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TECHNICAL MEMORANDUM. 56

VIBRATIONS OF AVIATION ENGINES.

By

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Taken from
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1. Simple system of a mass and a spring. - If we consider a simple system, consisting of a mass M resting on springs (Fig. 62), and if a force F acts on the system, it moves a certain distance dz. If the force ceases suddenly, the system resumes its position of equilibrium by a series of oscillations on either side of this position. The period of these oscillations is the particular period belonging to the system. Let us assume that the recoil force of the spring is proportional to its distortion and that the resisting force of friction, which damps the oscillations, is proportional to their velocity. If z is the value of the ordinate of the center of gravity of the mass $M = \frac{P}{g}$ (P, weight in kg.; g, acceleration due to gravity) at a given instant, its velocity is $\frac{dz}{dt}$, its acceleration is $\frac{d^2 z}{dt^2}$ and the corresponding force of inertia is $-\frac{P}{g} \times \frac{d^2 z}{dt^2}$. This force, according to the classic theorems of mechanics, is equal to the sum of the impressed forces, namely, the recoil force cz and the friction $2b \frac{dz}{dt}$ (in neglecting the weight). The general equation of the motion is accordingly

(1) $$M \frac{d^2 z}{dt^2} + 2b \frac{dz}{dt} + cz = 0$$

which is often put, in order to simplify the calculations, in the following form

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* From "Les Nouveaux Moteurs d'Aviation," (published by Berger-Levrault, Paris), Chap. V, pp. 277-293. 1921.

(2) $$\frac{d^2z}{dt^2} + 2h\frac{dz}{dt} + k^2 z = 0,$$

by making

$$h = \frac{b}{M} \text{ and } k = \sqrt{\frac{g}{M}}$$

The general solution of this equation takes the form $Z = e^{st}$, s being the root of the equation of the second degree:

$$s^2 + 2hs + k^2 = 0;$$

whence:

$$s = -h \pm \sqrt{h^2 - k^2}$$

$$Z_1 = A_1 e^{(-h + \sqrt{h^2 - k^2})t} + A_2 e^{(-h - \sqrt{h^2 - k^2})t}$$

If $h^2 - k^2 > 0$, z keeps on decreasing. If $h^2 - k^2 < 0$, Z is a sinusoidal function

$$Z_1 = Ae^{-ht} \cos(\sqrt{(k^2 - h^2)}t + \varphi),$$

which comes from

$$Z = A_1 e^{-ht} (D_1 \sin \varphi_1 t + D_2 \cos \varphi_1 t).$$

In the second case, the exponential term in e^{-ht} likewise decreases. The amplitude of the oscillations thus decreases rapidly with t and the principal period of these damped oscillations (Fig. 63) is equal to

$$\frac{2\pi}{\sqrt{k^2 - h^2}}$$

The coefficients h and k^2 characterize the suspension and can be determined experimentally, which makes it possible to find the period of this simple system.

2. Engine placed on an elastic support. - In reality, in the case of an airplane, the mass M is replaced by an engine in operation and the spring is replaced by a frame and a fuselage more or less elastic. The momentary force considered above is replaced by periodic impulses transmitted by the engine and due to variations in the engine couple and to the forces of inertia proportional to the square of the angular velocity. These impulses may be represented by the expression

$$A_1 \omega^2 \sin(\omega t + \alpha_1) + A_2 \sin(\omega t + \alpha_2)$$

by neglecting the terms of the succeeding orders in $2\omega t$, etc. These periodic impulses give place to what are called sustained vibrations (Fig. 63). If we assume ω to be constant and adopt a suitable origin for the arcs, this expression may be put under the general form $F\omega^2 \sin \omega t$.

By likening the engine frame to a single spring and assuming ω to be constant, the general equation for the motion of the engine in the vertical plane becomes:

(3) $$\frac{d^2z}{dt^2} + 2h\frac{dz}{dt} + k^2z = E \sin \omega t, \text{ in which } E = \frac{\omega^2 F}{M}$$

In order to have the resultant motion, we must consider the motions along the other two axes of the coordinates, the lateral and longitudinal axes of the engine, which yield analogous equations.

