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AIRPLANE PERFORMANCE AS INFLUENCED BY THE USE OF A SUPERCHARGED ENGINE.

by

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AIRPLANE PERFORMANCES AS INFLUENCED BY THE USE OF A SUPERCHARGED ENGINE.

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The question of the influence of the use of a supercharged engine on airplane performance will be treated here in a first approximation, but one which gives an exact idea of the advantage of supercharging. The method used may be directly extended to treat this problem without any of the simplifying assumptions made. These assumptions are made exclusively to allow an easier survey of the problem.

Let us consider an airplane which climbs first with an ordinary engine, not supercharged, (called in the following case I), and afterwards climbs with a supercharged engine (case II), and let us find the difference of the ceilings reached in the two cases.

We will assume in both cases the power $\mathcal{L}_{m_o}$ of the motor at sea level to be the same and the efficiency η of the propeller to be maintained constant all the time. This is quite possible, to a certain extent for a propeller with an adjustable pitch, a conclusion reached by theory and experimentally verified.

In case I, we can consider in a first approximation the power $\mathcal{L}_m$ of the motor to be proportional to the density, that is, to be expressed in the form

(1) $$\mathcal{L}_m = m\delta$$

where δ is the air density at a given height H

m a constant coefficient characteristic for the motor considered, assuming the number of revolutions of the motor to be kept nearly constant.

At sea level we have

(2) $$\mathcal{L}_{m_0} = m\,\delta_0$$

where δ_0 is the corresponding air density.

The power expended for horizontal flight at any altitude is equal to

(3) $$\mathcal{L}_n = \eta\,\mathcal{L}_m = \eta\, m\,\delta = QV$$

where η is the propeller efficiency

Q the propeller thrust

V the flying speed.

On the other hand the equations of the horizontal steady motion are of the form

(4) $$P = R_y = k_y\,\delta A V^2$$

(5) $$Q = R_x = k_x\,\delta A V^2$$

where P is the total weight of the airplane

k_x and k_y the drag and lift coefficients (functions of the angle of attack only)

A the wing area.

Comparing (3) and (5), we find

(6) $$QV = \eta\, m\,\delta = k_x\,\delta A V^3$$

and following

(7) $$\frac{\eta m}{A} = k_x V^3$$

an equation that fixes the relation between the angle of attack i and the speed V for horizontal flight at any altitude in case I. I call <u>climbing curve</u> (<u>or C curve</u>) the curve of V plotted i

against i according to equation (7).

Let us now plot on a system of (V, i) axes the system of curves (see equation (4))

(8) $$\frac{P}{\delta A} = k_y V^2$$

for different values of δ. I call the last curves <u>velocity curves</u> (see figure). As the height H reached by an airplane is a direct function of δ (depending upon atmospheric conditions) for the curve (8) we can use H as parameter instead of δ. If we plot on the same (V, i) axes the C curve (7), each point of intersection of a velocity curve with the C curve gives for the height H corresponding to the velocity curve considered, the velocity V and the angle of attack i of the horizontal flight at the height H of the airplane considered. That velocity curve which is tangent to the C curve gives the value H_I (case I) of the ceiling and the values of V and i corresponding to this ceiling.

The last value of the ceiling can also be found directly as follows: Eliminating V from (7) and (8), we find

(9) $$\delta_I = \frac{P\,\delta_0^{2/3}}{A^{1/3}\,\eta^{2/3}\,\mathcal{L}_{m_0}^{2/3}} \left(\frac{k_x}{k_y^{3/2}}\right)^{2/3}$$

that is, the density δ_I (for case I) in function of the angle of attack i. The minimum value of δ_I given by the last equation will correspond to the maximum of the height H, that is, to the ceiling. Thus the value i_M of the angle of attack corresponding to the ceiling in case I will be found from the relation

$$\frac{d(\delta_I)}{di} = 0 \quad \text{or} \quad \frac{d}{di}\left(\frac{k_x}{k_y{}^{3/2}}\right) = 0$$

and

(10) $$\delta_{I_{min}} = \delta_I(i_M)$$

Practically, the best way is to plot the curve (9) and find its minimum graphically, because k_x and k_y are empirical functions.

