

N.A.E.

R. & M. No. 2594
(10,628)
A.R.C. Technical Report



MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Calculated Aerodynamic
Characteristics of Two Infinite
Wings with Constant Chord

By

V. M. FALKNER, B.Sc., A.M.I.MECH.E.,
of the Aerodynamics Division, N.P.L.

Crown Copyright Reserved

LONDON: HER MAJESTY'S STATIONERY OFFICE

1952

THREE SHILLINGS NET

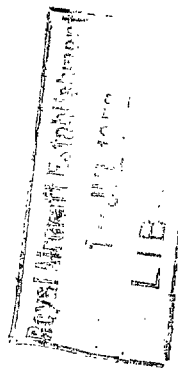
Calculated Aerodynamic Characteristics of Two Infinite Wings with Constant Chord

By

V. M. Falkner, B.Sc., A.M.I.Mech.E, of the Aerodynamics Division, N.P.L.

Reports and Memoranda No. 2594

May, 1947



Summary.—The report gives solutions obtained by the vortex lattice method for the aerodynamic loading of two infinite wings of constant chord with sweepback of 45 deg, one with a V-joint at the centre, the other rounded off with arcs of radius four times the chord. The true mathematical solution for these problems is exceedingly difficult to find, and the accuracy has been verified by considering the convergence of solutions of varying complexity.

The V-wing shows a reduction in circulation near the joint with accompanying backward movement of the local centre of pressure, while the rounded wing has increased circulation without appreciable variation of the centre of pressure from the 0.25-chord position.

The results will be used to modify loading functions used in vortex lattice theory in order to improve solutions for wings of small aspect ratio, particularly when the leading or trailing edges meet at an included angle which differs considerably from 180 deg.

1. *Introduction.*—The object of this investigation is to examine the effect on wing loading of sudden changes in direction of the leading or trailing edge, with particular reference to swept-back wings. The work has arisen as a result of doubts which have occasionally been expressed as to the accuracy of wing loading solutions obtained by the vortex lattice method, particularly where small aspect ratios are concerned when the leading and trailing edges may meet at the wing centre at an angle which differs considerably from 180 deg.

There are two separate three-dimensional wing loading effects involved in the case of swept-back wings. The first is the well-known effect due to the span being of finite length, and it is not questioned that this is covered by the loading functions, equivalent to a Fourier series, which are normally used. The second effect comes into prominence at large angles of sweep-back and is due to conditions at any position where the leading or trailing edge has a sudden change of direction. The effect is that there will usually be a sheet of vorticity shed from the wing in the region of such a change which is quite unconnected with aspect ratio effects. In the previous work by vortex lattice theory it has been assumed that this effect is either covered by the standard formulae or is so small that no appreciable error is involved by its neglect. The present work seeks to establish the nature and magnitude of the variation and to indicate what modifications are required to the standard loading formulae to allow for it.

The work is concerned solely with the second effect described above, and aspect ratio effects have therefore been eliminated by considering infinite wings only, of which two have been calculated (a) constant chord infinite V-wing, angle of sweep-back 45 deg, Fig. 1, (b) constant chord infinite wing, angle of sweep-back 45 deg, rounded off by circular arcs with radius four times chord, Fig. 2.

2. *General Considerations.*—McKinnon Wood¹ has shown that, when an infinite wing of constant chord is yawed, the chord c and incidence being adjusted to be the same parallel to the wind direction, the two-dimensional circulation K is reduced to $K \cos \beta$, where β is the angle

of yaw. It follows that, for the two infinite wings considered here, the circulation at a great distance from the wing centre, say K_∞ , will be $K \cos \beta$, where K is the circulation for a straight infinite wing of the same chord c . If lines of constant circulation $\int k dx$, k being the intensity of bound vorticity and the range of integration being from leading edge to a point x on the chord, are plotted on the wing surface they will form a parallel system of lines at a great distance from the wing centre. In the case of the V-wing there is no doubt that the lines must be rounded off at the centre, and, as considerations of downwash also suggest that the distance apart must be increased at the centre, some of the lines will break away from the surface, giving rise to a sheet of trailing vorticity and a loss of circulation near the centre region of the wing.

