CCD Data Processing Improvements for Star Cameras

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Abstract

This paper first introduces two methods to speed up the acquisition, namely, the Peak Finder (PF), which uses two integer vectors, and the Run Length Encode (RLE), which uses an iterative approach applied to an adjacent star segment table and performs acquisition and centroiding simultaneously. Regarding centroiding, the recursive functions are introduced. Recursive and ellipsoidal masks present centroiding accuracy gains with respect to the standard squared mask, which is more affected by the mask edge errors. Finally, two Gaussian Best-Fitting methods to increase centroiding accuracy, are introduced and compared versus standard Center-Of-Mass approach.

Introduction

Research aimed at improving spacecraft attitude estimation using star trackers has recently focused upon: 1) making faster and more reliable star pattern recognition algorithms and, 2) making faster and more accurate the centroiding process to evaluate the direction of the stars observed by a CCD/CMOS imager. This paper begins by introducing a method to simulate the “Point Spread Functions” (PSF), as observed by a given CCD imager, lens, defocusing value, and with a given attitude. Then, the acquisition and the centroiding processes are executed to evaluate the star directions. Centroiding standard geometrical approaches are here compared versus topological methods.

Centroiding highly depends on the PSF shapes. Since complex PSF shapes are not well fit by square or rectangular masks, two new approaches are introduced to tackle this problem. The “Run Length Encode” (RLE) method is the first one, while the second is based on the use of recursive functions. The recursive functions, initiated at the brightest pixel, visits all the pixels whose grey level is greater than the threshold and connected to the first one by a continuous path.

RLE, recursive, and ellipsoidal mask methods are less sensitive to electronic noise and are demonstrated to be more accurate than the standard square mask approach. The accuracy gain can reach 40\%, as shown by numerical comparison tests comparing geometrical and topological centroiding algorithms. Moreover, and maybe the most important aspect, the topological approaches allow us to accommodate different PSFs as those produced by the multiple fields-of-view star trackers, as StarNav...
II (Ref. [1]), and StarNav III (Ref. [2]), recently proposed. In particular, StarNav II, by introducing astigmatism, outputs ellipsoidal PSFs, while StarNav III, using a technique of pupil partition, outputs toroidal PSFs.

**Gaussian distribution**

A 1-D Gaussian distribution is characterized by the probability density function

\[
P_x = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}
\]

where \( \sigma \) is the standard deviation, and where the condition \( \int_{-\infty}^{\infty} P_x \, dx = 1 \) is satisfied. A 2-D Gaussian distribution is characterized by the probability density function \( P_{xy} \), that can be written as

\[
P_{xy} = P_x P_y = \frac{1}{2\pi \sigma_x \sigma_y} e^{-\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{(y-y_0)^2}{2\sigma_y^2}}
\]

(1)

**Virtual CCD**

Let us consider a star tracker FOV as seen by a lens of focal length \( f \) and with a \( N_H \times N_V \) rectangular CCD whose pixel dimensions are \( d_H \times d_V \). The angular aperture (\( \vartheta_H \) and \( \vartheta_V \)) of this FOV are derived from the equations \( 2f \tan \vartheta_H = N_H d_H \), and \( 2f \tan \vartheta_V = N_V d_V \). Let the four vectors \( \{S(\bullet) = \sin(\bullet), C(\bullet) = \cos(\bullet)\} \)

\[
\left\{ \begin{array}{c}
v_1^1 \equiv \{+S_{\vartheta_H} C_\alpha, -S_\alpha, +C_{\vartheta_H} C_\alpha\} \equiv \{+S_{\beta}, -S_{\vartheta_V} C_{\beta}, +C_{\vartheta_V} C_{\beta}\} \\
v_2^1 \equiv \{+S_{\vartheta_H} C_\alpha, +S_\alpha, +C_{\vartheta_H} C_\alpha\} \equiv \{+S_{\beta}, +S_{\vartheta_V} C_{\beta}, +C_{\vartheta_V} C_{\beta}\} \\
v_3^1 \equiv \{-S_{\vartheta_H} C_\alpha, +S_\alpha, +C_{\vartheta_H} C_\alpha\} \equiv \{-S_{\beta}, +S_{\vartheta_V} C_{\beta}, +C_{\vartheta_V} C_{\beta}\} \\
v_4^1 \equiv \{-S_{\vartheta_H} C_\alpha, -S_\alpha, +C_{\vartheta_H} C_\alpha\} \equiv \{-S_{\beta}, -S_{\vartheta_V} C_{\beta}, +C_{\vartheta_V} C_{\beta}\}
\end{array} \right.
\]

