{v_c} is termed the "critical T_v" and is defined as the value of T_v for $R_v = 0.60$. The value $R_v = 0.60$ was chosen as that corresponding to the speed for best climb. By this specification of the curves, values of T_v used in calculating the

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*The BEST PERFORMANCE PROPELLER is defined as a propeller having its diameter and blade angle chosen so that η_o lies on the envelope of the family of efficiency curves.

The PEAK EFFICIENCY PROPELLER is defined as a propeller having its dimater and blade angle chosen so that η_o lies on the peak of its efficiency curve.

These definitions are those adopted by Oswald in reference 1.

performance charts will agree with the actual values at maximum velocity and at a speed near to that for best climb for any propeller design and will approximate values for other speeds very closely. The curve for $T_{v_c} = 0.94$ was chosen to include the cases in which, in order to reduce the take-off run, it is desirable to use a propeller of diameter greater than that of a peak efficiency propeller (reference 3), which gives a T_v curve having values slightly greater than 1.00 at values of R_v near 1.00 and lying as a whole above the T_v curves for either the best performance or the peak efficiency propellers. For reasonable increases in propeller diameter, the curves used in calculating the charts will give a sufficiently accurate approximation and, in any case, will be on the conservative side. In the calculation of the charts, it is necessary to know the derivative dT_v/dR_v. This derivative was graphically obtained by using a large scale because it was impossible to find a convenient analytical expression that would represent the T_v curves used.

EXPRESSION FOR T_a

The T_a function represents the variation of $t.hp._a$ at different altitudes for the airplane flying at constant velocity. The $t.hp._a$ equation, in which the subscript h refers to altitude conditions, becomes

$$t.hp.a_h = b.hp._h \, \eta_h$$

Three curves for variation of b.hp. with altitude are shown in figure 4. The lower curve labeled "Oswald" and used in reference 1 (fig. 22) is a combination of data obtained from references 3 and 4 and is representative of unsupercharged engines. The upper curve labeled "N.A.F." is a curve obtained from altitude-chamber tests at the Naval Aircraft Factory and is believed to be more representative of present-day supercharged engines. The middle curve is a mean adopted in calculating the charts and is somewhat conservative for the higher altitudes. It is expressed by the equation

$$\frac{b.hp._h}{b.hp._o} = \frac{\sigma - 0.117}{0.883} = \frac{\sigma - a}{1 - a}$$

From the condition that N remain constant, the value of V/nD is constant for a given velocity at all altitudes. The value of C_s, however, varies with altitude according to the relation

$$C_{s_h} = \frac{0.638\ V\ \sigma^{1/5}}{(b.hp._h)^{1/5}\ N_o^{2/5}} = C_s\ (\sigma = 1.0)\left[\frac{\sigma^{1/5}}{\left(\frac{\sigma - a}{1 - a}\right)^{1/5}}\right]$$

The change in propulsive efficiency with altitude at constant velocity, due to the change in C_s, was investigated for the various C_{s_o}'s for both the best performance and the peak efficiency propellers, for values of R_v from 0.6 to 1.0. The maximum increase in efficiency at 30,000 feet for the best performance propeller was found to be about 3 percent for C_{s_o}, ranging from 1.7 to 2.0, and at operating velocities. The increase in efficiency for the peak efficiency propeller in the same range was from 1 to 2 percent. For Cs_o less than 1.7, the change in efficiency at operating speeds at 30,000 feet was always within 1 percent, being sometimes positive and sometimes negative.

This change in propulsive efficiency with altitude at constant velocity was not considered sufficient to take into account in calculating the performance charts. Hence it is assumed that η will be the same for all values of σ, and the T_a expression becomes

$$T_a = \frac{b.hp._h\ \eta_h}{b.hp._o\ \eta_o} = \frac{b.hp._h}{b.hp._o} = \frac{\sigma - 0.117}{0.883}$$

From the foregoing discussion it is seen that this assumption leads to somewhat conservative values for thrust horsepower at the higher altitudes for high C_{s_o}.

By the use of these T_a and T_v functions, performance charts have been calculated as explained in reference 1. Figures 5 to 9 give, respectively: maximum velocity at altitude; absolute and service ceiling; maximum rate of climb; velocity for maximum rate of climb; and minimum time to climb to altitude.

USE OF CHARTS

The first step in making a performance calculation is to estimate the value of the parasite area f and the airplane efficiency factor e. This estimation is most conveniently accomplished by comparison with similar airplanes of known performance or by the use of figure 25 and table III of reference 1. Then, by assuming a value of V_m and using the known engine data, one can find the value of C_{s_o}. By the selection of the propeller diameter, the values of J_o and η_o are determined. This determination gives sufficient data to compute the design parameters l_p, l_s, and l_t. The value of V_m may then be computed and the assumed value checked.

Before the charts can be employed, the value of T_{v_c} must be determined, which is accomplished by taking 60 percent of the C_{s_o} and J_o values and obtaining a new η corresponding to the new C_s and J. If a best performance or peak efficiency propeller is used, this new η may be found from figure 1 or figure 2 at $R_v = 0.6$ for the design C_{s_o}. Dividing this new η by η_o gives the value of T_{v_c}. If T_{v_c} is known and Λ is computed, the charts may be used to find the performance characteristics.

Should the airplane have a supercharged engine, these charts may be used to calculate the full-throttle performance above the critical altitude by either of two methods given in reference 5. The second of these methods, which will be used in the second example, consists of using a fictitious unsupercharged engine of increased b.hp. chosen to give the actual design b.hp. at the critical altitude. All performance characteristics below the critical altitude are invalid and should be discarded.

EXAMPLE I - UNSUPERCHARGED ENGINE

Given: W = 8,500 lb. S = 390 sq. ft.

b = 48 ft. f = 7.6 sq. ft.