In the case of the rounded-off wing, similar considerations apply, but it appears that the separation of the lines will not be enough to lead to breakaway, and, in fact, lines will break into the trailing edge, with consequent increase in circulation in the middle region of the wing. It is obvious that, for the V-wing, conditions are such that the local C.P. must move backwards, but prediction of the effect in the case of the rounded-off wing would be based on guesswork.

3. *Calculations.*—The calculations have been carried out by means of the vortex lattice, using the $\frac{1}{4}/\frac{3}{4}$ -method as well as a more elaborate process equivalent to the 126 vortex layout². No claim is made that the solutions are mathematically accurate. The writer is informed by a mathematician familiar with this subject that the accurate mathematical solution would be exceedingly difficult to find. Nevertheless, although simple functions, which cannot be expected to represent truly the conditions at the discontinuity, must be used in this work, general experience with the lattice method suggests that the solutions obtained may provide a good practical approximation to the accurate answer.

The solutions are based on the general formulae for the vorticity distribution

$$\frac{k}{2V} = A_0 \cot \frac{1}{2} \theta + A_1 \sin \theta + A_2 \sin 2\theta \quad \dots \quad \dots \quad \dots \quad \dots$$

where k is the vorticity distribution, and A_0, A_1 , etc., are functions of $\eta = y/c$.

If the solution is per radian incidence measured parallel to the wind direction, the limiting values of coefficients as $\eta \rightarrow \infty$ will be

$$A_0 = \cos \beta = 0.7071, \quad A_1 = A_2 = A_n = 0$$

The values of coefficients at $\eta = 0$ and ∞ will be denoted by suffices c and ∞ . The value of K/K_∞ is

$$\frac{A_0 + \frac{1}{2}A_1}{0.7071},$$

and the position of the centre of pressure as a fraction of the chord is

$$0.25 \frac{A_0 + A_1 - \frac{1}{2}A_2}{A_0 + \frac{1}{2}A_1}.$$

3.1. *V-Wing.*—A solution of the V-wing was first attempted using the equivalent of the $\frac{1}{4}/\frac{3}{4}$ -chord method. The formulae used were

$$k/2V = A_0 \cot \frac{1}{2} \theta$$

$$A_0 = (A_{0\infty} - A_{0c}) [R_1 e^{-\frac{1}{2}\eta^2} + R_2 e^{-\eta^2} + R_3 e^{-2\eta^2} + R_4 e^{-4\eta^2} + R_5 e^{-8\eta^2}],$$

where $\Sigma R = 1$.

It is only necessary to use the lattice for a range of $\pm \eta$ over which the exponential functions remain finite, as the integrals of downwash associated with the infinite part of the wing can be computed analytically for any given control point. The spanwise spacing of the vortices was 0.2 chord, and the solution obtained is given in Table 1 and Fig. 3. It is clear that the coefficients of the exponential functions have adjusted themselves so that the rounding off has been reduced to a peak and, as a result of this, a second solution was obtained by using

$$A_0 = (A_{0\infty} - A_{0c}) [R_1 e^{-\eta} + R_2 e^{-2\eta} + R_3 e^{-4\eta} + R_4 e^{-6\eta} + R_5 e^{-8\eta}]$$

where the exponential functions are used as symmetrical functions about

$$\eta = 0, \text{ or } e^{-\eta} \equiv e^{-\sqrt{\eta^2}},$$

with a discontinuity of slope. The solution obtained is given in Table 2 and Fig. 3. Finally, by a least squares process, the number of variables was reduced, giving

$$A_0 = (A_{0\infty} - A_{0c}) [R_1 e^{-\eta} + R_2 e^{-2\eta} + R_3 e^{-4\eta}]$$

The results, given in Table 3 and Fig. 3, show no appreciable loss in accuracy through reduction of the number of variables, and have the additional advantage of exceptional smoothness.

