Experimental Verification of Various Modelling Techniques for Piezoelectric Actuated Panels

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Abstract

The scientific equipment being used on-board satellites sometimes requires a high degree of pointing accuracy and a minimum level of disturbances i.e. vibrations. To this end, actively controlled structures are employed and one of the best methods of achieving this degree of control with the minimum of impact on the structure itself is to employ piezoelectric patches as sensors/actuators.

In this paper, various techniques for the mathematical modelling of the dynamics of a piezoelectric actuated panel are presented. The results obtained via the numerical simulations are then compared with test results in order to assess the accuracy of the various modelling techniques.

Attention is focused on a simply supported panel, with twin patches of piezoelectric material bonded on opposite faces of the panel, which act as actuators. The equations governing the dynamics of the system, which includes the panel and the actuators driven by electrical signals, are derived using Lagrange's equations of motion with vibration mode shapes of the bare panel as the Ritz functions. Other techniques, such as Mechanical Impedance based method and Finite Element method are also used to produce mathematical models of the system.

Introduction

The scientific equipment being used onboard satellites often requires a high degree of pointing accuracy and therefore it is necessary to minimise the level of mechanical disturbances i.e. vibrations. The equipment is often mounted on lightweight panels where vibrations produced by the functioning of other necessary subsystems have to be suppressed to achieve the required level of stability. One method, which allows very good performance to be achieved, in terms of vibration reduction, is to use active control [1, 2, 3, 4]. Control forces are applied to the structure, in order to change its response in the required manner. These control forces are produced by actuators mechanically coupled with the structure, which are driven by signals coming from a control unit. In turn the control unit receives signals from sensors placed on the structure. The most suitable type of actuators for this application are piezoelectric patches bonded on the faces of the panel [5, 6]. Furthermore piezoelectric patches can also be used as sensors for the control system. The main advantage in the use of piezoelectric patches is the minimum impact on the structure, and the fact that they do not require large power or a backing structure.
It is crucial to be able to quantify the impact that sensors and actuators have on the structure [7, 8]. In fact when these piezoelectric transducers are fixed on the structure, the dynamic behaviour of the assembly changes [9], and this change can be relevant for the design of the control system.

In this paper, various techniques for the mathematical modelling of the dynamics of a piezoelectric actuated panel are presented. The results obtained via the numerical simulations are then compared with test results in order to assess the accuracy of the various modelling techniques. Attention is focused on a simply supported panel, with twin patches of piezoelectric material bonded on opposite faces, which act as actuators. The panel is an acceptable compromise between problem complexity and the need to gain useful insights as to the benefits (and limitations) of various active control techniques. The equations governing the dynamics of the system, which includes the panel and the actuators driven by electrical signals, are derived using Lagrange’s equations of motion with vibration mode shapes of the bare panel as the Ritz functions. Other techniques, such as Mechanical Impedance based method and Finite Element method are also used to produce mathematical models of the system.

A test rig was designed, which was comprised of a simply supported aluminium alloy panel and supporting frame. Particular attention has been placed in designing the rig to reproduce as accurately as possible a simple support along all four edges. The panel was tested using an impact hammer, and the raw data was acquired using a miniature accelerometer in order to minimise interference with the plant dynamics. At a later stage, the piezoelectric patches were bonded to the panel and a new set of experimental data acquired in order to assess the impact of the patches on the panel dynamics. Various frequency response functions were computed from the experimental data and compared with the results from the numerical simulations.

