NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Tech. Memo. No. 33.

PROBLEM OF LANDING.

By

E. Pistolesi.

4.7.7

Translated from
"L'Aeronautica," Jan.-Feb., 1921.

August, 1921.

PROBLEM OF LANDING.

By

E. Pistolesi.

The problem of landing deserves more serious attention than it has thus far been given. In the following brief remarks, I will expound certain considerations which seem to me to throw some light on the subject.

First of all, it must be carefully determined asto how the airplane is to land. After descending to a certain height above the ground by gliding, the airplane "flattens out" and flies horizontally near the ground, without engine (or at "idling" speed) so as to be retarded rather than accelerated), during which time the airplane loses speed by increasing its incidence, until, being no longer able to support itself, it descends and rests its wheels on the ground. Thus the tail skid sometimes touches the ground before the landing gear.

The landing is therefore a fall from a low height, which takes place when the airplane, because of its reduced speed, is no longer supported. Naturally, it is not a free fall, since the lift, although no longer equal to the weight of the airplane, does not become zero all at once.

From the foregoing, it follows that the wheels do not touch the ground, in a skillful landing, until the airplane has reached its minimum speed, corresponding to a certain value of the thrust

* Taken from "L'Aeronautica," Jan.-Feb., 1921, pp. 22-27.

(K_y^*) which, as shown by experience, does not coincide with the geometric maximum of the polar, but lies, instead, between this and the value corresponding to the minimum traction. Since the practical and geometrical maxima of K_y^* do not coincide, it is probably due to considerations of stability. It suffices for us to have called attention to the fact.

1. Horizontal Flight Over Field.

The retarding force is given by

$$-K_x SV^2 \,{**}$$

and therefore the acceleration by

$$-\frac{K_x SV^2}{Q/g}$$

Granted that

$$Q = K_y SV^2$$

we have:

$$\frac{dV}{dt} = -\frac{g}{\eta} \qquad (1)$$

Indicating with η the ratio

$$\eta = \frac{K_y}{K_x}$$

(1) may be transformed as follows:

$$\frac{dV}{ds}\frac{ds}{dt} = -\frac{g}{\eta}$$

$$\frac{d(V^2)}{ds} = -\frac{2g}{\eta} \qquad (2)$$

** For simplicity of notation, the asterisks, which indicate that the K_x, K_y, etc., refer to the whole airplane, are omitted. For the notations, see "Il Bolletino Tecnico," No.17, of the "Dirzione Sperimentale d'Aviazione."

Hence

$$s = -\frac{1}{2g}\int_{V_1}^{V_2} \eta \, d(V^2) \tag{3}$$

in which the integral is included between the speed V_1 at which flight begins and the ultimate minimum speed V_2.

Equation (3) may, with easy transformations be written

$$s = \frac{Q}{2gS}\int \frac{1}{K_x} \frac{dK_y}{K_y} \tag{4}$$

being the integral limited by the initial and final values of K_y.

For integrating either (4) or (5), when it is not desired to make use of a graphic process, very easily applied to (4), it is necessary to know the equation of the polar in the space between the initial and final points P_1 and P_2 (Fig. 1).

As already mentioned, this interval is not very large, just because the flight over the field begins with an angle of incidence varying but little (often somewhat less) from that of minimum traction. It is therefore possible to substitute for the actual space $P_1 P_2$ a curve with a simple equation (parabola of n order) of the following type.

$$K_x = K_o + AK_y^{\ n} \tag{5}$$

in which K_o would be the value of the abscissa relative to the point where the prolongation of the curve substituted for the space $P_1 P_2$ encounters the axis of K_x.

By substituting in (4) and integrating, we obtain

$$s = \frac{Q}{2gSK_on} \log \frac{K_{x_1} \ K_{y_2}^{\ n}}{K_{x_2} \ K_{y_1}^{\ n}} \tag{6}$$

Or:

$$s = \frac{Q}{2gSK\ n} \log \frac{\eta_2 V_1^{2(n-1)}}{\eta_1 V_2^{2(n-1)}} \tag{7}$$

Hence, for $n = 1$

$$s = \frac{Q}{2gSK_o} \log \frac{\eta_2}{\eta_1} \tag{8}$$

For $n = 2$

$$s = \frac{Q}{4gSK_o} \log \frac{\eta_2 V_1^2}{\eta_1 V_2^2} \tag{9}$$

For $n = 3$

$$s = \frac{Q}{6gSK_o} \log \frac{\eta_2 V_1^4}{\eta_1 V_2^4} \tag{10}$$

All the above formulas lead, in practice, to very similar results, as is shown by the following example.