The solution was then elaborated by using the layout equivalent to the 126-vortex solution, firstly with 6 control points (2 chordwise), and secondly with 9 control points (3 chordwise). For the latter case the formulae used were:—

$$A_0 = A_{0\infty} - [A_{0\infty} - A_{0c}] [R_1 e^{-\eta} + R_2 e^{-2\eta} + R_3 e^{-4\eta}]$$

$$A_1 = A_{1c} [S_1 e^{-\eta} + S_2 e^{-2\eta} + S_3 e^{-4\eta}]$$

$$A_2 = A_{2c} [T_1 e^{-\eta} + T_2 e^{-2\eta} + T_3 e^{-4\eta}]$$

The results for these two solutions are given in Tables 4 and 5 and are plotted in Fig. 3.

3.2. Rounded-off V-Wing.—The solutions for the rounded-off wing were obtained by the use of functions of the type $e^{-\eta^2}$, as there is no discontinuity present to give rise to difficulties. The work of computing the downwash due to the infinite part of the solution was more laborious than for the V-wing, and the work has been limited to two $\frac{1}{4}/\frac{3}{4}$ solutions and one 6-point solution (2 points chordwise).

For the first solution the formula used was

$$A_0 = A_{0\infty} - [A_{0\infty} - A_{0c}] [R_1 e^{-\frac{1}{2}\eta^2} + R_2 e^{-\eta^2} + R_3 e^{-2\eta^2} + R_4 e^{-4\eta^2} + R_5 e^{-8\eta^2}]$$

and, after confirming that the terms could be reduced to three without loss of accuracy, the general formulae used were

$$A_0 = A_{0\infty} - [A_{0\infty} - A_{0c}] [R_1 e^{-\frac{1}{2}\eta^2} + R_2 e^{-\eta^2} + R_3 e^{-2\eta^2}]$$

$$A_1 = A_{1c} [S_1 e^{-\frac{1}{2}\eta^2} + S_2 e^{-\eta^2} + S_3 e^{-2\eta^2}]$$

The three solutions are given in Tables 6, 7, 8 and are plotted in Fig. 5.

4. Conclusions.—For the V-wing the closeness of the agreement between solutions 1, 2 and 3 demonstrates that it is unnecessary to use more than three terms of the series in the spanwise direction. The subsequent agreement between the solutions 4 and 5, which allow for variation

in the chordwise direction, suggests that a good approximation to the accurate solution has been obtained. It is observed that the movement backwards of the local C.P. is a phenomenon concentrated mainly near the wing centre, at which it reaches a maximum of 0.416 chord, with a corresponding reduction of circulation to 0.654 of the value at infinity. The locus of the C.P. has been plotted in Fig. 4 and suggests strongly that the curve is rounded off without discontinuity. If this condition were assumed and expressed mathematically in the original formula, there is little doubt that improved solutions could be obtained.

For the rounded-off wing, there is good agreement between the $\frac{1}{4}/\frac{3}{4}$ and the more elaborate solution. The deficit of circulation has been changed by the rounding off to an excess amounting to 1.123 of the value at infinity. The C.P., however, varies little from the two-dimensional value of 0.25 chord.

5. *Method of Use.*—A brief note is now given of the method by which the results will be applied to wing loadings of finite wings. If the effect of aspect ratio is combined with the effect described above, typical terms of a suitable series of loading functions would be

$$e^{-n}\sqrt{(1 - \eta^2)} \text{ and } e^{-n^2}\sqrt{(1 - \eta^2)}.$$

There is no doubt that the latter series can be adequately represented by the usual form

$$\eta^n\sqrt{(1 - \eta^2)}$$

where n is even. The former cannot, however, be so represented, at least by a few terms, because of the discontinuity at the origin. As it is convenient to retain the form of the loading function series, the term

$$\eta \sqrt{(1 - \eta^2)} \text{ treated as } \sqrt{\eta^2} \sqrt{(1 - \eta^2)}$$

was added, and a separate investigation has shown that terms of the series

$$e^{-n} \sqrt{(1 - \eta^2)}$$

likely to be used can be represented adequately by the series

$$a_0 \sqrt{(1 - \eta^2)} + b_0\eta \sqrt{(1 - \eta^2)} + c_0\eta^2 \sqrt{(1 - \eta^2)} + e_0\eta^4 \sqrt{(1 - \eta^2)}.$$

