# On Sun-Synchronous Orbits and Associated Constellations

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## Abstract

This paper contains the preliminary ideas on how to design orbits and constellations associated with extended Sun-Synchronicity concepts. The *Multi Sun-Synchronous Orbits* are introduced first. These orbits could be adopted by those Earth science missions whose aim is to study the local hour effects on the observed object. In particular, these orbits allow us to build the *Multi Sun-Synchronous Constellations*, whose mechanism is briefly introduced here. By applying the *Sun-Synchronicity* constraint to the *Flower Constellations* theory, the *Sun-Synchronous Flower Constellations* have been derived. For these constellations, however, the synchronization is referring to the relative trajectory in a reference frame that rotates with Earth's mean motion rather than to the Earth-Centered Inertial frame. Finally, by merging the concepts of *Multi Sun-Synchronicity* and the *Flower Constellations*, the *Multi Sun-Synchronous Flower Constellations* are obtained.

## Introduction

This paper defines a set of satellite orbits associated with classic and novel Sun-Synchronicity concepts. The intent of this paper is to summarize and present the basic ideas. The complete analysis on the proposed orbits and constellations will belong to a full paper that will contain the associated exhaustive study. In particular, the *Multi Sun-Synchronous Orbits* (MSSO), which generalize the Sun-Synchronous Orbits, are introduced. These orbits are built such that the difference between the orbit nodal precession rate and the Earth's mean motion is proportional to the orbit nodal precession rate or to the Earth's mean motion through a rational number. Based on these orbits, the idea of *Multi Sun-Synchronous Constellations* is presented. In this constellation, after every orbit nodal period, the constellation is shifted by a constant angle, such that the constellation appears identical with respect to the Sun direction.

Then, within the recently proposed *Flower Constellations* theory [1], it is also shown how to build the *Sun-Synchronous Flower Constellations*, with an adaptation of the definition for the Sun-Synchronous concept. The orbits of a *Flower Constellations* are compatible orbits, that is, all the spacecraft follow the same closed loop repeating trajectory in an *Earth-Centered Earth-Fixed* (ECEF) reference frame. In the *Sun-Synchronous Flower Constellations*, is the spacecraft relative trajectory in ECEF that track the Sun, rather than the inertial spacecraft orbit in *Earth-Centered Inertial* (ECI). Finally, by merging the ideas of the *Multi Sun-Synchronous Orbits* and the *Flower Constellations*, the *Multi Sun-Synchronous Flower Constellations* have been devised.

In order to proceed with the analysis some reference frames (see Figure 1) have to be introduced.

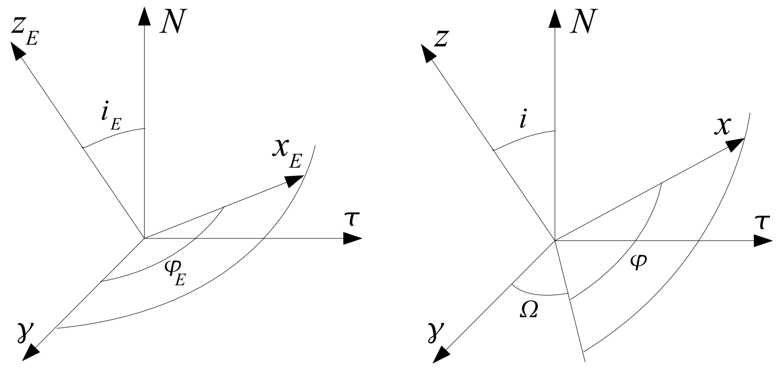


Figure 1: Reference Frames

The first reference frame is the ECI reference frame $[\gamma, \tau, N]$, where $\gamma$ is the vernal direction and $N$ is the Earth's spin axis. The second frame is the Earth's orbit reference frame $[x_E, y_E, z_E]$, where the $x_E$-axis is aligned with the Sun-Earth direction and the $z_E$-axis is the Earth's orbit normal (celestial pole). Finally, the third frame is the spacecraft orbit reference frame $[x, y, z]$, that has the $x$-axis aligned with Earth-Spacecraft direction and the $z_E$ axis is the S/C orbit normal. The orientation of the $[x_E, y_E, z_E]$ reference frame with respect to the ECI frame is identified by the orthogonal matrix