Let:

$K_{x_1} = 0.00225 \qquad K_{y_1} = 0.020$ (point P_1)

$K_{x_2} = 0.005 \qquad K_{y_2} = 0.047$ (point P_2)

The above values are obtained from the polar of Fig. 1. After making the calculations by means of (6), we find:

For $m = 1 \qquad s = 13.2\ \frac{Q}{S}$

" $m = 2 \qquad s = 13.8\ \frac{Q}{S}$

" $m = 3 \qquad s = 14.1\ \frac{Q}{S}$

The differences between the various cases do not reach 7%.

Taking therefore as the average

$$s = 14\ \frac{Q}{S}$$

for $\frac{Q}{S} = 40$, we obtain $s = 560$ m; for $\frac{Q}{S} = 50$, we obtain $s = 700$ m.

For practical purposes, the above formulas are a little complicated and, since it suffices to know the space s approximately, it is better to use simpler formulas, which may be obtained by applying the theorem of the mean value.

Thus we may obtain from (4)

$$s = \frac{Q}{2gS} \frac{1}{(K_x)_m} \log \frac{K_{y_2}}{K_{y_1}} \tag{11}$$

in which, for the mean value $1/(K_x)_m$, we may put simply the arithmetical mean:

$$\frac{1}{2}\left(\frac{1}{K_{x_1}} + \frac{1}{K_{x_2}}\right)$$

By applying the above to the example, we find:

$$s = 13.75 \frac{Q}{S}$$

which demonstrates the admissibility of this procedure.

But ~~the~~ a simpler and more expressive formula is obtained by applying the theorem of the mean value to (3)

$$s \cong \frac{(\eta)_m}{2g} (V_2^2 - V_1^2) \tag{12}$$

Since it has already been mentioned that the space $P_1 P_2$ has about the value of the angle of minimum traction, for which is minimum, we may (always in the way of a broad approximation) put $\eta_{min.}$ instead of $(\eta)_m$ and we shall have:

$$s = \frac{\eta_{min.}}{2g} (V_1^2 - V_2^2) \tag{13}$$

or, even assuming

$$\frac{V_1}{V_2} = \beta$$

$$s = \frac{\eta_{min.}}{2g} V_2^2 (\beta^2 - 1) \tag{14}$$

By applying this to the usual example, we have:

$$s = 15.6 \frac{Q}{S}$$

or a somewhat excessive value, as was to be expected, but still approximate to within 10%. The approximation would naturally be closer, if the field P_1 P_2 were more restricted, and in particular if P_1 were nearer the point of minimum traction.

If we apply it to the practical example:

V_2 (landing speed) = 32 m/sec.

$\eta_{min.} = 10$

$V_1 = 1.5\ V_2 = 48$ m/sec.

we shall have:

$$s = 640 \text{ m.}$$

It is interesting to observe that equation (13) expresses the work which would have been done by gravity, if the machine had descended with a constant inclination of $\frac{1}{\eta_{min.}}$

2. Dimensions of Landing Field.

The foregoing enables us to form an opinion on the dimensions of the landing field. It is evident that these dimensions are generally very great and depend essentially on two factors: the fineness of the airplane and the ratio between the speed with which it begins its flight over the field and the minimum speed of which it is capable.

For swift and very fine airplanes the landing problem becomes all-important and must be seriously considered. In fact, to avoid the necessity of immense fields, it will be necessary to descend on the field at a low speed, since the equation $\beta = \frac{V_1}{V_2}$ exerts a great influence; as can be judged from Fig. 2, in which $\beta^2 - 1$ is shown with relation to β.

But, in order to bring V_1 nearer to V_2, it is necessary to descend with a small gliding angle and hence to have a field free from all obstacles. In other words, while it would be convenient (in order to facilitate alighting, especially in case of a forced landing) to be able to descend on the field at a steep inclination, there is opposed to this the necessity of not striking the ground at too great speed.

If, in the last example of the preceding paragraph, we should simply make $\beta = 1.8$, instead of 1.5, the length of flight would pass from 640 to 1250 m.

