Optimal impulsive maneuvers in orbital transfers
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Abstract

In the present paper several strategies for impulsive orbit transfer between circular orbits are compared. From analytical formulas it turns out that $N = 3$ is the maximum number of impulses for optimal transfers between coplanar circular orbits. Partial analytical results and numerical evidence support the conclusion that three is also the maximum number of impulses in the non planar case. The authors are not aware of a direct analytical proof of this fact, and such a proof seems difficult because the equations of the optimal distribution of plane variations on several impulses are rather implicit.

Introduction

The problem of the optimal impulsive transfer between two orbits is almost seventy years old, but the question “How many impulses” [1] is still open despite of the theories and a lot of numerical work developed in this field (see for instance [2, 3] and the many references therein). Optimal transfer is considered here in the sense of minimizing the variation of velocity needed in the transfer, and some analytical results are known in the linearized case. This is when the initial and final orbit are close: namely three is the maximum number of impulses to perform a transfer between close planar orbits, and five is the maximum number of impulses in the non planar case [4]. These upper bounds on the number of impulses do not longer hold true if the initial and target orbits are not close. A counterexample is known about an optimal four impulse transfer between two coplanar (not close) elliptic orbits.

The present paper deals with optimal impulsive transfers between circular orbits: a result of the Lawden primer vector theory [5] is applied to show that the impulses between circular orbits must be always given at perigee or apogee of the elliptic transfer orbits in order to minimize the $\Delta V$: then these optimal transfer orbits must be ellipses with their apsidal line on a common straight line. This condition, called apsidal line rule, determines where the impulses must be given, but not the optimal number $N$ of impulses. However this result allows to prove optimality by a parametric analysis on the number $N$ or by analytical formulas. For planar circular orbits an analytical proof is here derived showing that the maximum number of impulses is three for any value of the ratio $x = R_f/R_0$ between the radii of the final ($R_f$) and the initial ($R_0$) orbits. Some analytical results and numerical evidences support the conclusion that three is also the maximum number of impulses for inclined orbits.
Two and three impulse optimal planar transfers

In this Section the N=2 and N=3 impulse strategies will be recalled and compared. The apsidal line rule together with the N=2 hypothesis imply that the transfer ellipse has perigee $R_0$ and apogee $R_f$, the so called Hohmann transfer.

For N=3 impulses the apsidal line rule implies that the transfer trajectory is composed by two ellipses, the first with perigee $R_0$ and apogee $R_2$, the second with apogee $R_2$ and perigee $R_f$, the so called bielliptic transfer. Note that for $R_2 = R_f$ the elliptic transfers degenerates into an Hohmann transfer. Moreover there is also the possibility to have $R_2 < R_f$: in this case the bielliptic transfer is called inner as opposite to the previous case (outer bielliptic transfer). Since the transfer ellipses are determined, the three impulses must generate the following variation of velocity:

$$
\Delta V_{BE}(y,x) = \sqrt{\frac{\mu}{R_0}} \left[ \left( \sqrt{\frac{2y}{y+1}} - 1 \right) + \left( \sqrt{\frac{2}{x+y}} \sqrt{\frac{x}{y}} - \sqrt{\frac{2}{y(y+1)}} \right) + \sqrt{\frac{2y}{x(y+x)}} \pm \frac{1}{\sqrt{x}} \right]
$$

(1)

where $x, y$ are the variables:

$$
x = \frac{R_f}{R_0}, \quad y = \frac{R_2}{R_0}
$$

and the sign $-$ holds for the outer and $+$ for the inner bielliptic transfer. For $y \to \infty$ the transfer ellipse becomes a parabola (parabolic transfer) and the variation of velocity depends only on $x$:

$$
\Delta V_{BP}(x) = \lim_{y \to \infty} \Delta V_{BE}(x,y) = (\sqrt{2} - 1) \left( \sqrt{\frac{\mu}{r_1}} \left( 1 + \frac{1}{\sqrt{x}} \right) \right)
$$

(2)

For $y = x$ the bielliptic becomes Hohmann transfer and

$$
\Delta V_{BE}(x,x) = \Delta V_H(x)
$$

(3)