$$A_E = \begin{bmatrix} \cos\varphi_E & \sin\varphi_E \cos i_E & \sin\varphi_E \sin i_E \\ -\sin\varphi_E & \cos\varphi_E \cos i_E & \cos\varphi_E \sin i_E \\ 0 & -\sin i_E & \cos i_E \end{bmatrix} \tag{1}$$

where $C \equiv \cos$ and $S \equiv \sin$, and the orientation of the $[x, y, z]$ reference frame in ECI has the form

$$A_{S/C} = \begin{bmatrix} C_\varphi C_\Omega - S_\varphi S_\Omega C_i & C_\varphi S_\Omega + S_\varphi C_\Omega C_i & S_\varphi S_i \\ -S_\varphi C_\Omega - C_\varphi \S_\Omega C_i & -S_\varphi S_\Omega + C_\varphi C_\Omega C_i & C_\varphi S_i \\ S_\Omega S_i & -C_\Omega S_i & C_i \end{bmatrix} \tag{2}$$

## Sun-synchronous orbits

A *Sun-Synchronous Orbit* (SSO) is defined as the orbit whose normal ($z$ axis) makes a constant angle with the Sun-Earth direction ($x_E$ axis), hence, the key property ($z^T x_E =$ constant) of being synchronized with the Sun, as the name implies, is derived. For circular orbits, this property implies that the local hour behavior is periodic in one orbit nodal period.

The Sun-Earth direction rotates about $z_E$ with an angular velocity equal to the Earth's orbit mean motion

$$n_E = \frac{2\pi}{T_E} \approx 1.991 \cdot 10^{-7} \text{ rad/s} \tag{3}$$

where the Earth orbit is $T_E \approx 365.25$ days. Therefore, the necessary and sufficient condition for a SSO is that it must rotate (precede) about $z_E$ with $n_E$ angular velocity. This cannot be achieved by natural motion. However, it is possible to approximate it by taking advantage of the $J_2$ effect that results in a precession of the inertial orbit about $N$. The nodal rate due to the Earth's gravitational coefficient, $J_2$, is

$$\dot{\Omega} = -\frac{3}{2} \frac{R_E^2}{a^2(1-e^2)^2} \, n J_2 \cos i \tag{4}$$

where $R_E = 6,378.1363$ km is the Earth radius, $a$ is the orbit semi-major axis, $e$ is the orbit eccentricity, $i$ is the orbit inclination, $J_2 = 1.0826269 \times 10^{-3}$ is a coefficient of an expansion representing the Earth's oblateness effect, while the expression of the orbit mean motion is $n = \sqrt{\frac{\mu}{a^3}}$ where $\mu = 398,600.4415$ km$^3$/s$^2$ is the Earth gravitational parameter. Therefore, the Sun-synchronicity approximated condition is achieved by setting

$$\dot{\Omega} = n_E \tag{5}$$

Substituting Equations 3-4 into Equation 5, we obtain the relationship

$$\cos i = -4.774 \cdot 10^{-15} \, a^{7/2} \, (1-e^2)^2 \tag{6}$$

Equation 5 does not fully comply with the Sun-Synchronicity definition ($z^{\mathsf{T}} x_E =$ constant) and the effect of this approximation are described in the following. While $z$ rotates about $N$, the $x_E$ direction rotates about $z_E$, and these two rotation axes are displaced by $i_E \approx 23.5°$ (ecliptic obliquity). This implies, for instance, that the angle between $z$ and $x_E$ (which should be constant), has a behavior that repeats in one year during which this angle experiences variations depending on the initial orbital condition in term of right ascension and inclination. Hence, the angle between $z$ and $x_E$ would experience a minimum variation that should be eliminated by orbit maintenance control. However, this would imply continuous plane change maneuvers that are, in turn, unacceptable because they prove to be fuel cost prohibitive. For this reason, Equation 5 is accepted as the (approximated) definition of the SSO, and the described unavoidable variations must, therefore, be accepted and tolerated. These variations affect the sub-satellite local hour. Figure 2 shows a typical local hour variation experienced in one year for a generic orbit ($\Omega_0 = 83.2°$ and $i = 98°$).