Mathematical Models of Piezoelectric Actuated Panel

Onboard spacecraft, aircraft or other vehicles, panels are, very common structural elements, through which vibrations, generated by various sources, propagate. Hence, the choice of a panel as the structural element to be modelled and investigated is both general and representative of practical applications. The development of the mathematical model is based on the assumptions that the panel is homogeneous and thin enough to be considered in a plane stress condition. These assumptions are justified by the fact that thin homogeneous panels are very commonly used and have the further advantage of simplifying the mathematical description of the dynamics. However both these assumptions can be relaxed. A schematic diagram of the arrangement considered is shown in Figure 1. The actuators for the active control systems are twin patches of piezoelectric material bonded onto opposite faces of the panel. The outer electrodes of the patches are electrically connected together and the plate, which is grounded, is used as the other electrode for both patches.
The mathematical model of the Actively Controlled Panel (ACP) is derived using the procedure described in reference [4]. The first step of this procedure consists of writing suitable expressions for the kinetic and potential energy associated with the elements of the ACP i.e. panel and piezoelectric patches.

The displacement field (out-of-plane displacements of the panel) is obtained using a Rayleigh Ritz approach as a superposition of shape functions $S_{m,n}$ multiplied by the time dependent modal co-ordinates $\phi_{m,n}$

$$w(x,y,t) = \sum_{n=1}^{Nm} \sum_{m=1}^{Nr} S_{n,m}(x,y)\phi_{n,m}(t) = \mathbf{s}^\mathbf{\phi}$$  \hspace{1cm} (1)

### Dimensions

<table>
<thead>
<tr>
<th>panel</th>
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<tbody>
<tr>
<td>a</td>
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<td></td>
</tr>
<tr>
<td>b</td>
<td>203.2 mm</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>1.524 mm</td>
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<th>actuator</th>
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<th></th>
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<tbody>
<tr>
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<td>50.8 mm</td>
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<td></td>
</tr>
<tr>
<td>xa₂</td>
<td>101.6 mm</td>
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<td></td>
</tr>
<tr>
<td>ya₁</td>
<td>25.4 mm</td>
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</tr>
<tr>
<td>ya₂</td>
<td>76.2 mm</td>
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<td></td>
</tr>
<tr>
<td>h_{pza}</td>
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### Material properties

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</tr>
<tr>
<td>ν</td>
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<td></td>
</tr>
<tr>
<td>η</td>
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<table>
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<th>E</th>
<th>63 10^9 Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ</td>
<td>7650 kg/m³</td>
<td></td>
</tr>
<tr>
<td>ν</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>1.66 10^{-10} m/volt</td>
<td></td>
</tr>
<tr>
<td>η</td>
<td>0.001</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Model properties

**LRR modelling technique**

The mathematical model of the Actively Controlled Panel (ACP) is derived using the procedure described in reference [4]. The first step of this procedure consists of writing suitable expressions for the kinetic and potential energy associated with the elements of the ACP i.e. panel and piezoelectric patches.
where \( s \) is the vector of the shape functions, \( \phi \) is the vector of the modal co-
ordinates and \( x \) and \( y \) are the in-plane co-ordinates on the panel. The kinetic
energies associated with the panel and the piezoelectric patches can easily be
written as a function of the modal co-ordinates as:

\[
T_{pl} = \frac{1}{2} \phi^i M_{pl} \phi \quad \text{and} \quad T_{pz} = \frac{1}{2} \phi^i M_{pz} \phi
\]  

(2a,b)

where \( M_{pl} \) and \( M_{pz} \) are the appropriate mass matrices (see reference [4] for full
details). Similarly the potential energy associated with the panel can be written as:

\[
U_{plate} = \frac{1}{2} \phi^i K_{pl} \phi
\]  

(3)

where \( K_{pl} \) is the generalised stiffness matrix. The potential energy of the
piezoelectric patches can be expressed as the sum of three energy components

\[
U_{pz} = U_{pz}^{\text{elast}} + U_{pz}^{\text{elastelect}} + U_{pz}^{\text{elect}}
\]  

(4)

where \( U_{pz}^{\text{elast}} \) is the elastic energy stored due to the elasticity of the material,
\( U_{pz}^{\text{elastelect}} \) represents the further elastic energy due to the voltage driven
piezoelectric effect, and \( U_{pz}^{\text{elect}} \) is the electric energy stored due to the dielectric
rigidity of the piezoelectric material and

\[
U_{pz}^{\text{elast}} = \frac{1}{2} \phi^i K_{pz}^{\text{elast}} \phi; \quad U_{pz}^{\text{elastelect}} = \mathbf{v}^T K_{pz}^{\text{elastelect}} \phi; \quad U_{pz}^{\text{elect}} = \frac{1}{2} \mathbf{v}^T K_{pz}^{\text{elect}} \mathbf{v}
\]  

(5a,b,c)

where the various \( K \) matrices are defined accordingly and \( \mathbf{v} \) contains the generalised
co-ordinates representing the voltages at the piezoelectric patches [4].