locate the four FOV corners, then we can write the relationships \( S_{\vartheta_H} C_\alpha = S_{\beta}, S_\alpha = S_{\vartheta_H} C_{\beta}, \) and \( C_{\vartheta_H} \cos \alpha = C_{\vartheta_V} \cos \beta \) which allows an explicit definition for \( \alpha \) and \( \beta \), which are \( \tan \alpha = \tan \vartheta_V \cos \vartheta_H \), and \( \tan \beta = \tan \vartheta_H \cos \vartheta_V \). These relationships, and Eq. (1), allows us to obtain a virtual CCD image of the stars as observed from any attitude. This has been done for a single Field of View star tracker as well as for the recently proposed Multiple Fields of View Star trackers, such as StarNav II and StarNav III.

The star magnitude \( m \) is related to the star flux density \( F \) through the relationship

\[
m = m_0 - 2.5 \log F,
\]

where \( m_0 \) is a scale constant (e.g. the magnitude of Sirio \( m_0 = -1.5 \)). This allows us to write

\[
F = e^{-\frac{(m_0-m)}{2.5}} = E/(\Delta t),
\]

where \( E \) is the star energy (volume of the Gaussian observed star), and where \( \Delta t \) is the CCD integration time.
All the above allows us to develop a Virtual CCD image using the equation

\[ T(x, y) = \frac{\Delta t}{2\pi\sigma_x\sigma_y} e^{-\frac{(m_0 - m)^2}{2.5}} e^{-\frac{(x - x_0)^2}{2\sigma_x^2}} e^{-\frac{(y - y_0)^2}{2\sigma_y^2}} \]  

The Virtual CCD image has been developed with a graphical interface (command panel) that allows an easy use of the program. In the command panel it is possible to choose the type of sensor (StarNav I, II, or III), and to define the driver parameters for the program. For the CCDs with less than 1 Megapixels, the processing time with MATLAB on an Intel Pentium III, 128 Mb RAM with Windows Operating System, is less than 5 seconds. The control parameters of this interface are: 1) Type of sensor (StarNav I, II, or III), 2) ratio between the defocusing standard deviation (Active only for StarNav II), 3) star magnitude threshold, 4) focal length (mm), 5) optical axis direction (right ascension and declination), 6) random choice of the attitude, 7) star identification numbers option (YES/NO), and 8) the noise parameters (mean and standard deviation).

**Peak Finder Acquisition**

As for the speed, a fast approach has been proposed which, by scanning all the CCD pixels just once, determines all the needed info about the PSFs location. This is accomplished by introducing two integer vectors, \( I_{val} \) and \( I_{col} \), both of \( n_r \) elements, where \( n_r \) is the number of the CCD row pixels. These two vectors keep record of value and location of the relative maximum in the corresponding row. This means, for instance, that if 4000 is the gray level value associated with the brightest element of the 5\(^{th}\) row, located at the 123\(^{th}\) column, then \( I_{val}(5) = 4000 \), and \( I_{col}(5) = 123 \). These two integer vectors allow us, then, a fast way to find out all the \( n \) first brightest pixels (most likely, where the star spot lights are located) on the CCD, by avoiding subsequent CCD pixel scans, which are, in turn, time consuming.

**Run Length Encode Acquisition**

An algorithm to perform acquisition, that is, to locate the clusters of bright pixels, should have some basic properties in order to be useful in real applications. It must be fast, robust, and it should use as little memory as possible. An algorithm that owns these properties is here proposed and explained in detail. This new algorithm, uses a “Run Length Encode” (RLE) technique to avoid memorization of all image pixels, thus saving a memory. RLE approach scans the entire image once, and only once, in order to find clusters of adjacent pixels which lies in the same image row, and then merges the clusters if they are adjacent. When no more adjacent clusters remain to be merged, the blocks of adjacent pixels are isolated and then the centroiding process can be applied.
This algorithm, which is capable of identifying PSFs of any complex form (topologic method), is more suitable than a recursive approach for acquisition because the stack-overflowing problem is eliminated. The RLE algorithm can be summarized by the following steps: 1) finds all the clusters and evaluates the cluster parameters (energy, number of pixels, etc.), 2) merges all adjacent blocks, and repeats this step until there are no more adjacent blocks to be merged, and 3) proceed with centroiding.