Another important parameter for SSO is the angle $\Delta\lambda$ between two successive ascending nodes. This angle, after each orbit nodal period, is represented by $\Delta\lambda = (\dot{\Omega} - \dot{\alpha}_E) T_\Omega$, where $\dot{\alpha}_E = 7.2921 \cdot 10^{-5}$ rad/s is the Earth's spin rate and $T_\Omega$ is the nodal period, that is defined [2] as

$$T_\Omega = \frac{2\pi}{n + \dot{M}_0 + \dot{\omega}} \tag{7}$$

where the rates of change for the initial mean anomaly and the perigee argument due to $J_2$ perturbation, have the expressions

$$\dot{M}_0 = \frac{3R_E^2 J_2}{4p^2} n \sqrt{1-e^2} \left(3\cos^2 i - 1\right) \tag{8}$$

and

$$\dot{\omega} = \frac{3R_E^2 J_2}{4p^2} n \left(5\cos^2 i - 1\right) \tag{9}$$

respectively. Equations 4, 8, and 9, come from geopotential perturbation theory [2] by considering second order zonal effects, only. Therefore, the distance along the equator between successive node crossing points is $\Delta\xi = R_E\,\Delta\lambda$.

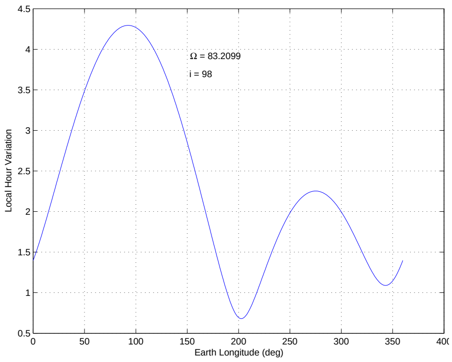


Figure 2: Local Hour Example

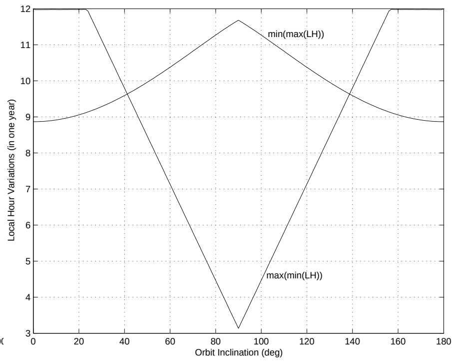


Figure 3: Local Hour Variations

Finally, in accordance with the notation adopted in Figure 1, the range of the sub-satellite point local hour $h$ is

$$6\left[3 - \frac{2}{\pi}\cos^{-1}\left(|z^{\mathrm{T}} x_E|\right)\right] \leq h \leq 6\left[3 + \frac{2}{\pi}\cos^{-1}\left(|z^{\mathrm{T}} x_E|\right)\right] \tag{10}$$

where

$$x_E = \begin{Bmatrix} \cos\varphi_E \\ \sin\varphi_E \cos i_E \\ \sin\varphi_E \sin i_E \end{Bmatrix} \quad \text{and} \quad z = \begin{Bmatrix} \sin\Omega \sin i \\ -\cos\Omega \sin i \\ \cos i \end{Bmatrix} \tag{11}$$

In particular, the maximum variation of $h$ is

$$\Delta h_{\max} = \frac{24}{\pi}\cos^{-1}\left(|z^{\mathrm{T}} x_E|\right) \tag{12}$$

Figure 2 shows a typical local hour variation during a year for a generic orbit ($\Omega_0 = 83.2°$ and $i = 98°$), while Figure 3 summarizes the results obtained by using Equation 12, in term of minimum of maximum and maximum of minimum values of $\Delta h_{\max}$ experienced in the entire range of orbit inclinations.