The general solution of the preceding equation is the sum of an integral of the equation, without the second member, and a particular solution of the complete equation. Since the integral of the equation, without the second member, tends rapidly toward 0 when t increases, as we have already seen, we will consider only the particular solution of the complete equation. This solution is a periodic function of the form $z = B \sin \omega t + C \cos \omega t$, the coefficients B and C being determined by substituting this value for z in equation 3, and by treating as identical the coefficient of each term in both members. We thus find that z may be put under the form

$$z = \frac{E}{\rho} \sin(\omega t - \alpha),$$

in which

$$E = \frac{A\rho}{\cos\alpha} = -\frac{B\rho}{\sin\alpha} \qquad \rho = \sqrt{(k^2 - \omega^2)^2 + 4h^2\omega^2}$$

$$\text{tang } \alpha = \frac{2h\omega}{k^2 - \omega^2} \qquad A = \frac{E(k^2 - \omega^2)}{(k^2 - \omega^2)^2 + 4h^2\omega^2}$$

$$B = -\frac{2h\omega E}{(k^2 - \omega^2)^2 + 4h^2\omega^2}.$$

When h is negligible, the denominator of A becomes 0 and when $\omega^2 = k^2$, the amplitude of the vibrations tends to increase indefinitely, giving us a synchronism. In practice, the friction is appreciable and the amplitude remains finite. Nevertheless, since h is small, the amplitude is perceptible, when $\omega^2 = k^2$, which renders tang α infinite, $\alpha = \frac{\Pi}{2}$. For this critical velocity, the maximum of z is displaced 90° with reference to the engine couple.

In reality, since the velocity ω is not strictly constant and since the variations of the couple and the forces of inertia are large, the general solution is a periodic function decomposable into elementary series of sinusoidal form, admitting many harmonics, whose periods are functions of ω and its multiples and whose first term is given below:

$$z = \frac{E}{\rho} \sin (\omega t - \alpha).$$

The engine transmits the vibrations to the support by means of a spring, in reality with the interpolation of a more or less elastic joint. The corresponding force can be represented at each instant by the tension of the spring, which is equal to $cz = k^2 M \frac{E}{\rho} \sin (\omega t - \alpha)$, that is, on replacing the letters by their values in terms of the coefficients of the primitive equations (1) and (3) (formula given by Mr. Lecornu)

$$\Phi = \frac{\omega^2 F c}{\sqrt{(c - M\omega^2)^2 + 4b^2\omega^2}} \sin (\omega t - \alpha) = E' \sin (\omega t - \alpha)$$

Such is the variable and periodic force with which the spring presses on the engine support. This force admits the same period as the forces of inertia of the engine.

For an engine with K cylinders, we saw that the forces transmitted are periodic functions of $K \omega t$ and $\frac{K}{2} \omega t$, which modifies the values of ω corresponding to the critical periods.

It is advantageous to reduce to a minimum the amplitude F' of this force Φ and, for this purpose, to reduce the forces of inertia F and the angular velocity ω, and to increase M and the coefficient of damping. In an extreme theoretical case, M is very large, the coefficients b of damping and c of the recoil tension of the spring are negligible, in comparison with M, and the maximum value of Φ tends toward $\frac{Fa}{M}$ and is independent of the velocity. The engine is mounted on a very light and flexible support, constituted, for example, by two overhung girders. In the case when the engine support is rigid, the recoil tension of the spring becomes predominant and the maximum value of Φ tends toward $\omega^2 F$. Aside from these extreme cases, the amplitude Φ varies with ω.

Under the influence of this periodic force, the engine support has therefore a tendency to vibrate, which makes it important that the period corresponding to the number of explosions or revolutions of the engine should not coincide with the natural oscillation period of its support. The vibrations of the support react in turn on the engine and are susceptible of varying the critical speed of certain revolving parts, for example, the crank shaft.