Lagrange’s equations of motion for the system can now be written in the form:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i
\]  

(6)

where:

\[
T = T_{pl} + T_{pz} \quad \text{and} \quad U = U_{pl} + U_{pz}
\]  

(7a,b)

and \( Q_i \) are the generalised forces and \( q_i \) the generalised co-ordinates (representing
both \( \phi \) and \( v \)). Assuming that the external excitation consists of \( N_f \) point forces \( F_j \)
acting on the panel at arbitrary locations, the generalised forces will have the form:
where $f$ is the vector of the forces and $S_f$ is a matrix containing the modal shape vector $s$ evaluated at the force locations. Equation (6) yields:

$$Q_i = \sum_{j,l} F_j \frac{\partial W}{\partial \phi_i} \quad \text{or} \quad Q = S_f f$$ \hfill (8a,b)

Equation (9a) describes the dynamics of the system when the piezoelectric patches are used as actuators (externally driven) and can be rewritten as [4]

$$M_{acp} \ddot{\phi} + C_{acp} \dot{\phi} + K_{acp} \phi = V_a v_a + S_f f$$ \hfill (10)

where damping has been introduced via the matrix $C_{acp}$. Equation (9b) can be rewritten as:

$$v_s = - \left(K_{s}^{elect}\right)^T K_{s}^{elastoelec} \phi$$ \hfill (11)

Mechanical Impedance modelling technique

The effect produced on the panel when the piezoelectric patch is driven as an actuator is very similar to that produced by line moments acting along the edge of the piezoelectric patch, which is how the piezoelectric input is modelled using the MI method.

This modelling technique is based on an approximation to mechanical impedance matching between the piezoelectric patches and the underlying structure, in order to account for the dynamic interaction between the two subsystems. The method used consists of calculating the dynamic output at the edge of a piezoelectric patch which is coupled to the mechanical impedance of a thin panel. In this way it is possible to calculate the line moments at the edges of a rectangular patch and, subsequently, the response of the panel excited by these line moments. The main stages of this modelling technique are briefly reported next, with full details given in reference [8].

The equations of motion of the piezoelectric patch, along the $x$ and $y$ axes, are taken as:
\[
\rho_{pz} \frac{\partial^2 u}{\partial t^2} = E_{pz} \frac{\partial^2 u}{\partial x^2} \quad \rho_{pz} \frac{\partial^2 v}{\partial t^2} = E_{pz} \frac{\partial^2 v}{\partial y^2}
\]

(12)

and their solution:

\[
u = A \sin(k_{pz} x)e^{j\omega t} \quad \nu = B \sin(k_{pz} y)e^{j\omega t}
\]

(13)

The constants A and B can be calculated from the constitutive relationships of the piezoelectric material:

\[
\frac{\partial u}{\partial x} = \frac{\sigma_x}{E_{pz}} - \frac{\nu_{pz} \sigma_y}{E_{pz}} + d_{31} E \quad \frac{\partial v}{\partial x} = \frac{\sigma_y}{E_{pz}} - \frac{\nu_{pz} \sigma_x}{E_{pz}} + d_{32} E
\]

(14)

The stresses can be obtained from the respective forces, which can be calculated using

\[
F_y = Z_{yy} \dot{v} + Z_{yx} \dot{u} \quad F_x = Z_{xx} \dot{u} + Z_{xy} \dot{v}
\]