Comparison is now made between the different strategies presented: first it is shown that outer bielliptic transfers are always more economical than inner ones. Namely, let $R_2$ be equal to $R_f + h$ in the outer and $R_f - h$ in the inner case (where $0 < h < R_f$). The difference $\Delta$ in the two cases is:

$$
\Delta = \Delta V_{outer} - \Delta V_{inner} = \sqrt{\frac{R_f + h}{2R_f + h}} + \sqrt{\frac{R_f - h}{2R_f - h}} - \sqrt{2}
$$

Such a difference $\Delta$ is less or equal to zero if

$$
\frac{4R_f^2 - 2h^2}{4R_f^2 - h^2} + 2 \sqrt{\frac{R_f^2 - h^2}{4R_f^2 - h^2}} \leq 2
$$

which holds true for any $h < R_f$. 
Comparison is now made between $N = 2$ and $N = 3$ strategies. Let us consider $y = x + h$, $h > 0$. Then one has the expansion:

$$\Delta V_{BE}(y,x) = \Delta V_{BE}(x+h,x) = \Delta V_{BE}(x,x) + \frac{\partial \Delta V_{BE}}{\partial y} \Big|_{y=x} h + o(h^2)$$

If $\frac{\partial \Delta V_{BE}}{\partial y} \Big|_{y=x} < 0$ the bielliptic transfer will be more economical than the Hohmann transfer, regardless of the apogee distance $R_2$. The critical value $x_c$ such that $\frac{\partial \Delta V_{BE}}{\partial y} \Big|_{y=x} = 0$ satisfies the equation

$$\frac{\partial \Delta V_{BE}}{\partial y} (x,x) = \sqrt{\frac{\mu}{R_0}} [2(2x)^{-\frac{1}{2}} (x+1)^{-\frac{3}{2}} + \frac{1}{\sqrt{2}} (1+x)^{-\frac{1}{2}} x^{-\frac{3}{2}}]$$

which has solution $x_c \sim 15.6$. That is for $x > 15.6$ any bielliptic transfer is more economical than the Hohmann transfer, regardless of the apogee distance $R_2$ (that is for any $h > 0$).

**N=4 impulsive optimal planar transfers**

Four impulse optimal transfers are composed by three coaxial ellipses (apsidal line rule) with perigee/apogee equal to $(R_0, R_2)$, $(R_3, R_2)$, $(R_f, R_3)$ respectively. Two options are possible: $R_2 > R_3 > R_f$ and $R_f > R_3 > R_0$. The total velocity variation $\Delta V^{IV}$ is the sum of the four $\Delta V$:

$$\Delta V_1 = \sqrt{\frac{\mu}{R_0}} \sqrt{\frac{2y}{1+y} - 1}, \Delta V_2 = \sqrt{\frac{\mu}{R_0}} \sqrt{\frac{1}{y} \left[ \sqrt{\frac{2y}{z+y}} - \sqrt{\frac{2}{1+y}} \right]}$$

$$\Delta V_3 = \sqrt{\frac{\mu}{R_0}} \sqrt{\frac{1}{z} \left[ \sqrt{\frac{2y}{z+y}} - \sqrt{\frac{2x}{z+x}} \right]}, \Delta V_4 = \sqrt{\frac{\mu}{R_0}} \sqrt{\frac{1}{x} \left[ \sqrt{\frac{2z}{z+x}} - 1 \right]}$$

where

$$x = R_f/R_0, \ y = R_2/R_0, \ z = R_3/R_0$$

Comparison is now made between $N = 4$ and $N = 3$ strategies: let us fix the value $R_2$ and choose $R_3 = R_f + H$: note that for H=0 the N=4 strategy coincides with the N=3. The difference

$$D(h) = \Delta V^{IV} - \Delta V_{BE}$$

vanishes for $h = H/R_0 = 0$, and the derivative

$$\frac{dD}{dh} \bigg|_{h=0} = \sqrt{\frac{\mu}{2R_f R_f}} [\sqrt{2} - \sqrt{\frac{R_2}{R_2 + R_f}}] = \sqrt{\frac{\mu}{R_0}} \frac{1}{\sqrt{xx}} \left[ 1 - \sqrt{\frac{y}{2(y+x)}} \right]$$

is always positive. Then $D$ vanishes on $h = 0$ and it is an increasing function of $h$: it follows that $\Delta V^{IV} > \Delta V_{BE}$ and this inequality holds true also for $R_3 < R_f$ and $R_2 < R_f$.