**Centroiding Geometrical Approach**

Centroiding process is usually accomplished by square masks whose size is given as a function of the brightest pixel value. This choice, however, limits the centroiding accuracy. In fact, it is possible to increase the accuracy by using geometrical masks (i.e. ellipsoidal) which, respect to the standard squared masks, more nearly match the shape of the PSF and therefore minimize the contributions due to electronic noise variation in the negligibly illuminated pixels located far from the PSF peak. It is important to outline that the centroiding process depends strongly on the shapes of the CCD PSFs. Since complex PSFs shapes (as those proposed for StarNav III) may not be well fit by the assigned geometrical masks, two new approaches to tackle this problem are proposed.

**Centroiding Topological Approach**

A recursive function approach is represented by a recursive function that identifies all the pixels, with grey level over a given threshold, within an ellipsoidal mask. The recursive function starts the process from a given pixel (i.e. the brightest), and visits all the pixels connected to the first one by means of a continuous path and having a grey level greater than the given threshold, evaluated as a function of the noise’s maximum and variance. This method allows to avoid to include, from centroiding computation, many pixels far from the brightest one, and which contain lower signal to noise (electronic) ratio. This new approach, together with that using ellipsoidal masks, is demonstrated to be more accurate than the rectangular masks based standard approach.

In Fig. 1, the gain obtained by numerical test using recursive functions constrained by ellipsoidal mask is compared to a standard square-mask approach. Figure 1 shows an increase of the centroiding accuracy from 0.138 to 0.08 mean values, that implies a 40% gain in accuracy.

Figure 2 shows the speed test results obtained by comparing four different methods to acquire and performing centroiding. The figure shows that RLE and PF with recursive function approaches are the fastest. The compared methods are: 1) *Square Mask*, which provides peak detection by subsequent CCD scans, and performs centroiding by square mask, 2) *Recursive*, which performs multiple CCD scans, while centroiding is evaluated by a recursive function starting from the peak, 3) *RLE*, which
scans the CCD just once to find clusters of adjacent pixels, and merges adjacent clusters. Blocks’ centroiding parameters are then evaluated, 4) $PF$, which performs peak detection using two integer vectors (with row’s maximum and column index), and centroiding by recursive function.

Gaussian Best-Fitting: first method

Centroiding accuracy can be improved by applying Gaussian best-fitting technique to the discretized PSF. Input data are described by the PSF grey levels $t_i$ at pixel coor-
Coordinates \((x_i, y_i)\). Performing the logarithm of Eq. (2), and setting \(z_0 = \ln \frac{\Delta t}{2\pi\sigma_x\sigma_y} + \frac{(m - m_0)}{2.5}\), and \(z_i = \ln t_i\), where \(t_i\) is the pixel grey level value, we obtain

\[
z_i \approx z_0 - \frac{(x_i - x_0)^2}{2\sigma^2_x} - \frac{(y_i - y_0)^2}{2\sigma^2_y} = f(x_0, y_0, z_0)
\]

Now, introducing the error \(\varepsilon_i = z_i - f(x_0, y_0, z_0)\), the problem implies to minimize the lost function \(L = \sum_i \alpha_i \varepsilon_i^2\), where \(\alpha_i\) are relative weights. Stationary condition implies \(\frac{\partial L}{\partial x_0} = 0\), \(\frac{\partial L}{\partial y_0} = 0\), and \(\frac{\partial L}{\partial z_0} = 0\), which assume the closed forms

\[
\begin{cases}
\Sigma_i[2(z_i - z_0) + \sigma^2_x(x_i - x_0)^2 + \sigma^2_y(y_i - y_0)^2](x_i - x_0) = 0 \\
\Sigma_i[2(z_i - z_0) + \sigma^2_x(x_i - x_0)^2 + \sigma^2_y(y_i - y_0)^2](y_i - y_0) = 0 \\
\Sigma_i[2(z_i - z_0) + \sigma^2_x(x_i - x_0)^2 + \sigma^2_y(y_i - y_0)^2] = 0
\end{cases}
\]

respectively. This nonlinear set of equations can be solved through linearization and iterative procedure. The linearization implies

\[
x_0 = \hat{x}_0 + \delta x_0, \quad y_0 = \hat{y}_0 + \delta y_0, \quad \text{and} \quad z_0 = \hat{z}_0 + \delta z_0
\]