The SSO are typical of Earth observation missions that desire to minimize the effects of different solar ray inclinations (local hour) on the observed object (clouds, ice, specific atmosphere component or layer, etc). A complementary Earth Science can be accomplished when the science target is precisely the measurement of the effect(s) of the local hour on the specific observed object. The *Multi Sun-Synchronous Orbits*, introduced in the next section, have been developed specifically to address this complementary Earth science mission.

## Multi sun-synchronous orbits

Equation 5 characterizes the SSO: the Sun-Earth direction $x_E$ and the orbit normal direction $z$ rotate about $z_E$ and $N$, respectively, by the same angular speed ($\dot{\Omega} = n_E$). This is a continuous property. By extension, the *Multi Sun-Synchronous Orbits* (MSSO), can be defined based on a discrete time characterization. This paper introduces two distinct definition of the MSSO.

First, consider multiplying Equation 5 through by the orbit nodal period $T_\Omega$. We obtain $T_\Omega \dot{\Omega} = T_\Omega n_E$, which allows one to view the Sun-Synchronicity concept from a different perspective but in an equivalent way. This implies that, in the time $T_\Omega$, the directions $x_E$ and $z$ rotate by the same angle. Based upon this view, the MSSO are then defined as satisfying the equation

$$T_\Omega (\dot{\Omega} - n_E) = 2\pi\xi \tag{13}$$

where $\xi = \dfrac{I_n}{I_d}$ is the MSSO rational parameter, and $I_n$ and $I_d$ are two independent integer parameters (positive or negative). In fact, this equation states that, in the orbit nodal period $T_\Omega$, the relative rotation angle between $x_E$ and $z$ is changed by a rational amount of the $2\pi$ angle. Equation 13 generalizes Equation 5 which, in turn, can be obtained by setting $I_n = 0$. For any rational value of $\xi$, Equation 13 represents the following relationship among $a$, $e$, and $i$.

$$A\cos^2 i + B\cos i + C = 0 \tag{14}$$

where, setting $\chi \equiv \dfrac{3R_E^2 J_2}{4}$, the expression for $A$, $B$, and $C$, become

$$\begin{cases} A = (5 + 3\sqrt{1-e^2})\,\xi \\ B = -a^{7/2}(1-e^2)^2/(\chi\sqrt{\mu}) \\ C = [\,a^2(1-e^2)^2/\chi - \sqrt{1-e^2} - 1\,]\,\xi - 2n_E/(\mu\chi^2) \end{cases} \tag{15}$$

Equation 14 is a quadratic function in $\cos i$ that can be numerically solved. For the case of circular orbits, the expressions for $A$, $B$, and $C$ reduce to

$$\begin{cases} A = 8\xi \\ B = -a^{7/2}/(\chi\sqrt{\mu}) \\ C = (a^2/\chi - 2)\,\xi - 2n_E/(\mu\chi^2) \end{cases} \tag{16}$$