Using a proof by induction on the number $N$ of impulses it can be proved that $\Delta V^N > \Delta V_{BE}$ for any $N \geq 4$. 
N=2 impulsive optimal transfers between inclined circular orbits

The velocity variation formula to move the spacecraft from an orbit with velocity \( v_i \) to an orbit with velocity \( v_f \) and inclined by the angle \( i \) is

\[
\Delta V = [v_i^2 + v_f^2 - 2v_i v_f \cos(i)]^{\frac{1}{2}}
\]  
(4)

Formula (4) suggests to perform the plane variation when \( v_i \) is lower, that is at apogee of the transfer orbit. If the plane variation is entirely performed at the apogee of the transfer orbit, the \( \Delta V_2 \) velocity variation must change into

\[
\Delta V_2 = [\frac{\mu}{R_f} \frac{2R_0}{R_0 + R_f} + \frac{\mu}{R_f} - 2 \frac{\mu}{R_f} \sqrt{\frac{2R_0}{R_0 + R_f} \cos(i)}]^{\frac{1}{2}}
\]

\[
= \sqrt{\frac{\mu}{R_0}} [\frac{2}{1+x} \frac{1}{x} + \frac{1}{x} - \frac{2}{x} \sqrt{2 \cos(i)}]^{\frac{1}{2}}
\]

Of course the velocity variation is a function of two variables: \((x, i)\):

\[
\Delta V_H(x, i) = \Delta V_1(x) + \Delta V_2(x, i)
\]

For \( i = 0 \), \( \Delta V_H(x, 0) \) is an increasing function of \( x \): however such a property does not hold true for any value of \( i \); in fact there are values of inclination such that \( \Delta V_H \) may decrease with \( x \). Namely, writing the derivative \( \frac{\partial \Delta V_H}{\partial x}(x, i) = \frac{\partial \Delta V_1}{\partial x} + \frac{\partial \Delta V_2}{\partial x} \), where:

\[
\frac{\partial \Delta V_1}{\partial x}(x) = \frac{1}{\sqrt{2x(x+1)^3}}
\]

\[
\frac{\partial \Delta V_2}{\partial x}(x, i) = \frac{[2 \sqrt{2} (1+x)^{-\frac{1}{2}} \cos i + \sqrt{2} \frac{(-1)}{x} (1+x)^{-\frac{3}{2}} \cos i - 2 (x^2 + x)^{-2} (2x + 1) - \frac{1}{x^2}]}{(2 \frac{2}{(x+1)x} + \frac{1}{x} - \frac{2}{x} \sqrt{2 \cos i})^{\frac{1}{2}}}
\]

one can find values of \( i \) so that such a derivative is a negative function of \( x \). In fact, let us find the critical values of inclination \( i \) which solve the equation

\[
\frac{\partial \Delta V_H}{\partial x}(x, i) = 0
\]

with \( i \) as unknown and \( x \) as parameter. For instance, for \( x = 1 \) one has

\[
\frac{\partial \Delta V_H}{\partial x}(x, i) = \frac{\partial \Delta V_H}{\partial x} = \frac{1}{4} (1 - \frac{5}{\sqrt{2}} \sqrt{1 - \cos i}) = 0
\]

which has solution \( i^* \sim 23^0 \). It follows that for \( i > i^* \), the velocity variation \( \Delta V_H \) decreases with \( x \) in the neighborhood of \( x = 1 \). Figure 1 shows the curve \( i = i^*(x) \) obtained solving the equation (5) : note that for \( i > 43^0 \) the function \( \frac{\partial \Delta V_H}{\partial x} \) never vanishes and \( \Delta V_H \) decreases for any value of \( x \). It follows that if the inclination
Figure 1: Values of $i$ (in deg) where $DV$ becomes decreasing in $x = R_f/R_0$

between the initial and the final orbits is greater than $43^0$ a lower $\Delta V$ is needed if a more distant orbit has to be reached.