Substituting Eq. (5) into Eq. (4), we obtain

\[
\begin{bmatrix}
\Sigma_i m_{11}^{(i)} & \Sigma_i m_{12}^{(i)} & \Sigma_i m_{13}^{(i)} \\
\Sigma_i m_{12}^{(i)} & \Sigma_i m_{22}^{(i)} & \Sigma_i m_{23}^{(i)} \\
\Sigma_i m_{13}^{(i)} & \Sigma_i m_{23}^{(i)} & \Sigma_i m_{33}^{(i)}
\end{bmatrix}
\begin{bmatrix}
\delta x_0 \\
\delta y_0 \\
\delta z_0
\end{bmatrix}
= \begin{bmatrix}
\Sigma_i c_{1}^{(i)} \\
\Sigma_i c_{2}^{(i)} \\
\Sigma_i c_{3}^{(i)}
\end{bmatrix}
\]

### Gaussian Best-Fitting: second method

Another way to perform Gaussian smoothing consists of a direct linearization of Eq. (3) using Eq. (5) (about \(\hat{x}_0, \hat{y}_0, \hat{z}_0\)), and to evaluate the unknown \((\delta x_0, \delta y_0, \delta z_0)\), by disregarding the higher order terms. Thus, obtaining

\[
r_i = z_i - f_i(\hat{x}_0, \hat{y}_0, \hat{z}_0) = \mathbf{F}_i^T \Delta = \begin{bmatrix}
\frac{\partial f_i}{\partial x} & \frac{\partial f_i}{\partial y} & \frac{\partial f_i}{\partial z}
\end{bmatrix}
\begin{bmatrix}
\delta x_0 \\
\delta y_0 \\
\delta z_0
\end{bmatrix}
\]

Hence, the resulting solving system is

\[
R = \begin{bmatrix}
r_1 \\
\vdots \\
r_n
\end{bmatrix}
= \begin{bmatrix}
f_{1x} & f_{1y} & f_{1z} \\
\vdots & \vdots & \vdots \\
f_{nx} & f_{ny} & f_{nz}
\end{bmatrix}
\begin{bmatrix}
\delta x_0 \\
\delta y_0 \\
\delta z_0
\end{bmatrix}
= \mathbf{F} \Delta
\]
This system allows an iterative least-square solution in the form

\[ \Delta_{k+1} = [F_k^T F_k]^{-1} F_k^T R_k \]

where \( k = 1, 2, \ldots \) indicates the iteration step. Once the correction \( \Delta_{k+1} \) has been evaluated, the process can be iterated accordingly to Eq. (5).

The two proposed Gaussian Best-Fitting (GBF) methods, whose convergence is numerically demonstrated, present different characteristics in the noisy real case. Both methods converge to the solution with no more than 2 iterations. The second method, however, seems to be more reliable than the first one which, in turn, presents some convergence failing in the noisy real case. In particular, the accuracy gain of the GBF with respect to the Center Of Mass (COM) standard approach, increases with the signal-to-noise ratio and as the star magnitude decreases (toward brightest stars). This latter effect is shown in Fig. 3 by 1000 numerical tests, and with star magnitude ranging from -1.5 (Sirius) to 6. This Figure suggest to adopt the second GBF method for the stars with magnitude less than four.

![Figure 3: GBF versus COM](image)

**Conclusions**

Some improvements in the data processing for CCD star tracker, are here presented. The new methods are proposed for both the acquisition, and the centroiding problems. Two methods to speed up the acquisition are introduced. The Peak Finder, uses two integer vectors while the Run Length Encode, uses an iterative approach applied to an adjacent star segments table and performs acquisition and centroiding simultaneously. As for centroiding, the use of the recursive functions, is found useful to estimate Point Spread Functions of any shape. The proposed methods present advantages
with respect to the most standard approaches, in both the speed and the accuracy obtainable, as shown in Figs. 1, and 2. Finally, two Gaussian best fitting methods are presented to increase centroiding accuracy. In particular, it is shown that for stars with magnitude less than four, the second Gaussian best fitting provides more accurate results than the standard Center Of Mass approach.

Acknowledgments

The authors thank the Italian Space Agency and the Centro di Ricerca Progetto San Marco for supporting this research. Special thanks to our colleagues at Texas A&M University and at Officine Galileo (Alenia Difesa) for their numerous discussions on these subjects.

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