P_o = 700 hp. at 2,000 r.p.m. e = 0.82

Assume:

V_m = 220 m.p.h. at sea level

C_{s_o} = 1.8 for 220 m.p.h., 700 hp., at 2,000 r.p.m.

η_o = 0.86, for best performance propeller (fig. 1).

Calculate:

$l_p = 1{,}118$

$l_s = 4.5$ $\qquad l_s\, l_t = 63.5$

$l_t = 14.1$ $\qquad l_p\, l_t = 79.3$

Find:

V_m = 218.5 m.p.h. (fig. 29, reference 1)

If this calculation does not check the assumed value of V_m closely enough, a new choice must be made, and the procedure repeated.

$\eta = 0.747$ at $R_v = 0.6$ for $C_{s_o} = 1.8$ (fig. 1)

If the propeller were not a best performance or a peak efficiency propeller, J_o corresponding to the design diameter would be found:

J_o = 1.035 (in this case for C_{s_o} and best performance setting)

Then find 0.6 C_{s_o} = 1.085 and 0.6 J_o = 0.621 and, from figure 14 of reference 2, read $\eta = 0.747$ at $C_s = 1.08$ and $J = 0.621$.

Calculate $T_{v_c} = \eta\ (R_v = 0.6)/\eta_o = 0.747/0.860 = 0.87$

Calculate $\Lambda = 14.75$

Enter charts with $\Lambda = 14.75$ and $T_{v_c} = 0.87$

Results:

Standard altitude	V_{m_h}	V_c	C_H	T
ft.	m.p.h.	m.p.h.	ft./min.	min.
0	218.5	129.3	1,210	0
10,000	206.0	135.5	592	11.6
20,000	164.5	146.0	42.6	88.0
Service ceiling				
18,800	. . .	. . .	100	. . .
Absolute ceiling				
20,900	147.0	147.0	0	. . .

EXAMPLE II - SUPERCHARGED ENGINE

Given:

$W = 8{,}500$ lb. $\quad$ $S = 390$ sq. ft.

$b = 48$ ft. $\quad$ $f = 9.0$ sq. ft.

$P_o =$ 600 hp. at 2,000 r.p.m. at 10,000 ft. $\quad$ $e = 0.82$

(The subscript f denotes fictitious conditions at sea level.)

Assume:

$V_{m_f} = 220$ m.p.h.

$F = 0.705$ at 10,000 feet (fig. 4)

$P_{o_f} = 600/0.705 = 852$ hp.

$C_{s_{o_f}} = 1.74$ for 220 m.p.h., 852 hp. at 2,000 r.p.m.

$\eta_{o_f} = 0.853$, for best performance propeller (fig. 1)

$\eta = 0.737$ at $R_v = 0.6$ for $C_{s_{o_f}} = 1.74$ (fig. 1)

Calculate $T_{v_c} = 0.737/0.853 = 0.865$

Calculate:

$l_p = 945$

$l_s = 4.5$ $\qquad l_s\ l_{t_f} = 52.7$

$l_{t_f} = 8500/(0.853)(852) = 11.7 \qquad l_p\ l_{t_f} = 80.7$

Find $V_{m_f} = 221.5$ m.p.h. and check assumption

Calculate $\Lambda = 12.2$

Enter charts with $\Lambda = 12.2$ and $T_{v_c} = 0.865$

Results:

Standard altitude ft.	V_{m_h} m.p.h.	V_c m.p.h.	C_H ft./min.
0	(221.5)		
10,000	211.0	134.5	798
20,000	183.0	144.0	171
Service ceiling 21,200	. . .	. . .	100
Absolute ceiling 22,900	147.5	147.5	0

CONCLUSION

Because insufficient flight-test data are available for airplanes having constant-speed propellers, it has been impossible to determine the accuracy of the charts by comparison with flight-test data. It is reasonable to expect the same order of accuracy as is obtained from Oswald's similar charts for fixed-pitch propellers.

California Institute of Technology,
Pasadena, California, March 1936.