It will be now shown that optimality of $N=2$ transfer between circular inclined orbits requires the splitting of the plane variation on both the impulses. Let $i_1$ be the variation of inclination on the first impulse and $i - i_1$ in the second. Then

$$\Delta V_1 = \sqrt{\frac{\mu}{R_0}} \left[ \frac{2x}{x+1} + 1 - 2 \sqrt{\frac{2x}{1+x} \cos (i - i_1)} \right]^2$$

$$\Delta V_2 = \sqrt{\frac{\mu}{R_0}} \left[ \frac{2}{x(x+1)} + \frac{1}{x} \frac{2}{x} \sqrt{\frac{2}{1+x} \cos (i - i_1)} \right]^2$$

The sum of the two velocity variation is the total velocity variation $\Delta V_H(x, i_1, i - i_1)$. This strategy is now compared to the previous one, where all the plane variation is on the second impulse ($i_1 = 0$). It is reasonable to think that $i_1$ is a small value, hence to write, up to $i_1^2$:

$$\Delta V_H(x, i_1, i - i_1) - \Delta V_H(x, 0, i) = \frac{\partial \Delta V_H}{\partial i_1} \bigg|_{i_1=0} i_1$$

Now

$$\frac{\partial \Delta V_H}{\partial i_1} = \frac{\mu}{R_0} \frac{1}{\Delta V_1(x)} \sqrt{\frac{2R_f}{R_0 + R_f} \sin (i_1)} - \frac{\mu}{R_f} \frac{1}{\Delta V_2(x)} \sqrt{\frac{2R_f}{R_0 + R_f} \sin (i - i_1)} \quad (6)$$
and for $i_1 = 0$:

$$\frac{\partial \Delta V_H}{\partial i_1}_{i_1=0} = -\frac{\mu}{R_f} \frac{1}{\Delta V_2(x)} \sqrt{\frac{2r_2}{r_1 + r_2}} \sin(i)$$

which is negative for any value of $x$ and $i$ (since the relative inclination between the initial and target orbits has range in the interval $[0, 90^\circ]$).

Then it is always preferable to split the plane variation on the two impulses. To find the value $i_{1\text{opt}}$ so to minimize $\Delta V$ one has to solve the equation

$$\frac{dV_H}{di_1}(i_{1\text{opt}}) = 0$$

with $x$ and $i$ as fixed parameters. This equation can be easily solved approximating in

(6) $\sin(i_1) \sim i_1$ and $\sin(i - i_1) \sim \sin(i)$; one thus obtain

$$i_1 = \frac{1}{x^2} \frac{\left(\sqrt{\frac{2x}{1+x}} - 1\right)}{H(x, i)} \sin(i)$$

where

$$H(x, i) = \left[\frac{2}{x(x+1)} + \frac{1}{x} - \frac{2}{x} \sqrt{\frac{2}{1+x}} \cos(i)\right]^\frac{1}{2}$$

Figure 2 shows the value $i_{1\text{opt}}$ as function of $x$ and $i$; Figure 3 shows the $\Delta V$ saved

Figure 2: First variation of inclination $i_1$ in Hohmann transfers

performing the $i_{1\text{opt}}$ plane variation at the first impulse:

$$\Delta V = (\Delta V_H(x, 0, i) - \Delta V_H(x, i_{1\text{opt}}, i - i_{1\text{opt}}))$$
**N=3 and N=4 impulsive optimal transfers between inclined circular orbits**

In the bielliptic transfers between inclined orbits the optimal plane variation has to be split on the three impulses: however let us consider first the case where the plane variation is entirely performed at the apogee of the transfer, that is (in terms of the adimensional variables $x = R_f/R_0$, $y = R_2/R_0$):

$$\Delta V_2 = \sqrt{\frac{\mu}{R_0}} \left[ \frac{2x}{y(y+x)} + \frac{2}{y(y+1)} - \frac{2}{y} \sqrt{\frac{2x}{y+x}} \sqrt{\frac{2}{y+1}} \cos(i) \right]^{\frac{1}{2}} \quad (7)$$

The comparison between the N=2 and the N=3 strategies with plane variation performed at the apogee gives:

$$\Delta V_{BE}(y = x+h, x, i) - \Delta V_H(x, i) = \frac{\partial \Delta V_{BE}}{\partial y}_{y=x} h + o(h^2) \quad (8)$$