It is easy to see that the angle of attack i_M, for which δ_I is minimum, is the same angle for which the power $\mathcal{L}_n$ expended for flight at sea level is minimum. In fact we have

(11) $$\mathcal{L}_n = QV = k_x \delta_o A V^3$$

and replacing in the last equation V by its value taken from (8) we get

(12) $$\mathcal{L}_n = \frac{P^{3/2}}{A^{1/2}\,\delta_o^{1/2}}\;\frac{k_x}{k_y{}^{3/2}}$$

The minimum of $\mathcal{L}_n$ takes place for an angle of attack given by

$$\frac{d(\mathcal{L}_n)}{di} = 0$$

that is

$$\frac{d}{di}\left(\frac{k_x}{k_y{}^{3/2}}\right) = 0$$

which thus is the same angle i_M

On the annexed figure are represented the velocity curves and the C_I curve for a good actual airplane, as well as the δ_I curve for case I, which curves fully illustrate all the foregoing. The ceiling is reached at an angle of attack of 13°, at a speed of 120 ft./sec. and has a value of 25,000 ft.

In case II we will have the power $\mathcal{L}_{m_o}$ maintained constant by the supercharger, up to a certain altitude, say 2 0,000 ft., for example. Afterwards the power of the motor will again drop in a first approximation as the density. Let us first assume the limit possibility of

$$\mathcal{L}_{m_o} = Const.$$

up to any altitude.

Proceeding quite similarly to case I, we will find

$$\mathcal{L}_n = \eta \mathcal{L}_{m_o} = QV = Const \tag{13}$$

following

$$QV = \eta \mathcal{L}_{m_o} = k_x \delta A V^3 \tag{14}$$

and dividing by (4) we get

$$\frac{\eta \mathcal{L}_{m_o}}{P} = \frac{k_x}{k_y} V \tag{15}$$

an equation which represents the curve in the limiting case II.

Plotting this C_{II} curve on the velocity curves, we will directly see the enormous increase of ceiling that an unlimited supercharging would give. The fact to be noted is that even in the case of an unlimited supercharging we reach a ceiling.

In this last case the density curve has for expression

$$\delta_{II} = \frac{P^3}{A \eta^3 \mathcal{L}_{m_o}^2} \left(\frac{k_x}{k_y^{3/2}} \right)^2 \tag{16}$$

and its minimum, corresponding to the ceiling, takes place for the same angle of attack i_M as in the preceding case.

$$\delta_{II_{min}} = \delta_{II}(i_M) \tag{17}$$

But the supercharging maintains the power only up to a certain altitude, and after this altitude is reached the power of the motor will vary according to the law

(18) $$\mathcal{L}_m = m_c \delta$$

where the value of m_c has to be taken from the relation

(19) $$\mathcal{L}_{m_0} = m_c \delta_c$$

δ_c being the density at the limit height up to which the supercharger maintains the power. The airplane will start to climb from this altitude as if δ_c were the sea level.

After the density δ_c has been reached, there must accordingly arise a sudden change in the course of the C_{II} curve. Its second branch C'_{II} will be given, as is easy to see, by the relation

(20) $$\frac{\eta m_c}{A} = k_x V^3$$

and the corresponding δ'_{II} density curve will be

(21) $$\delta'_{II} = \frac{P\, \delta_c^{2/3}}{A^{1/3}\, \eta^{2/3}\, \mathcal{L}_{m_0}^{2/3}} \left(\frac{k_x}{k_y{}^{3/2}}\right)^{2/3}$$

and its minimum takes place, as it is easy to see, for the same angle of attack i_M , which minimum fixes the value of the ceiling in this last case of supercharging,