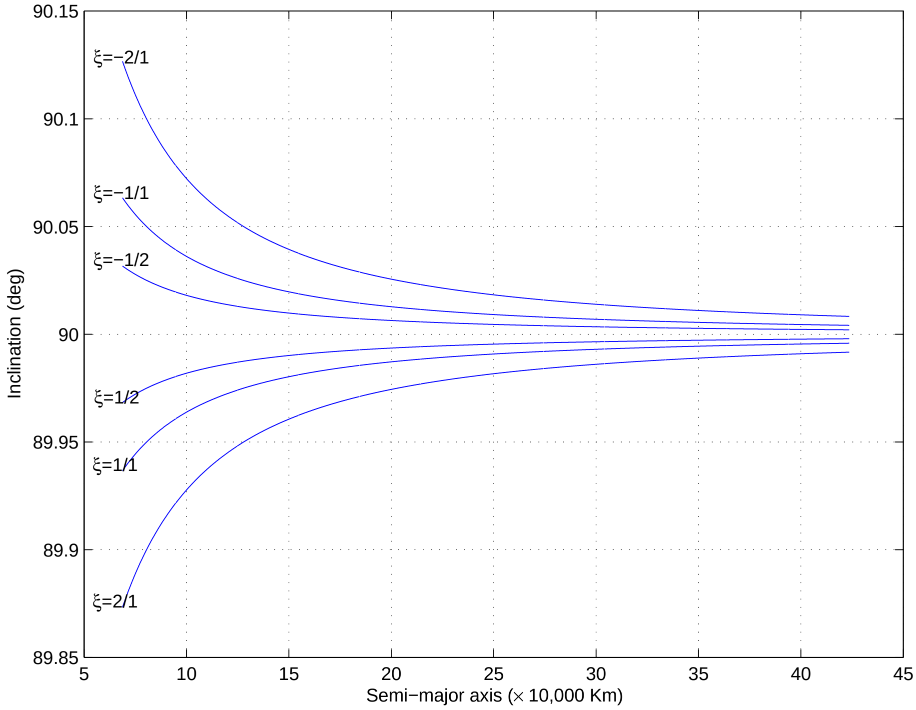


Figure 4: Multi Sun-Synchronous Circular Orbits: Type I

Figure 4 shows the relationship between $a$ and $i$ for various values of the integer parameters $I_d$ and $I_n$, for the case of multi Sun-Synchronous circular orbits of this kind. The correct solution is $\cos i = (-B - \sqrt{B^2 - 4AC})/(2A)$.

Alternatively, a second way to introduce the MSSO is by the relationship

$$K_d\,\dot{\Omega} = K_n\,n_E \tag{17}$$

where $K_n$ is a positive integer parameter and $K_d$ is an independent (positive or negative) integer parameter. This equation states that, in $K_d$ Earth orbit periods, the orbit nodal line precesses (clockwise or counter-clockwise) $|K_n|$ times. Equation 17 also generalizes Equation 5 that, in turn, can be obtained by setting $K_n = K_d = 1$. Equation 6 can be rewritten for the MSSO obtaining

$$\cos i = -4.774 \cdot 10^{-15}\,\frac{K_n}{K_d}\,a^{7/2}(1-e^2)^2 \tag{18}$$

which enlarges the possibilities given by Equation 6. Notice that the parameter $\eta = \dfrac{K_n}{K_d}$ can be negative, which avoids the SSO constraint of a retrograde orbit.