The bielliptic transfer is more economical if $\frac{\partial \Delta V_{BE}}{\partial y}_{y=x} < 0$ and one has critical values for $x, i$ solving the equation

$$\frac{\partial \Delta V_{BE}}{\partial y}_{y=x} = f(x) + \frac{[\alpha(x) + \beta(x) \ast \cos(i)]}{\sqrt{a(x) + b(x) \cos(i)}} \quad (9)$$

The computation of the functions of $x$: $f, \alpha, \text{etc.}$ is not difficult and the previous equation becomes a second order equation on $\cos(i)$:

$$[\alpha(x) + \beta(x) \ast \cos(i)]^2 = (f(x) + g(x))^2(a(x) + b(x) \cos(i)) \quad (9)$$
which is satisfied by a solution lower than 1 for \( x \) lower than a certain value \( x_c \). Viceversa (9) has no solution lower than 1 for \( x > x_c \). This implies that for \( x > x_c \) the bielliptic transfer is more economical than the Hohmann transfer regardless of the value of the inclination \( i \). For \( x < x_c \), the curve \( i = i(x) \), obtained as solution of (9), determines the following condition: if \( i > i(x) \) then any bielliptic is more economical than Hohmann transfer. Figure 4 shows the curve \( i(x) \). Optimal bielliptic transfers

![Figure 4: Values of \( x \) and \( i \) such that any bielliptic is more economical than Hohmann transfer](image)

will be now analyzed: these have plane distribution on the three impulses:

\[
i = \gamma_1 + \gamma_2 + \gamma_3
\]

The optimal distribution of the out of plane impulses can be obtained applying the formulas (40)-(43) of Reference [6] to the \( N=3 \) case: these formulas relate the primer vector components to the two transfer ellipses of the bielliptic transfer which have eccentricities:

\[
e_1 = \frac{R_2 - R_0}{R_2 + R_0}, \quad e_2 = \frac{R_2 - R_f}{R_2 + R_f}
\]

The following equations on the out of plane angles \( \gamma_1, \gamma_3 \) can be derived from [6]:

\[
\sin(\gamma_1) = \frac{e_1 - 1}{e_1 + 1} \frac{\sin(\gamma_2)}{\Delta V_2} \Delta V_1 = k_1 \Delta V_1 \\
\sin(\gamma_3) = \frac{e_2 - 1}{e_2 + 1} \frac{\sin(\gamma_2)}{\Delta V_2} \sqrt{1 - e_1^2} \Delta V_3 = k_3 \Delta V_3
\]

These generate the following two equations:

\[
\sin(\gamma_1)^4 + \sin(\gamma_1)^2[4k_1^2(1 + e_1) - 2k_1^2(2 + e_1)] + k_1^4[(2 + e_1)^2 - 4(1 + e_1)] = 0 \quad (10)
\]
\[ \sin(\gamma_3)^4 + \sin(\gamma_3)^2[4k_3^4(1 + e_2) - 2k_3^2(2 + e_2)] + k_3^4[(2 + e_2)^2 - 4(1 + e_2)] = 0 \] (11)

These equations do not select a unique solution, since they are of fourth order, however the optimal solution must satisfy the following unitary norm condition on the primer vector having components \((\lambda, \mu, \nu)\), that is (since \(\lambda = 0\), see [5]):

\[ \mu^2 + \nu^2 = 1 \]

which implies the two conditions:

\[
\left( \frac{\nu_{11}}{\nu_{21}} \right)^2 = \left( \frac{e_1 + 1}{e_1 - 1} \right) = \frac{1 - \mu_{21}^2}{1 - \mu_{11}^2}, \quad \left( \frac{\nu_{32}}{\nu_{22}} \right)^2 = \left( \frac{e_2 - 1}{e_2 + 1} \right) = \frac{1 - \mu_{32}^2}{1 - \mu_{22}^2}
\]

where \(\mu_{ij}\) are:

\[
\mu_{11} = \frac{1}{\Delta V_1} (\sqrt{1 + e_1 - \cos \gamma_1}), \quad \mu_{21} = \frac{1}{\Delta V_2} (\cos \gamma_2 \sqrt{1 - e_2} - \sqrt{1 - e_1})
\]

\[
\mu_{22} = \frac{1}{\Delta V_2} (\sqrt{1 - e_2 - \cos \gamma_2} \sqrt{1 - e_1}), \quad \mu_{32} = \frac{1}{\Delta V_3} (\cos \gamma_3 - \sqrt{1 + e_2})
\]