The combined engine and its supporting airplane in reality represent a sort of double pendulum consisting of the engine with a mass M, connected more or less elastically with its frame, and of the airplane itself, which may be likened to a certain mass M' resting on fixed points, by means of an intermediate complex spring. These fixed points are materialized either in the contact points of the wheels with the floor of the hangar, or in the points of application of the forces of sustentation to the wings, when the airplane is in flight. The critical speed may therefore differ in the two cases. In practice, the vibrations are greater on the ground, than during flight.

Under the influence of the engine vibrations, the airplane tends to assume a vibratory motion, whose equation has the same form as the one already indicated for an engine supported only by springs. If we let y represent the displacements of the engine, we will then have, in order to determine its value, after making the same hypotheses for the sake of simplification,

$$M' \frac{d^2 y}{dt^2} + 2b' \frac{dy}{dt} + c' y = E \, (\sin \omega t - \alpha) \qquad (3')$$

the same equation as above (3) in which the corresponding letters are accentuated.

The same conclusions apply therefore to the supporting airplane. The coefficients of this equation are much more complex to determine, but the general solution is still a complex periodic function, with the same period as that of the engine, also resolvable into elementary sinusoidal functions.

The period of the oscillations proper of the support is shortened in proportion to the rigidity of the system and the shortness of the displacements allowed by the connections.

The critical speeds corresponding to equations (3) and (3') are not generally the same, but there are nevertheless critical speeds for the whole system of this sort of double pendulum constituted by the engine and the airplane.

The foregoing general equations occur in the study of all oscillatory and periodic phenomena, especially in alternating electric currents, which originate in a circuit containing a self-induction coil and a condenser. The coefficients simply change their signification.

Since the mass of the supporting frame is small, it is important to make it either as rigid as possible, so that it will have its own vibration periods, much shorter than those due to the explosions and the forces of inertia, or very flexible, so that it will have very long periods, different from those of the disturbing forces. In the first case, the engine is supported by a very rigid frame of wood or metal, with the interposition of elastic pads to act as shock absorbers, and is held in position by Belleville washers designed to preserve the contact. In the second case, the engine is supported by two flexible wooden beams. In reality, the whole airplane forms the support and is subjected to the vibration of the engine It is important to determine the vibration nodes, in order that they may not become dangerous in any vicinity by producing distortions beyond the elastic limit of the substance. We know that the vibration period of simple systems (such as cables, struts, spars and levers of simple geometric form and practically uniform cross-section and of small size in comparison with their length) depends on the cross-section and the distance between their fixed points. These fixed points are generally the points of support determined by the construction of the airplane itself, and which consequently, cannot be shifted. In order to change the vibration period, it is sometimes possible to create supplementary nodes by the introduction of intermediate stays and struts.

3. Vibrometers and accelerometers. - In order to determine experimentally the amplitude of the vibrations, the comparative method is employed, either by utilizing empirically the impression received by holding the hand on the vibrating part or, more scientifically, by examining the indications of a vibrometer.

A vibrometer should give not only the period, but also the value of the maximum local acceleration at any instant and the maximum amplitude of the oscillations, indispensable elements for characterizing numerically the destructive power of any shaking in the airplane. Most of the vibrometers utilize the inertia pendulum. We will only mention the one of Auclair and Boyer-Guillon and the one of Bourlet and A. de Gramont de Guiche.