Hence, it is concluded that, when the wing is appreciably rounded off, no modifications to the standard loading functions are required, while, when leading or trailing edges meet at an angle, the loading series

$$a_0 \sqrt{(1 - \eta^2)} + c_0\eta^2 \sqrt{(1 - \eta^2)} + e_0\eta^4 \sqrt{(1 - \eta^2)}$$

should be modified by the addition of the term

$$b_0\eta \sqrt{(1 - \eta^2)} \text{ treated as } b_0 \sqrt{\eta^2} \sqrt{(1 - \eta^2)}.$$

The effect of this modification on wing loading will be given in a later report. It can be said, however, that the effect on spanwise loading is small, the main result being a movement backwards of the local C.P. in the region of the discontinuity. A further modification is required when the discontinuity is along the span, as, for instance, in the case of the Pterodactyl, but there should be no difficulty about this.

6. *Acknowledgement.*—The writer wishes to acknowledge the work of Misses S. D. Brown and P. I. Bond, who were responsible for the computing of the solutions given in this report.

REFERENCES

No.	Author	Title, etc.
1	R. McKinnon Wood	Notes on Swept-back Wings for High Speeds. A.R.C. Report 8806. (Unpublished).
2	V. M. Falkner	Calculations of the Aerodynamic Loading of a Delta Wing. A.R.C. Report 9830. (Unpublished).

TABLE 1

$\frac{1}{4}/\frac{3}{4}$ Solution for V-Wing, No. 1

$$K_0/K_\infty = 1 - 0.5870 e^{-2\eta^2} + 0.9508 e^{-\eta^2} - 0.9472 e^{-2\eta^2} + 0.5634 e^{-4\eta^2} - 0.2406 e^{-8\eta^2}.$$

C.P. 0.25 chord.

η	K_0/K_∞	η	K_0/K_∞
0	0.739	2.4	0.970
0.4	0.811	2.8	0.989
0.8	0.854	3.2	0.996
1.2	0.888	3.6	0.999
1.6	0.905	4.0	1.000
2.0	0.938		

TABLE 2

$\frac{1}{4}/\frac{3}{4}$ Solution for V-wing, No. 2

$$K_0/K_\infty = 1 - 0.649 e^{-\eta} + 0.948 e^{-2\eta} - 1.446 e^{-4\eta} + 1.436 e^{-6\eta} - 0.550 e^{-8\eta}.$$

C.P. 0.25 chord.

η	K_0/K_∞	η	K_0/K_∞
0	0.739	2.4	0.949
0.4	0.807	2.8	0.964
0.8	0.852	3.2	0.975
1.2	0.880	3.6	0.983
1.6	0.905	4.0	0.988
2.0	0.929		

TABLE 3

$\frac{1}{4}/\frac{3}{4}$ Solution for V-wing, No. 3

$$K_0/K_\infty = 1 - 0.446 e^{-\eta} + 0.276 e^{-2\eta} - 0.089 e^{-4\eta}.$$

C.P. 0.25 chord.

η	K_0/K_∞	η	K_0/K_∞
0	0.741	2.4	0.962
0.4	0.807	2.8	0.974
0.8	0.852	3.2	0.982
1.2	0.890	3.6	0.988
1.6	0.921	4.0	0.992
2.0	0.945		

TABLE 4

Solution for V-wing, two points chordwise, No. 4

$$A_0 = 0.7071 - 0.4702 e^{-\eta} + 0.4768 e^{-2\eta} - 0.5433 e^{-4\eta}.$$

$$A_1 = 0.2386 e^{-\eta} - 0.5212 e^{-2\eta} + 0.8773 e^{-4\eta}.$$

$$K/K_\infty = 1 - 0.4963 e^{-\eta} + 0.3058 e^{-2\eta} - 0.1481 e^{-4\eta}.$$

η	K/K_∞	C.P.	η	K/K_∞	C.P.
0	0.661	0.409	2.0	0.938	0.254
0.2	0.732	0.308	2.4	0.957	0.253
0.4	0.775	0.273	2.8	0.971	0.252
0.8	0.833	0.258	3.2	0.980	0.252
1.2	0.877	0.256	3.6	0.987	0.251
1.6	0.912	0.256	4.0	0.991	0.251