From the above equations the values \(\gamma_1, \gamma_2, \gamma_3\) can be determined. The N=2 and N=3 total velocity variation with optimized distribution of inclination in two and three impulses respectively are compared in Figure 5. The optimal plane variation in four impulses

\[ i = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 \]
can be found applying formulas (40)-(43) of [6] to N=4:

\[
\sin(\gamma_1) = \frac{e_1 - 1}{e_1 + 1} \sqrt{1 - e_2} \sin(\gamma_2) \frac{\Delta V_1}{\Delta V_2} = k_1 \Delta V_1 \\
\sin(\gamma_3) = \frac{e_2 - 1}{e_2 + 1} \sqrt{1 - e_3} \sin(\gamma_2) \frac{\Delta V_3}{\Delta V_2} = k_3 \Delta V_3 \\
\sin(\gamma_4) = \frac{e_3 + 1}{e_3 - 1} \sqrt{1 + e_2} \sin(\gamma_3) \frac{\Delta V_4}{\Delta V_3} = k_4 \Delta V_4
\]

These can be regarded as equations on the optimal \(\gamma\) angles:

\[
\sin(\gamma_1)^4 + \sin(\gamma_1)^2 [4k_1^4(1 + e_1) - 2k_1^2(2 + e_1)] + k_1^4[(2 + e_1)^2 - 4(1 + e_1)] = 0
\]

\[
\sin(\gamma_3)^4 + \sin(\gamma_3)^2 [4k_3^4(1 + e_3)(1 + e_2) - 2k_3^2(2 + e_2 + e_3)] \\
+ k_3^4[(2 + e_2 + e_3)^2 - 4(1 + e_2)(1 + e_3)] = 0
\]

\[
\sin(\gamma_4)^4 + \sin(\gamma_4)^2 [4k_4^4(1 - e_3) - 2k_4^2(2 - e_3)] + k_4^4[(2 - e_3)^2 - 4(1 - e_3)] = 0
\]

The optimal solutions satisfy the further conditions:

\[
\left(\frac{V_{21}}{V_{11}}\right)^2 = \left(\frac{e_1 + 1}{e_1 - 1}\right)^2 = \frac{1 - \mu_{21}^2}{1 - \mu_{11}^2}
\]

\[
\left(\frac{V_{32}}{V_{22}}\right)^2 = \left(\frac{e_2 - 1}{e_2 + 1}\right)^2 = \frac{1 - \mu_{32}^2}{1 - \mu_{22}^2}
\]

\[
\left(\frac{V_{43}}{V_{33}}\right)^2 = \left(\frac{e_3 + 1}{e_3 - 1}\right)^2 = \frac{1 - \mu_{33}^2}{1 - \mu_{33}^2}
\]

where \(\mu_{ik}\) :

\[
\mu_{11} = \frac{1}{\Delta V_1} (\sqrt{1 + e_1} - \cos\gamma_1) \quad \mu_{21} = \frac{1}{\Delta V_2} (\sqrt{1 - e_2 \cos\gamma_2} - \sqrt{1 - e_1})
\]

\[
\mu_{22} = \frac{1}{\Delta V_2} (\sqrt{1 - e_2} - \sqrt{1 - e_1 \cos\gamma_2}) \quad \mu_{32} = \frac{1}{\Delta V_3} (\sqrt{1 + e_3 \cos\gamma_3} - \sqrt{1 + e_2})
\]

\[
\mu_{33} = \frac{1}{\Delta V_3} (\sqrt{1 + e_3} - \sqrt{1 + e_2 \cos\gamma_3}) \quad \mu_{43} = \frac{1}{\Delta V_4} (\cos\gamma_4 - \sqrt{1 - e_3})
\]

Figure 6 compares the N=4 and N=3 strategies with equal \(R_2\) and \(R_3 = R_2 + h\) with \(h = 0.5\) (for \(h = 0\) N=4 degenerates to \(N = 3\)). Note that the \(\Delta V\) required by the N=4 strategy is bigger than the N=3 case, and the difference becomes bigger as \(h\) increases. From numerical tests this occurs also for higher number of impulses \(N > 4\).
Figure 6: DV saved using bielliptic vs four impulse optimized transfers

References