The essential part of the inertia pendulum is a mass partially solid with the system and guided in such manner as to be able to move in only one given direction. This mass, under the influence of the effect of inertia and opposing forces, takes a relative motion with reference to the system. The pendulum and its damping device must have an inertia as small and a coefficient of damping as large as possible, in order to minimize the "displacements" of the inscriptions and the "lancers." In general the opposing force is produced by a spring or by the weight. We may adopt, for the displacements of the pendulum, the law already indicated for the vibrations proper of the simple system. The relative displacement x of the inertia mass and the absolute displacement y of the pendulum support are both assumed to be motions of translation. The force transmitted to the pendulum by its support is proportional to the acceleration of the support, that is to $\frac{d^2y}{dt^2}$ which is a function of the time t. The differential equation binding x and y is consequently the following, according to the one already given at the beginning of the chapter for the motions of an engine resting on a spring:

$$a \frac{d^2x}{dt^2} + 2b \frac{dx}{dt} + cx = \frac{d^2y}{dt^2} \qquad \frac{d^2y}{dt^2} = F(t).$$

in which a is the coefficient of inertia of the pendulum, b the coefficient of damping and c the coefficient of recoil. These coefficients are characteristic of the instrument.

What we observe are the displacements x of the pendulum. From them we deduce by means of the above equation, the function F(t) of the displacements of the support. The pendulum thus becomes an amplifier for revealing the scarcely visible motions of the support.

The pendulum is generally provided with a recording mechanism which automatically traces a curve representing, as a function of the time, the relative motion of the inertia mass. This curve is a graphic representation of the solution of the differential equation binding y to x.

As a particular solution of this equation, we may consider the one which satisfies the initial conditions of the motion. We have already seen that the general integral of the equation in the second member tends towards 0 at the end of a certain time, so that, after the speed has been once established, the inertia pendulum follows the law given by the particular integral. Consequently, whenever the initial conditions of the relative motion of the inertia mass are fixed, the diagrams obtained are comparable, when the function $F(t)$ remains the same.

Every periodic motion of the support, the frequency of which has become established, can be recorded by the inertia pendulum and the same analyzed. The complex periodic motion of the support may be considered, according to Fourier's theory, as the resultant of the simple sinusoidal motions, $f(t)$, $f_2(t)$ .... $f_n(t)$, with periods equal to sub-multiples of the period of the resultant motion $T \frac{T}{2} .. \frac{T}{n}$:

$$F(t) = f_1(t) + f_2(t) + \quad + f_n(t).$$

The search for an integral of the complete equation returns to that of the simple elementary equations. Now, an integral of the differential equation:

$$f(x) = f_n(t) = K_n \cos (\alpha_n t + \beta_n) \qquad \alpha = \frac{2\pi n}{T}$$

is a simple sinusoidal function, displaced with reference to it. Hence, if the motion of the pendulum support is a simple sinusoidal motion, the motion of the inertia pendulum will also be a simple sinusoidal one with the same period, but with a certain difference in phase. Their amplitudes will bear a definite ratio, provided there is no synchronism. If the motion of the support is a complex periodic motion, the motion of the pendulum will also be complex and have the same period. Thus we have the theoretical means for recording the vibrations. The diagram obtained makes it possible to determine, for each value of t, that of $F(t)$, by measuring on it the value of x, of $\frac{dx}{dt}$ and of $\frac{d^2x}{dt^2}$.

In order to be able to obtain practical results, the coefficient of damping must remain nearly constant, the recording device must introduce no disturbance, nor be subjected to shocks from its support, and the vibration period of the pendulum must not be the same as that of its support.

The manufacture of these instruments is difficult. In fact, we must keep the recoil force strong enough so the pendulum will not be sensitive to slight external perturbations. The damping, on the contrary, must be so slight as not to injure its sensitiveness. In order that its period may be large, that is, that the expression $\frac{ac - b^2}{a}$ be large, it is advantageous to have a long or heavy pendulum.

The accelerometer of Auclair and Boyer-Guillon (Fig. 64), sometimes called a maximum accelerometer, has a mass supported by a spring, whose tension can be regulated and which rests on a shoulder. The mass remains stationary so long as the force of inertia is less than the sum of the tension of the spring and the weight of the mass. The maximum ordinate is noted by an amplifying device with a recording drum. The instrument may be adjusted so that the mass will leave the shoulder only at the instant of this maximum, by utilizing, for example, an electric contact. At this instant the mass has the same acceleration as its support and we have the equation $a' \frac{d^2y}{dt^2}$ (force of inertia) $= b'R$ (tension of spring) $+ c'P$ (weight), a', b', and c' being the constants of the instruments.