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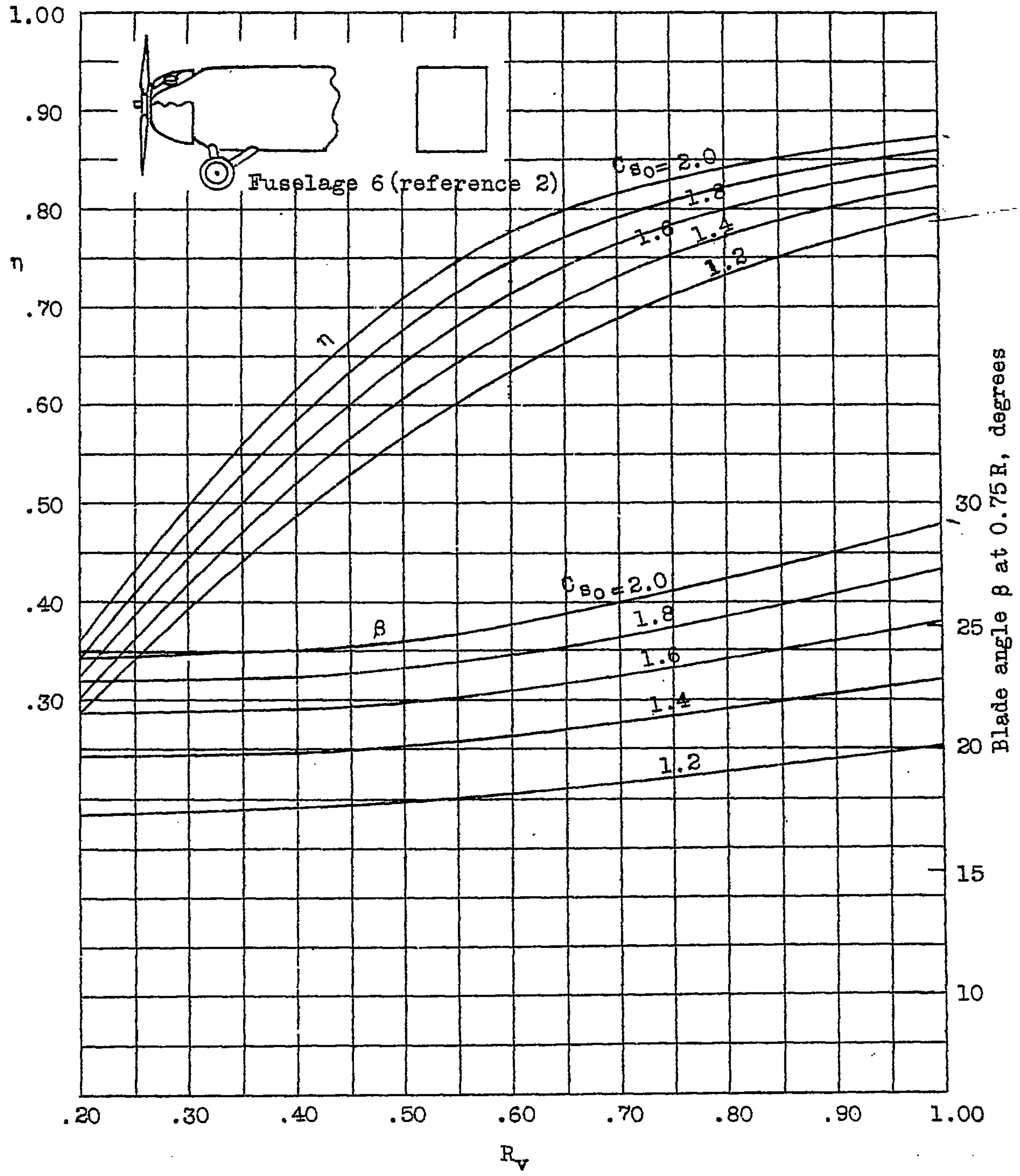

Figure 1.- Characteristics of two-bladed, constant-speed best performance propeller.

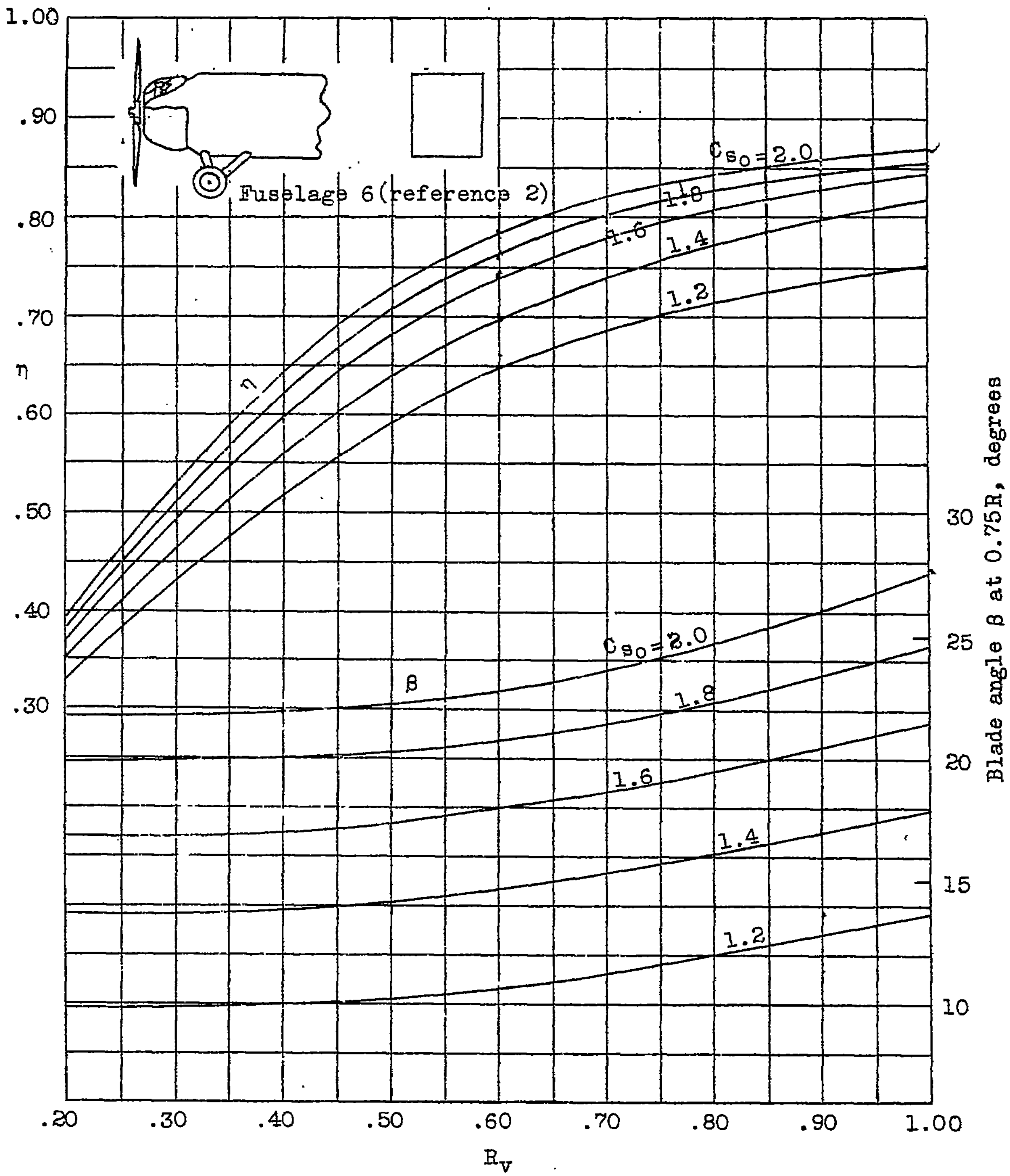

Figure 2.- Characteristics of two-bladed constant-speed peak efficiency propeller.