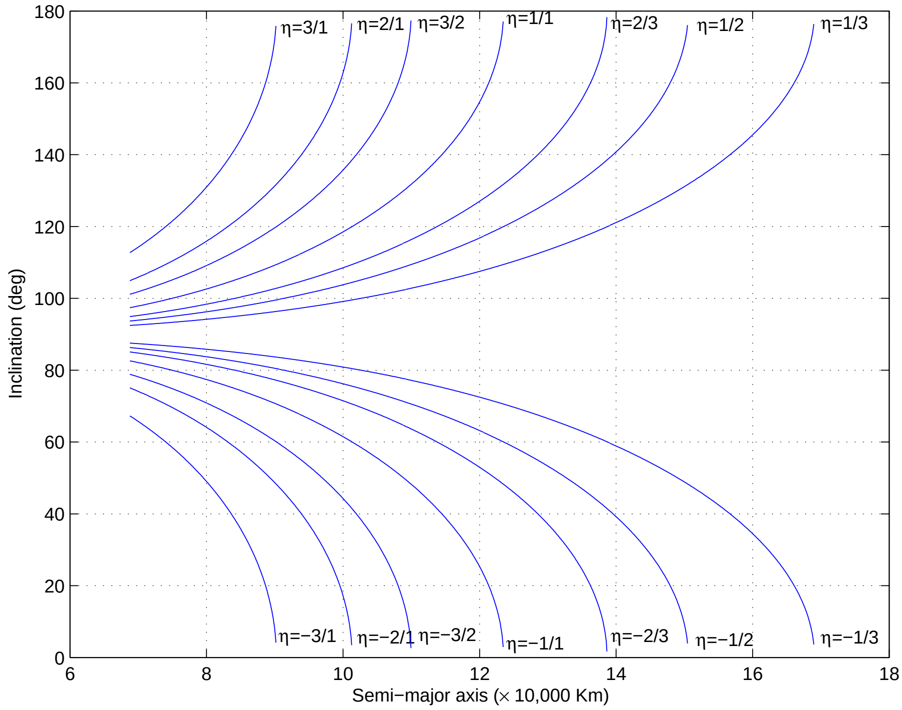


Figure 5: Multi Sun-Synchronous Circular Orbits: Type II

Figure 5 shows the relationship between $a$ and $i$ for various values of the integer parameters $K_d$ and $K_n$, for the case of multi Sun-Synchronous circular orbits of the second kind.

## Multi sun-synchronous constellations

Let us consider a satellite constellation consisting of $N_s$ satellites whose orbits have identical inclination, argument of perigee, eccentricity, and semi-major axis. As for the right ascensions, these orbits are evenly and equally separated by the angle

$$\Delta\Omega = \frac{2\pi}{N_s} \tag{19}$$

In consideration of the time-varying parameters, say the mean anomalies for each spacecraft, there are no particular constraints. However, particular attention is given to those values complying with the theory of *Flower Constellations* [1]. Based upon these assumptions, we can introduce the *Multi Sun-Synchronous Constellations* (MSSC), as follows.

In the time $(J_d T_\Omega)$, where $T_\Omega$ is the orbit nodal period and $J_d$ is a given integer number, the relative angle between node and Sun direction increases/decreases by a given angle. The MSSC are defined such that this angle is exactly $m$ times the angle $\Delta\Omega$ given in Equation 19, that is, is $(J_n \Delta\Omega)$, where $J_n$ is another integer parameter. Now, since the relative velocity between the node and Sun directions is $(n_E - \dot{\Omega})$, then we can write the relationship

$$J_d T_\Omega (n_E - \dot{\Omega}) = J_n \Delta\Omega = J_n \frac{2\pi}{N_s} \tag{20}$$

that defines the *Multi Sun-Synchronous Constellations*. By introducing the MSSC rational parameter

$$\eta = \frac{J_d}{J_n} \tag{21}$$

By comparing Equation 13 with Equation 20 we can derive the relationship between the integer parameters

$$I_d = N_s J_d \qquad \text{and} \qquad I_n = J_n \tag{22}$$

Equation 20 constitutes a relationship between inclination, semi-major axis, eccentricity, and the MSSC rational parameter $\eta$.

## Sun-synchronous flower constellations

The *Flower Constellations* (FCs), introduced in [1], are built using *compatible orbits*. A compatible orbit (often called repeated ground track orbit) has the orbital period such that the relative trajectory in an Earth-Centered Earth-Fixed (ECEF) system of coordinates constitutes a repeated closed-loop trajectory (the name *Flower Constellation* is due to the petal-like shape of the closed-loop relative trajectories as viewed from an ECEF rotating frame). The relative trajectory can be designed such that it repeats after the satellite completes $N_p$ revolutions (orbit nodal periods) over $N_d$ sidereal days. If $T_r$ is the period of repetition, then

$$T_r = N_p T_\Omega = N_d T_{\Omega G} \tag{23}$$

where $T_{\Omega G}$ is the nodal period of Greenwich, which is defined by Carter [3] as

$$T_{\Omega G} = \frac{2\pi}{\dot{\alpha}_E - \dot{\Omega}} \tag{24}$$

In the theory of the FCs, which includes the $J_2$ gravitational perturbation, it is shown that there are a set of *admissible* orbital positions (admissible mean anomalies) that allow each spacecraft to follow the same identical closed-loop relative trajectory in ECEF. The FCs are built on this property and on a particular procedure (phasing rule) to distribute the satellites on the admissible positions. In particular, in an axial-symmetric FCs, the node lines of each satellite are equally displaced along the equatorial plane.