We thus have the means for studying the vibration phenomena. In a simple sinusoidal motion, the maximum acceleration and the amplitude of the motion are combined by a simple law resulting from the properties of the sinusoidal functions:

$$x = A \sin \omega t,$$
$$\frac{dx}{dt} = A\omega \cos \omega t,$$
$$\frac{d^2x}{dt^2} = - A\omega^2 \sin \omega t.$$

The maximum amplitude is A and the maximum acceleration is equal to

$$A\omega^2 \text{ when } \omega = \frac{2\pi n}{60}$$

n being the number of revolutions or double oscillations per min.

When the vibrations are very complex, which is the case with engines, it is necessary to increase the number of measurements, with varying tensions of the spring, which constitutes a difficult operation.

In the vibrometer of Carlo Bourlet and A. de Gramont de Guiche (Fig. 65), it has been sought to obtain the minimum inertia and the maximum coefficient of damping, while conserving a very great sensitiveness. The very light rod is attached to the very taut membrane of a Marey air capsule. The oscillations of this rod, mounted on a vibrating part of the engine support, acquire an oscillatory motion with the same period as the support, and with an amplitude which is a function of that of the support. The oscillatory motion produces varying air pressures in the capsule, which are transmitted by a flexible tube to a second identical capsule, insulated from the vibrations of the engine frame and connected with a recording stylus. The instrument is completed by a tuning-fork whose synchronous vibrations are recorded on the same strip of paper, unrolled at a uniform speed by a recording drum, and give the measurement of the time.

The diagrams in Fig. 66 show the interesting results obtained. They correspond, of course, to the vibratory condition produced by the engine at the point of the support to which the capsule is applied.

The instrument has three capsules, placed in three different rectangular planes, which consequently, give the three component vibrations.

In order to enable a comparative study of the vibrations of two engines, it is necessary to employ the same support and the same revolution speed. By placing it on various parts of the engine support, or of the airplane it enables the determination of the critical speeds, that is, the maximum vibrations at any point. It has not yet been possible to determine, for each speed, the absolute values of the vibrations. The indication of their relative size already constitutes, however, a valuable indication.

These have been made on the theory of the vibrations of blades and of frequency meters, which give information on the number of revolutions, but not on the amplitude and acceleration of the motions. As a vibrometer without inertia and of great sensitiveness, it will perhaps be possible sometime to utilize lamps with grills, which will enable the measuring of the vibrations of a small metal cylinder, solid with the engine frame, by the vibrations of the electric field produced in an annular solenoid, independent of the engine frame.

Accidental causes of vibrations. - Any failure in the equilibrium of the rotating masses or any accidental variation of the couple or torque causes vibrations or tremblings, according to the character (periodic or not) of these actions.

Among the moving masses, especial importance must be attached to the propeller, not only from the point of view of its static but also of its dynamic equilibration, to its perfect symmetry from the aerodynamic point of view and to its mounting on the engine. The propeller axis must be perpendicular to that of the engine, so that it will describe a disk or a cone. The breaking of a propeller blade, from a lack of balancing of the centrifugal forces, may result in wrenching the engine from its supporting frame. The propeller must be attached to the engine shaft in such manner that its motion will be perfectly regular. Any displacement of the propeller about its hub causes, by reason of its inertia and variations of the engine couple, dangerous vibrations and friction, capable of raising its temperature even to setting the wood on fire.

Variations of the couple, due either to the fuel supply or its carburetion, or to the filling of the cylinders (distribution, intake, valves, tightness of the piston rings) or to the ignition, cause serious disturbances from the point of view of the vibrations. These variations must be closely watched and reduced to the minimum, before making any alteration in the engine or its support, for the sake of reducing the vibrations.

Translated by the National Advisory Committee for Aeronautics.