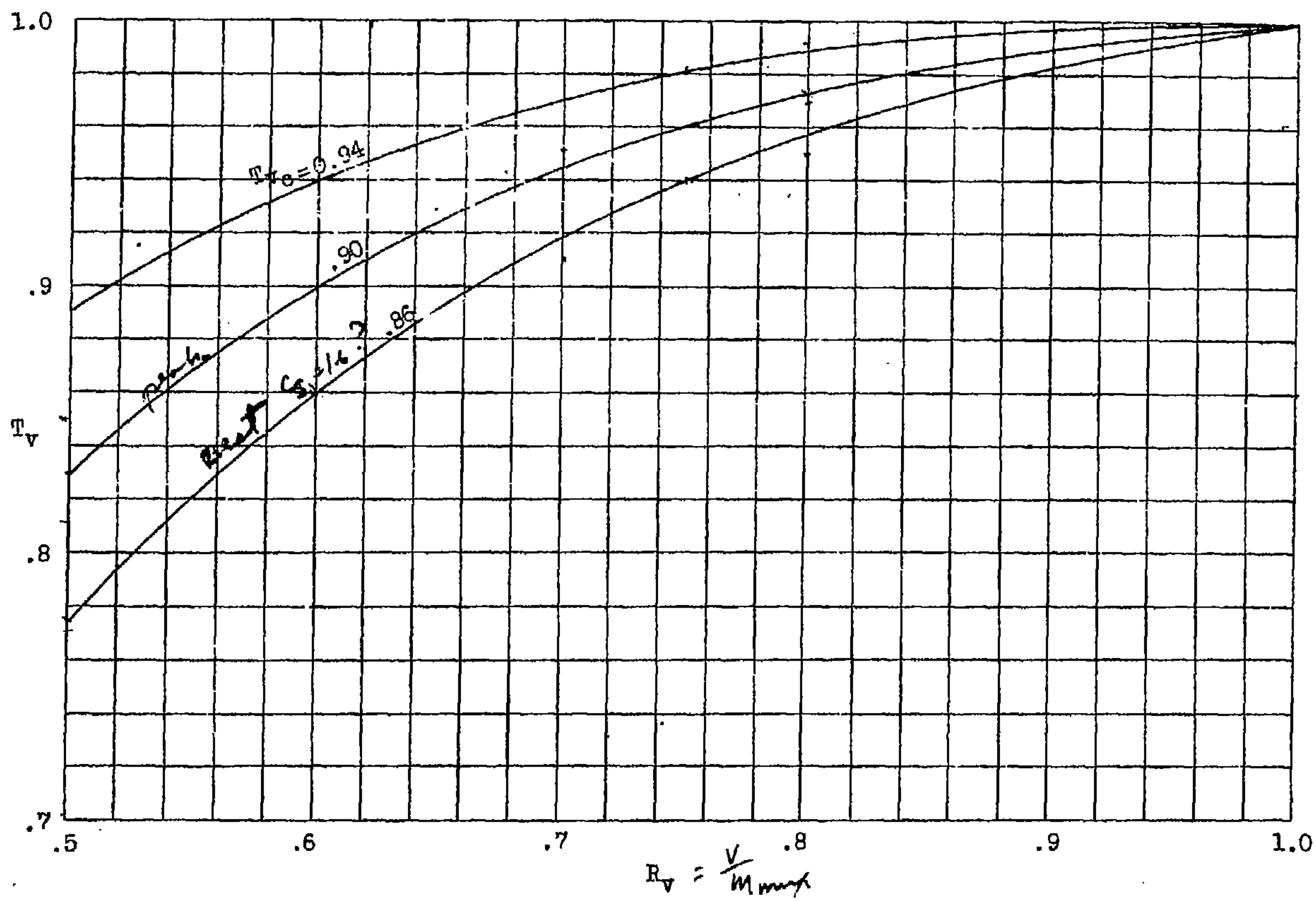

Figure 3.- Variation of thrust horsepower with velocity at constant engine speed.