In order to reduce the length of flight over the field, without imposing on the gliding angle limitations which would prove inopportune in practice, it would be necessary to diminish artificially the fineness of the machine, by creating a counter-pull or brake, either by means of resisting surfaces opposed to the wind, or by means of the propeller itself, by reversing the pitch or the rotation.

By thus increasing the structural resistance, η is diminished and consequently s. Thus, in the case of the polar in Fig. 1, if there is added a structural resistance of $k = 0.002$, η passes from the value of 10.8 to 6.8 and, with equality of initial and final speeds, the distance s is reduced to $\frac{6.3}{10}$.

In order to produce a like resistance with a surface normal to the wind, it would be necessary to have a surface of

$$\frac{1}{40} S$$

or, for example, for an airplane of 40 sq.m. wing surface to have a resisting surface of 1 sq.m. If the resisting surface could be tripled, η would be reduced to 4.25 and s to about 0.4.

But the introduction of such a resistance would be much superior, if it were applied during the descent, either because it would enable the descent with a steep inclination at a low speed, or because, by increasing the angle of minimum pull, it would bring V_1 nearer to V_2 and therefore diminish β by bringing it nearer to unity.

In this connection, experiments have been tried in America with a parachute for a brake, but thus far there have been only preliminary experiments designed to show whether the parachute was strong enough. The parachute was broken at 80 miles per hour.

Instead of a resisting surface, we may use, as already mentioned, the propeller with either the pitch or the rotation direction reversed. Experiments were begun at Naples with reversed propeller by Col. Bongiovanni, which could not be continued on account of an accident.

As for the case of the propeller with inverted pitch, it is not easy to say, in the present status of the experiments, what force it would give, since the propeller (aside from the exchange of the front and rear) would work in a direct current in the opposite direction to its thrust, and this working condition is very little understood. In order to obtain an approximate value, we may, however, assume that the propeller gives, under this condition, a counter-thrust nearly equal to that of an ordinary stationary propeller and that this is about 1.5 times that in flight with a speed of V_1. Thus it is found that the effect of the propeller would be about that of introducing a nearly constant counter thrust equal to

$$1.5\, K_{x_1} V_1^2 S = 1.5 \frac{Q}{\eta_1}$$

In order to obtain an idea of the influence of such constant resistance, it will suffice to rewrite equation (1) thus modified:

$$\frac{dV}{dt} = - \frac{g}{\eta} + \frac{g}{Q} \times 1.5 \frac{Q}{\eta_1}$$

$$\frac{dV}{dt} = - g \frac{1}{\eta} + \frac{1.5}{\eta_1} \qquad (15)$$

By applying, as before, the theorem of the mean value and making $\eta = \eta_1 = \eta_{min.}$ we shall have:

$$\frac{dV}{dt} = -2.5 \frac{g}{\eta_{min.}}$$

which leads to the conclusion that the space s, with equality of initial and final speeds, is reduced according to the ratio of 1 to 2.5.

Lastly, we observe that, even without any special device, there is always a braking resistance due to the propeller, its pitch and direction of rotation remain constant.

It is known that when the rotation speed is small with reference to the forward motion (or $\frac{V}{N\ D}$ is large), the propeller may develop a considerable braking force.

Experiments performed at the Aeronautic Experimental Institute on an S.V.A. propeller demonstrated that, for a suitable value of $\frac{V}{N\ D}$, the ratio $\frac{\text{brake}}{V^2}$ can attain a value (negative) nearly equal to the value of the same speed ratio in horizontal flight.

If the propeller is stationary, the value of this ratio becomes, for the propeller under consideration, about 2/3 of the above value.

This denotes the introduction of a specific structural resistance, equal in the two cases to $K_{x\ min.}$ and 2/3 $K_{x\ min.}$ It is therefore a question of an influence far from negligible.

Thus in the case of Fig. 1, assumed for a stationary propeller, there is an increase in K_x of about 0.0015, causing a diminution of η from 10.8 to 7.4 $\frac{6.8}{10}$.

Having assumed moreover the beginning of horizontal flight at the incidence of minimum traction, we find that K_y passes from 0.033 to 0.038. Taking $K_{y_2} = 0.047$, β^2 will be equal in the two cases to 2.025 and 1.53. From this it follows that s varies in the equation

$$0.68 \times \frac{0.53}{1.025} = 0.3$$

The length of the landing flight is reduced to 1/3 and the gliding angle in the ratio of about 1 : 1.5.

3. The Fall and the Strength of the Landing Gear.

The second phase of landing is the "fall" which begins when the machine has arrived at the minimum practical speed below which its inertia will no longer support it.