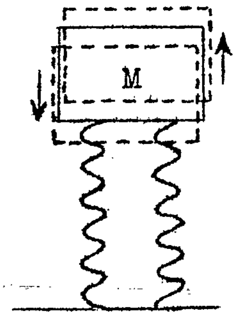

Fig. 62. - Vibrations of a mass supported on two springs.

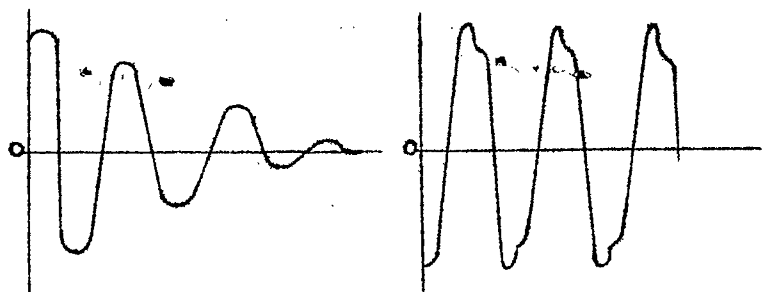

Fig. 63. - Oscillations.

a) Damped oscillations. b) Sustained oscillations.

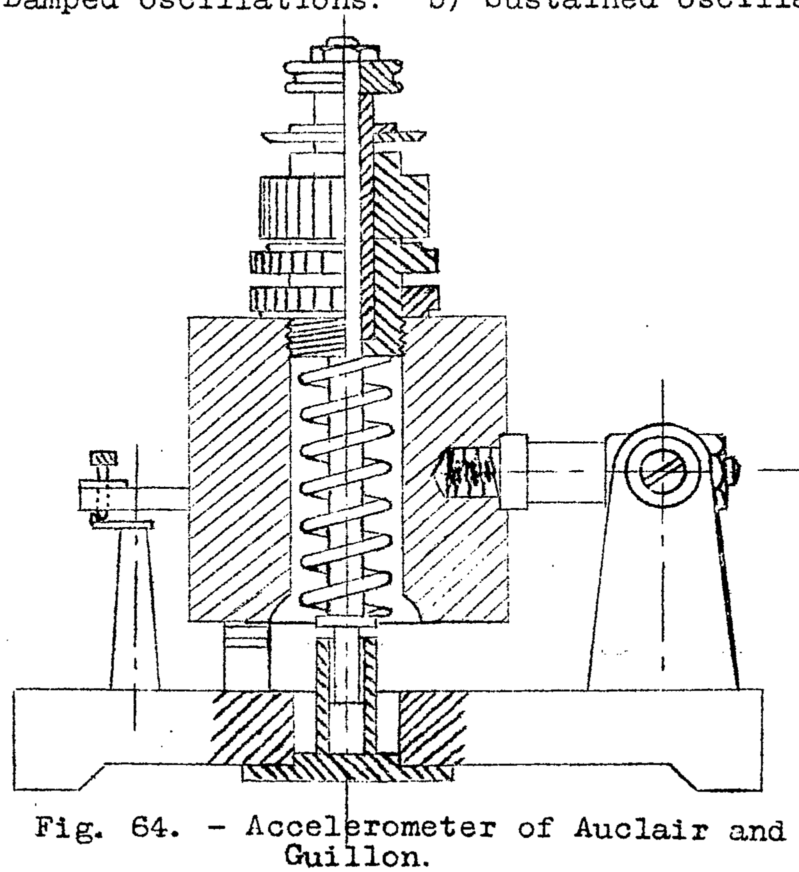

Fig. 64. - Accelerometer of Auclair and Boyer-Guillon.