$$T_v = \frac{\text{t. hp. at } R_v}{\text{t. hp., at } R_v = 1.0}$$

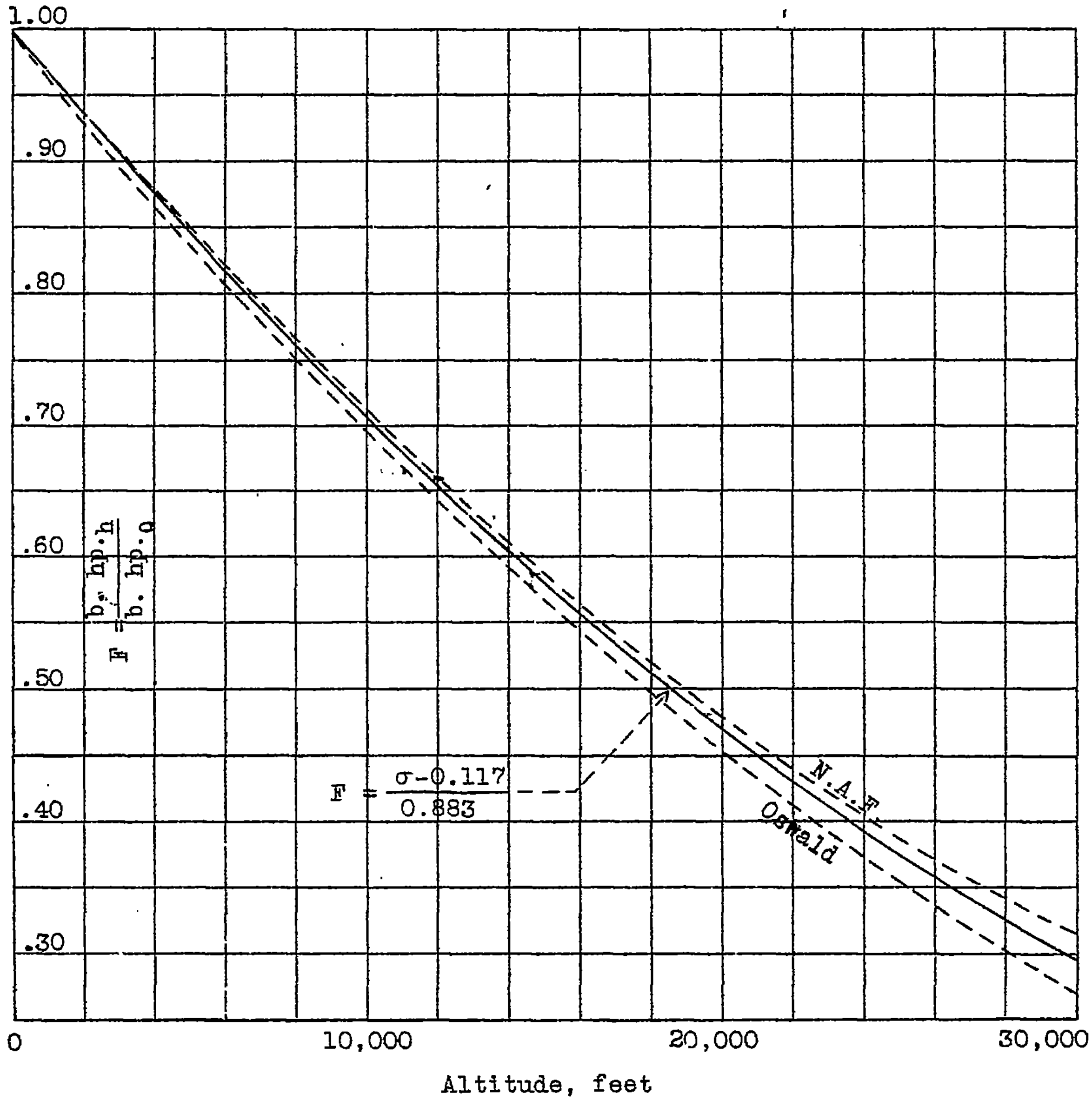

Figure 4.- Variation of full-throttle brake horsepower at constant r.p.m. with altitude.