(22) $$\delta'_{II\,min} = \delta_{II}(i_M)$$

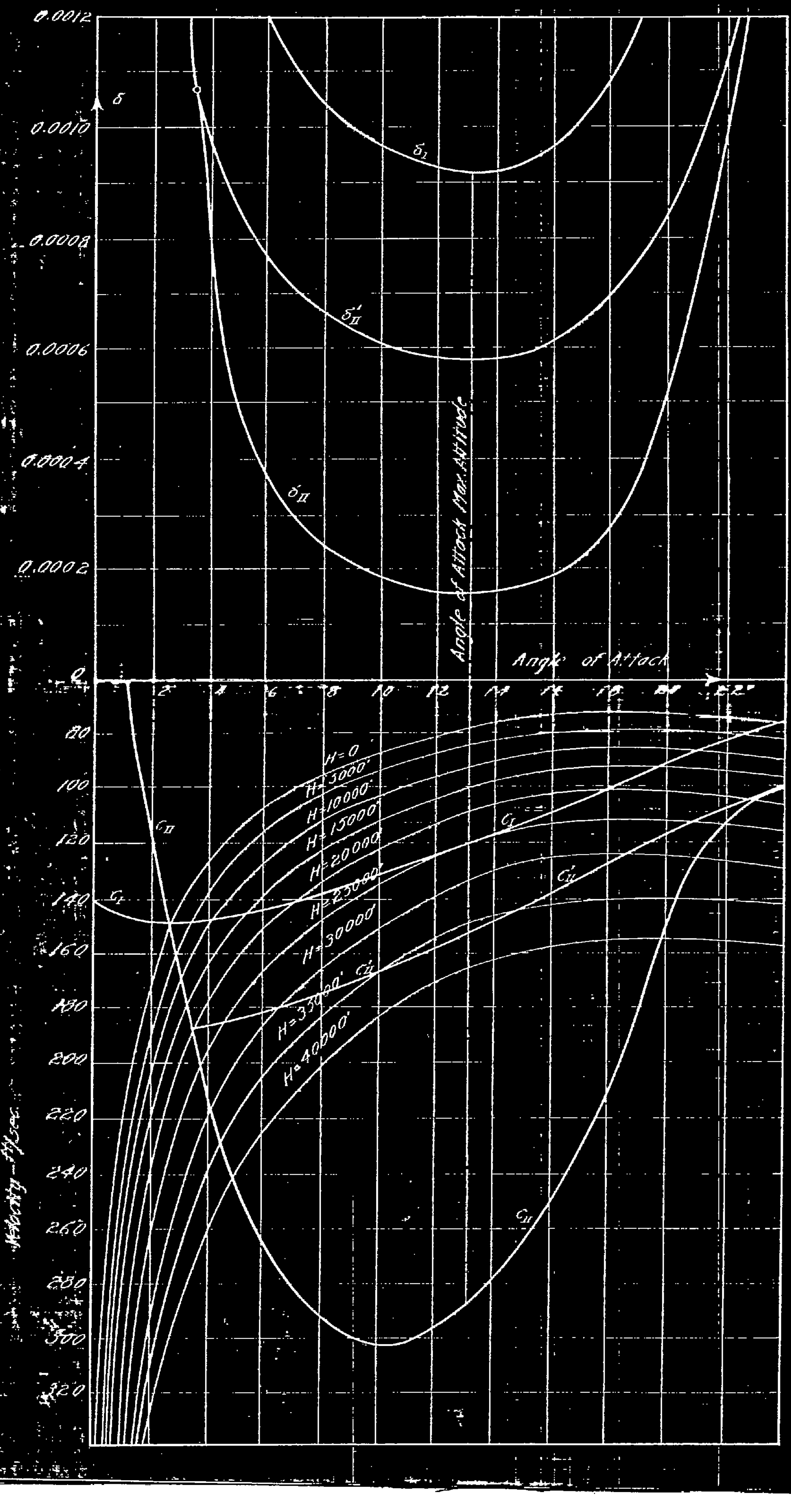
The C'_{II} curve and the C_{II} curve necessarily intersect on the velocity curve

(23) $$\frac{P}{\delta_c A} = k_y V^2$$

corresponding to the value δ_c of the density up to which the supercharging maintains the motor power.

In the case of our figure the ceiling from 25,000 ft. is increased to 37,000 ft., the supercharging maintaining the power only up to 20,000 ft. This makes, in comparison with case I of an engine without supercharging, an increase of the ceiling of about 50 per cent.

We thus see the whole importance of engine supercharging, which has for general result so sensible an increase of ceiling.


δ
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0
δ_I
δ'_{II}
δ_{II}
Angle of Attack Max. Altitude
Angle of Attack
2
4
6
8
10
12
14
16
18
20
22°
80
100
120
140
160
180
200
220
240
260
280
300
320
Velocity - ft/sec
H=0
H=5000'
H=10000'
H=15000'
H=20000'
H=25000'
H=30000'
H=35000'
H=40000'
C_I
C_{II}
C'_{II}