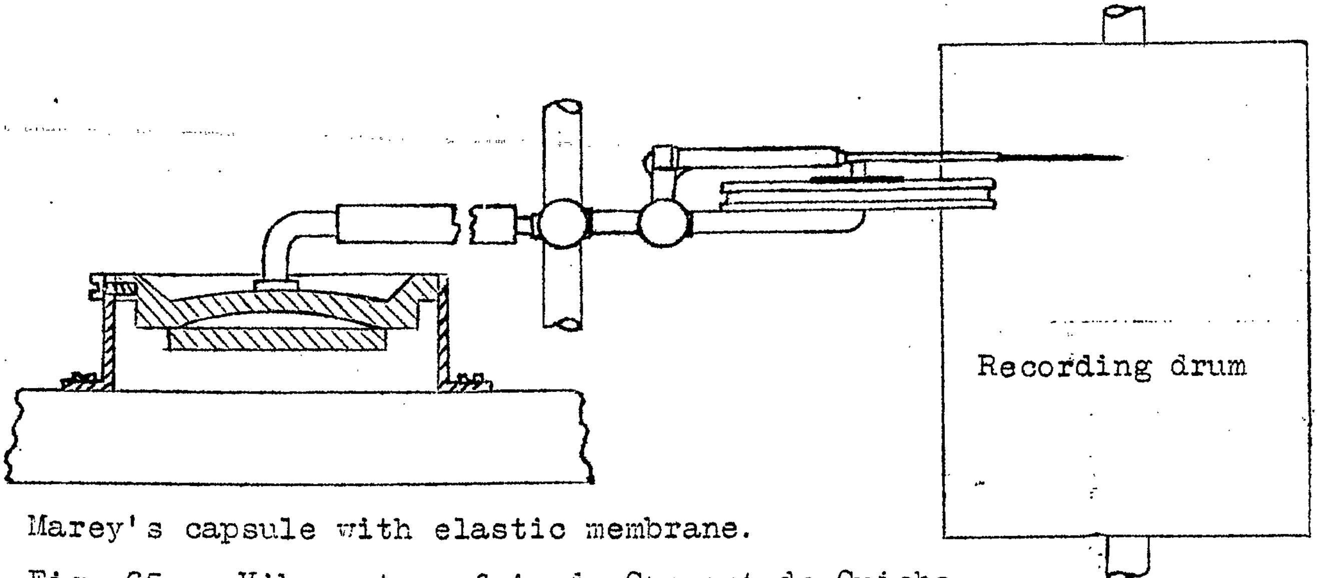

Fig. 65. - Vibrometer of A. de Gramont de Guiche and Carlo Bourlet.

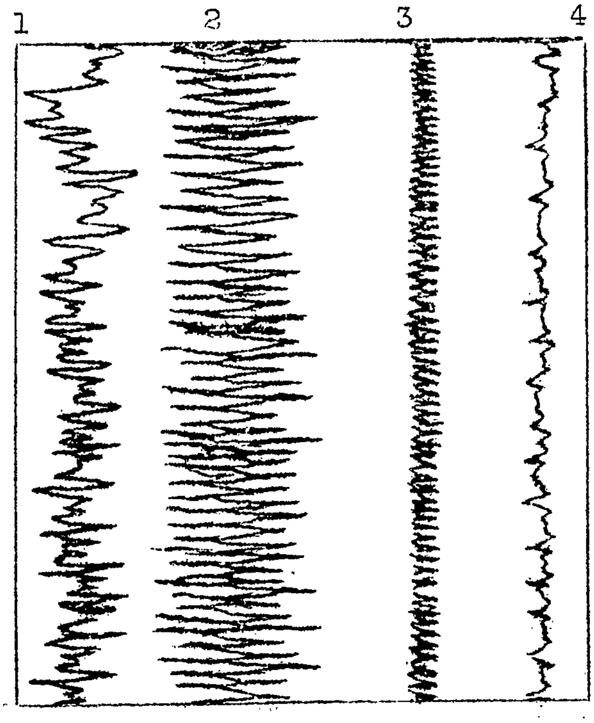

Fig. 66. - Vibrations recorded by a Guiche vibrometer on a 140 HP Dion engine mounted on a testing bench and running at 1860 r.p.m.

1. Lateral vibrations; 2. Vertical vibrations;
3. Vibrations of tuning fork; 4. Longitudinal vibrations.