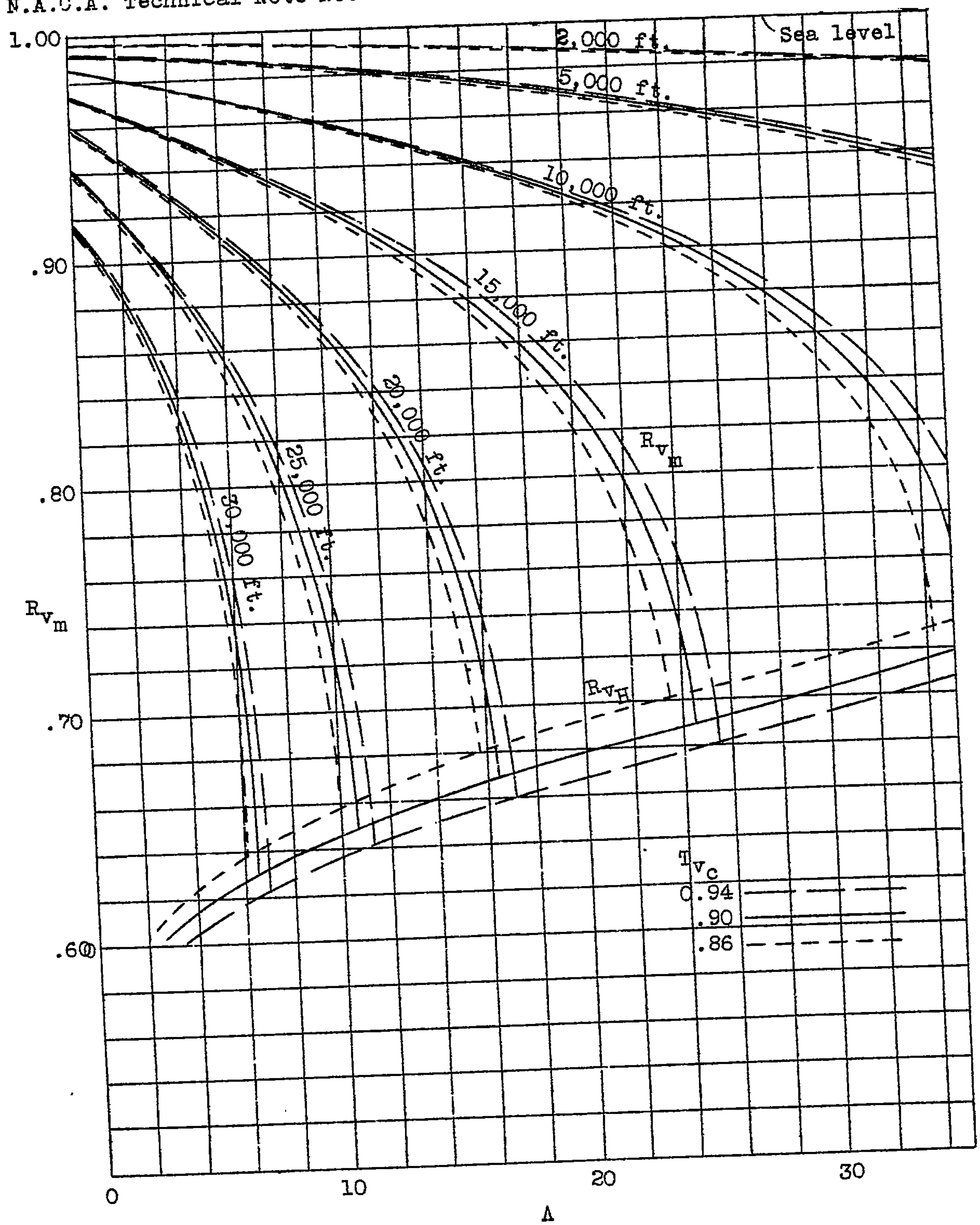

Figure 5.- R_{v_m} and R_{v_H} as functions of Λ.

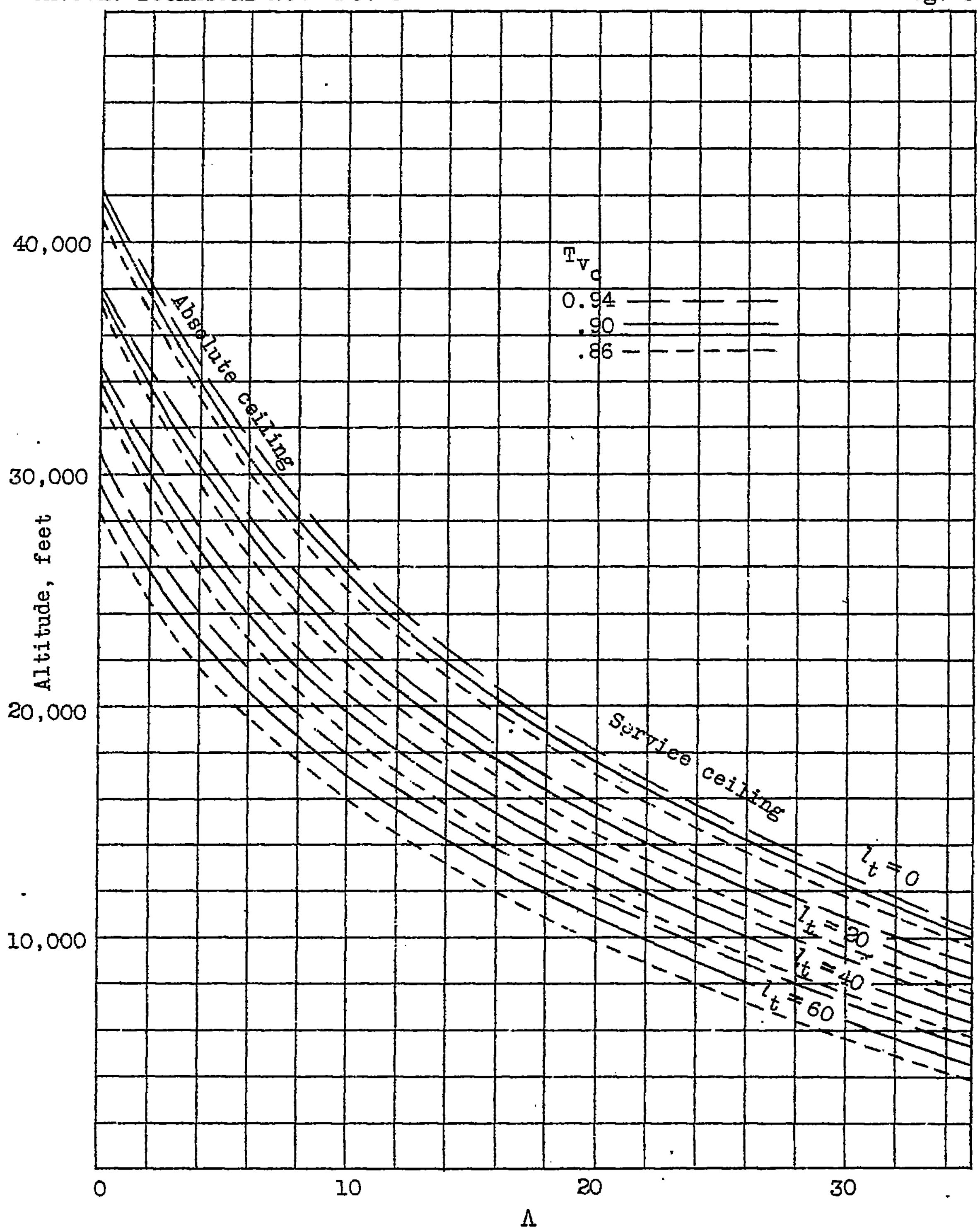

Figure 6.- Absolute and service ceiling as functions of Λ.