The machine then assumes very steep angles of incidence for which K_y remains nearly constant, while K_x continues to increase with the increasing incidence.

The differential equations of the motion in this case are the following, in which v indicates the vertical rate of descent:

$$\frac{dv}{dt} = g \quad 1 - \frac{V^2}{V_2^2} \times \frac{Ky}{Ky_2} \qquad (16)$$

$$\frac{dV}{dt} = - g \frac{V^2}{V_2^2} \quad \frac{Kx}{Ky_2} \qquad (17)$$

A simplification may be made in the above equations by supposing $Ky_2 = Ky$, a supposition justified by the ordinary behavior of the polar which, after having reached its maximum, bends toward the axis of the abscissas and then assumes a direction almost parallel to said axis.

But even thus simplified, the two equations in question do not lead to practical results, it being impossible to determine how the incidence varies with the time and hence how K_x varies. Therefore it is necessary to resort to an artifice.

That is, we assume that the vertical acceleration during the fall increases lineally (This may be done on account of the brevity of the falling phase under consideration), by passing from the zero value at the beginning to the value αg at the end. It would be $\alpha = 1$, if, at the end of the fall, the vertical lift should completely disappear, or else $V = 0$. If t_o is the falling time, we have

$$\frac{dv}{dt} = \alpha g \frac{t}{t_o} \qquad (18)$$

$$v = \frac{1}{2} \alpha g \frac{t^2}{t_o} \qquad (19)$$

and assuming $v = \frac{dy}{dt}$, we obtain

$$y = \frac{1}{6} \alpha g' \frac{t}{t_o} \qquad (20)$$

The time t_o and the speed v_o, with which the airplane touches the ground, are therefore readily expressed in terms of the distance fallen, y_o :

$$t_o = \sqrt{\frac{6y_o}{\alpha g}} \qquad (21)$$

$$v_o = \sqrt{\frac{3}{2} \alpha g y_o} \qquad (22)$$

α remains to be determined. For this we shall use formula (17), in which we shall put, as already mentioned, $K_{y_2} = K_y$ and hence $\frac{K_x}{K_{y_2}} = \frac{1}{\eta}$. By applying the theorem of the mean value, we shall find:

$$t_o = \frac{V_2 \eta_m}{g} \quad \frac{V_2}{V_o} - 1 \tag{23}$$

in which η_m indicates a mean value of the ratio η.

Now, in the hypothesis $K_{y_2} = K_y$, $\frac{V_2}{V_o} = \frac{1}{\sqrt{1 - \alpha}}$

and therefore, by comparing (21) and (23):

$$\frac{\alpha \left(1 - \sqrt{1 - \alpha}\right)^2}{1 - \alpha} = \frac{6 y_o g}{V_2^{\,2} \cdot \eta_m^{\,2}} \tag{24}$$

The first member is a function increasing with α and therefore the maximum value of α will occur for the maximum value of the second member. In this case, it is allowable to substitute for the mean value η_m a minimum, which may be held equql to 2 (the mean value corresponding to an incidence of 20°).

The same reasoning leads us to assume for y (distance fallen) a high value (certainly not exceeded in practice, at least in normal landings) of, for example, 1 m.

Hence:

$$\frac{\alpha \left(1 - \sqrt{1 - \alpha}\right)^2}{1 - \alpha} = \frac{15}{V_2^{\,2}} \tag{25}$$

Assuming V_2 = 18 m/sec. we obtain $\alpha = \sim 0.43$

V_2 = 36 m/sec. we obtain $\alpha = \sim 0.3$

In practice we may therefore take α between 3/7 and 3/10, according to the minimum speed of the airplane.

By substituting in (22) and putting it under the form

$$v_o = \sqrt{\epsilon \times 2 g y_o}$$

we have, in the two cases, $\epsilon = 0.32$, $\epsilon = 0.225$, which means that the freely falling distance, to be considered for the strength of the landing gear lies, according to the minimum speed of the airplane, between 1/3 and 2/9 of the actual height at which the final phase of the descent begins.

Thus, having taken $y_o = 1$ m., we have $\epsilon y = 0.225$ to 0.32 m.

The figure of 0.5 m. which it is customary to consider as the freely falling height, in determining the strength of the landing gear, is therefore fully justified by the foregoing considerations.

(Translated by the National Advisory Committee for Aeronautics.)