(15)

once the impedances \(Z_{xx}, Z_{xy}\) of host structure at the edges of the patch are known. The forces at the edges at the piezoelectric patch can be related to the moments by the relation:

\[
M = F (h + h_{pz})
\]

(16)

as illustrated in Figure 2 and once the moments at the edge of the patch are known, the response of the panel can be calculated from:

\[
\sum_{m=1}^{N_m} \sum_{n=1}^{N_n} \left( \rho h \omega_{m,n}^2 w_{m,n} + \rho h \ddot{w}_{m,n} \right) = \frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y}
\]

(17)

where \(\omega_{m,n}\) is the resonance frequency of the \(m,n\) mode, \(w\) is the out of plane displacement and \(M_x\) and \(M_y\) can be obtained from (15) and (16).
The out of plane displacement $w$ can be represented by using a mode superposition technique as for the Lagrange Raleigh Ritz Model (LRRM). This model has been implemented in MATLAB, and the results are compared with those obtained with the LRR technique and FEM in the next section.

**FE Model**

The FE method is one of the most used methods for structural analysis. In Figure 3 the FE model of the piezoelectric actuated panel is shown. The FE model was built using the commercial software package Ansys. The type of element used for the panel and piezoelectric patch is Shell99 (multi layered shell element), where in the area covered by the patch three layers where used (Piezoelectric material top patch / Aluminium / Piezoelectric material bottom patch). In the FE model the effect of the piezoelectric patches working as actuators is modelled by applying line moments along the edge of the patch. The value of the moments to be applied can be calculated following the theory presented by Brennan et al. in [10].

![Figure 3: FE model of the piezoelectric actuated panel](image)

The FRF obtained using the FE model are shown in Figure 7

**Experimental verification**

The experimental verification of the response predicted by the mathematical models requires the availability of an experimental rig able to recreate as accurately as possible a simple support along the edges of the panel. The simple supports are produced by using vertical shims bonded to the edge, as illustrated in Figure 4. More details about this set-up can be found in [11]. The FE model of the experimental rig composed of the panel and frame reproducing the simple supports is shown in Figure 5. In this FE model, the box structure of the frame is modelled using shell elements, whilst the corner segments which support the shims are modelled using solid elements. The experimental implementation is shown in Figure 6.
Figure 4: Simple support along the edge of the panel

The response of the panel was retrieved using a B&K accelerometer (2g) positioned at x=50.8 mm and y=152.4 mm. Data were recorded using a Signal Processing Ltd four-channel data acquisition suite connected to a personal computer which operated using the Matlab software. The force and acceleration data were post-processed using the Matlab “spectrum” function. The piezoelectric patches were excited with a sinusoidal signal of 2 V (peak to peak) over the frequency range 50 Hz to 500 Hz. The lowest limit of the frequency range was imposed to omit the rigid body motion of the system. The upper limit was fixed to avoid exciting the flexible modes of the supporting frame.

Figure 5: FE model of the SS piezoelectric actuated panel

From the comparison between the results given by the mathematical model and the experimental data, it is possible to see that all models are able to predict the dynamic response of the panel driven by the piezoelectric patch with good accuracy. However, the MI method tends to overestimate of the natural frequency of vibration, due to the fact the out-of-plane inertia of the patch is not included in the model.
In this work, various methods that can be used for the production of mathematical models of piezoelectrically actuated panels for active control design studies have been briefly presented. The dynamic response of the panel predicted using each of these models is then compared with experimental results. The impact of the patch
on the dynamics of the panel is very small, therefore in order to be able to resolve it, the experimental set up needs to be very accurate. The experimental set up used to reproduce the simple support along the edges of the panel is briefly illustrated. From the comparison of the results given by the mathematical model, with experimental data, it is possible to see that both FEM and LRRM are able to predict the dynamic response of the panel driven by the piezoelectric patch with high accuracy. The MIM although able to predict the overall pattern of the FRF very well, was less accurate in the prediction of the natural frequencies.

References