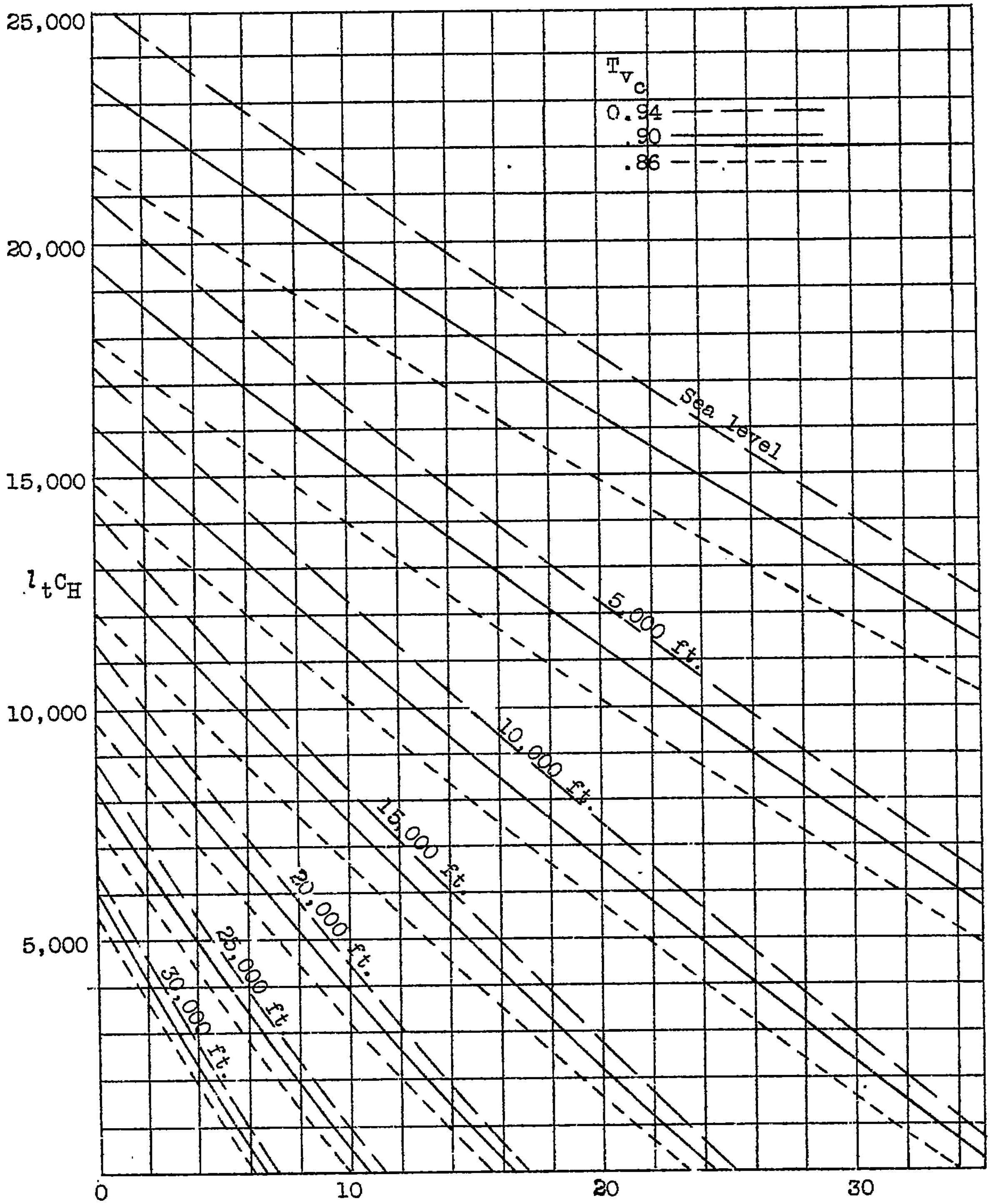

Figure 7.- l_tC_H as a function of Λ at various altitudes. C_H is the maximum rate of climb in feet per minute.