The *Sun-Synchronous Flower Constellations* (SSFCs) that are introduced here are FCs whose orbits are compatible with respect to a fictitious Earth that spin with angular velocity equal to the Earth's mean motion $n_E$. This is the description of an Earth that is gravity gradient stabilized with respect to the Sun. In other words, this fictitious body is fixed on the $[x_E, y_E, z_E]$ reference frame. This implies a new way to see the Sun-synchronicity concept. The inertial spacecraft orbit is not synchronized with the rotation of the solar ray direction (Sun-Synchronous classical definition), but rather it is the spacecraft relative trajectory (with respect to a ECEF rotating with angular velocity $n_E$) that is synchronized with the Sun. In order to build such Sun-Synchronous relative trajectories, the Earth spin rate $\dot{\alpha}_E$ must be substituted by the Earth mean motion $n_E$ in Equation 24.

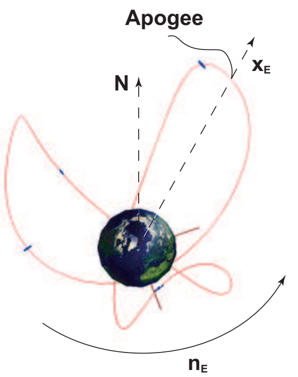


Figure 6: SSFC

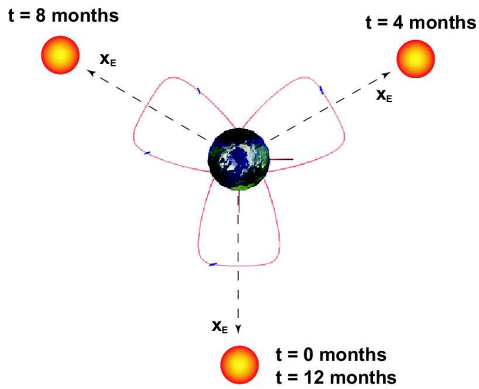


Figure 7: MSSFC

Figure 6 shows an isometric view of a 3-1 *Flower Constellation* where one apogee has been aligned with the Earth-Sun axis ($x_E$). The entire relative path is set to rotate about the Earth's polar axis ($N$) such that that particular apogee stays aligned with $x_E$. Hence, this is a *Sun-Synchronous Flower Constellation*.

## Multi sun-synchronous flower constellations

The concept of *Multi Sun-Synchronicity* can be applied to the *Flower Constellations* theory, thus obtaining the *Multi Sun-Synchronous Flower Constellations*. The synchronization is that the FC relative trajectory rotates in the inertial space with an angular velocity such that, every time a petal (relative apogee) becomes aligned with the Sun rays direction then a spacecraft of the constellation passes to the aligned relative apogee.

Figure 7 shows a polar view of a 3-1 Flower Constellation where one apogee has been initially aligned with the Earth-Sun axis ($x_E$). The entire relative path rotates about the Earth's polar axis ($N$) such that the petals (relative apogees) will sequentially align with $x_E$ at a prescribed times (0, 4, and 8 months). Hence, this is a *Multi Sun-Synchronous Flower Constellation*.

## Conclusion

In this paper, we have presented the summary of the basic ideas on how to design orbits and constellations associated with extended Sun-Synchronicity concepts. In particular, the *Multi Sun-Synchronous Orbits* have been introduced. These orbits have been devised for Earth science missions that want to study the local hour effects on the observations. Built on these orbits, the *Multi Sun-Synchronous Constellations* have been proposed. Then it has been shown how to apply the *Sun-Synchronicity* constraint to the *Flower Constellations* theory, and the *Sun-Synchronous Flower Constellations* have been derived. The main characteristics of these constellations is that the *Sun-synchronization* is associated with the motion (in ECI) of the spacecraft relative trajectory that appear fixed into a reference frame that rotates with Earth's mean motion, while the classic *Sun-Synchronicity* concept is a particular precession of the inertial orbit. The paper then presents the *Multi Sun-Synchronous Flower Constellations* that are obtained by exporting the *Multi Sun-Synchronicity* concept into the *Flower Constellations* theory. All the ideas presented in this paper are preliminary. A detailed and complete analysis of these orbits and constellations is presently under preparation and it will appear in a future paper.

## References


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