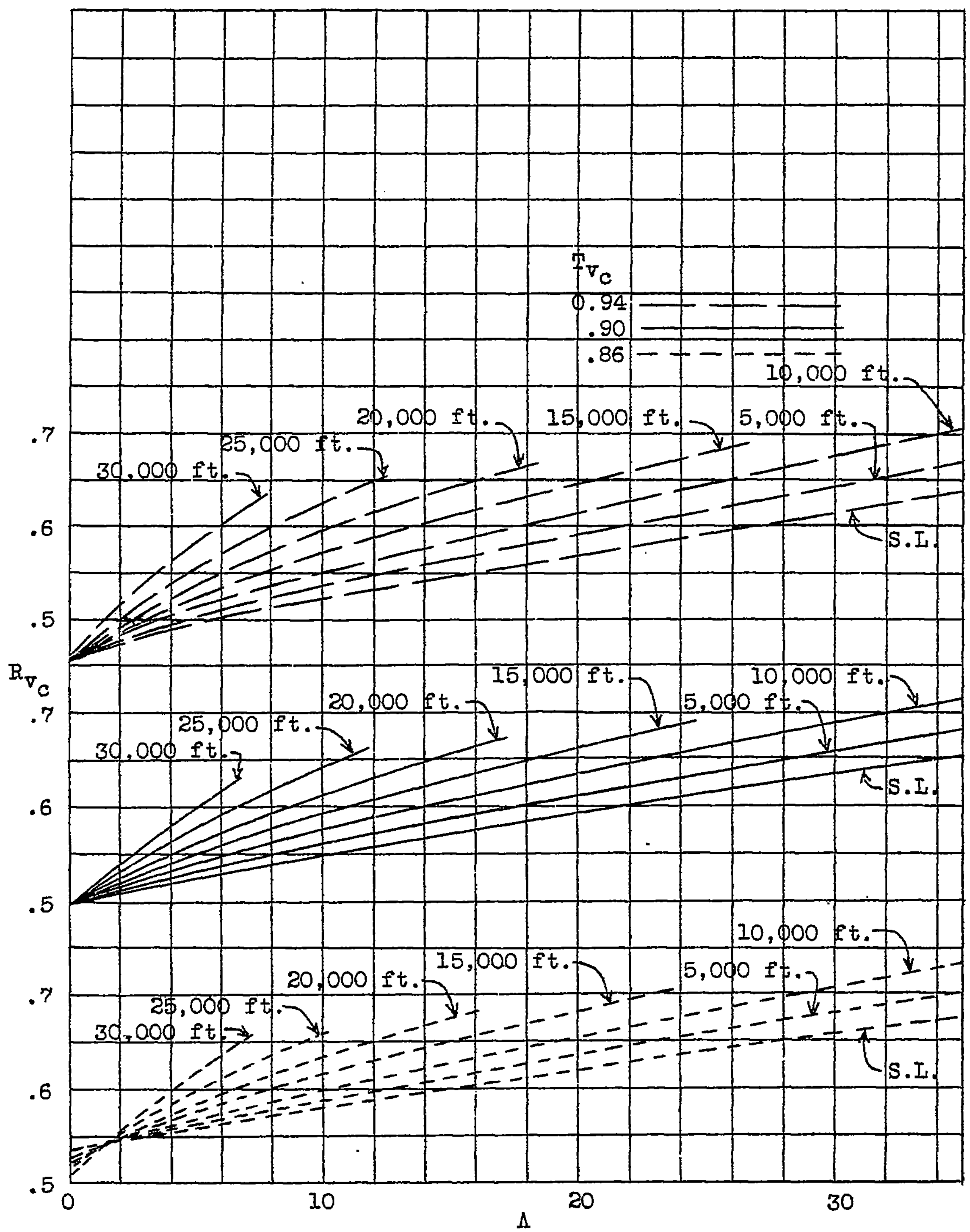

Figure 8.- R_{V_c} as a function of Λ at various altitudes.

$$R_{V_c} = \frac{\text{velocity for maximum rate of climb}}{\text{maximum velocity at sea level}}$$

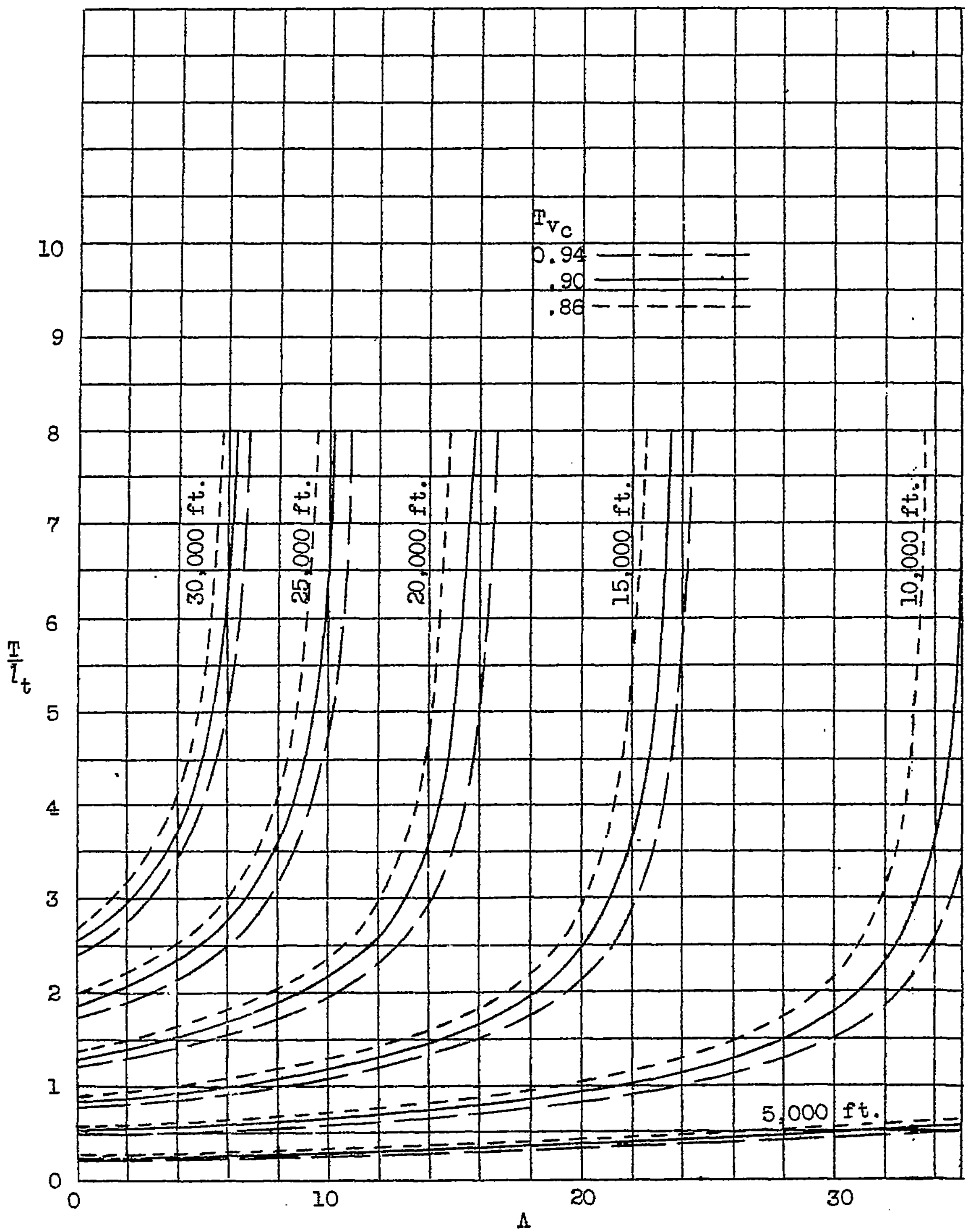

Figure 9.- T/l_t as a function of A at various altitudes. T is the minimum time required to climb to altitude (minutes)