TABLE 5

Solution for V-wing, three points chordwise, No. 5

$$A_0 = 0.7071 - 0.4177 e^{-\eta} + 0.3223 e^{-2\eta} - 0.4778 e^{-4\eta}.$$

$$A_1 = 0.1317 e^{-\eta} - 0.2416 e^{-2\eta} + 0.7670 e^{-4\eta}.$$

$$A_2 = -0.0824 e^{-\eta} + 0.2294 e^{-2\eta} - 0.1057 e^{-4\eta}.$$

$$K/K_\infty = 1 - 0.4976 e^{-\eta} + 0.2850 e^{-2\eta} - 0.1334 e^{-4\eta}.$$

η	K/K_∞	C.P.	η	K/K_∞	C.P.
0	0.654	0.416	2.0	0.938	0.254
0.2	0.724	0.311	2.4	0.957	0.253
0.4	0.768	0.275	2.8	0.971	0.252
0.8	0.829	0.258	3.2	0.980	0.251
1.2	0.875	0.256	3.6	0.987	0.251
1.6	0.911	0.255	4.0	0.991	0.251

TABLE 6

 $\frac{1}{4}/\frac{3}{4}$ solution for rounded-off V-wing, No. 6

$$K/K_\infty = 1 + 0.4157 e^{-\frac{1}{2}\eta^2} - 0.4952 e^{-\eta^2} + 0.3090 e^{-2\eta^2} - 0.1239 e^{-4\eta^2} + 0.0262 e^{-8\eta^2}.$$

C.P. = 0.25 chord.

η	K/K_∞	η	K/K_∞
0	1.132	2.0	1.047
0.4	1.128	2.4	1.022
0.8	1.117	2.8	1.008
1.2	1.102	3.2	1.002
1.6	1.079		

TABLE 7

1/4/3/4 solution for rounded-off V-wing, No. 7

$$K/K_\infty = 1 + 0.1998 e^{-\frac{1}{2}\eta^2} - 0.0044 e^{-\eta^2} - 0.0661 e^{-2\eta^2}.$$

C.P. = 0.25 chord.

η	K/K_∞	η	K/K_∞
0	1.129	2.0	1.027
0.4	1.133	2.4	1.011
0.8	1.124	2.8	1.004
1.2	1.092	3.2	1.001
1.6	1.055		

TABLE 8

Solution for rounded-off wing, two points chordwise, No. 8

$$A_0 = 0.7071 + 0.1722 e^{-\frac{1}{2}\eta^2} - 0.1346 e^{-\eta^2} + 0.0320 e^{-2\eta^2}.$$

$$A_1 = 0.0414 e^{-\frac{1}{2}\eta^2} + 0.0036 e^{-\eta^2} - 0.0099 e^{-2\eta^2}.$$

$$K/K_\infty = 1 + 0.2729 e^{-\frac{1}{2}\eta^2} - 0.1878 e^{-\eta^2} + 0.0383 e^{-2\eta^2}.$$

η	K/K_∞	C.P.	η	K/K_∞	C.P.
0	1.123	0.256	0.9	1.106	0.254
0.1	1.123	0.256	1.0	1.102	0.254
0.2	1.122	0.256	1.2	1.090	0.253
0.3	1.121	0.256	1.4	1.077	0.253
0.4	1.120	0.255	1.6	1.062	0.252
0.5	1.118	0.255	2.0	1.034	0.251
0.6	1.116	0.255	2.4	1.015	0.250
0.7	1.113	0.255	2.8	1.005	0.250
0.8	1.110	0.255	3.2	1.002	0.250

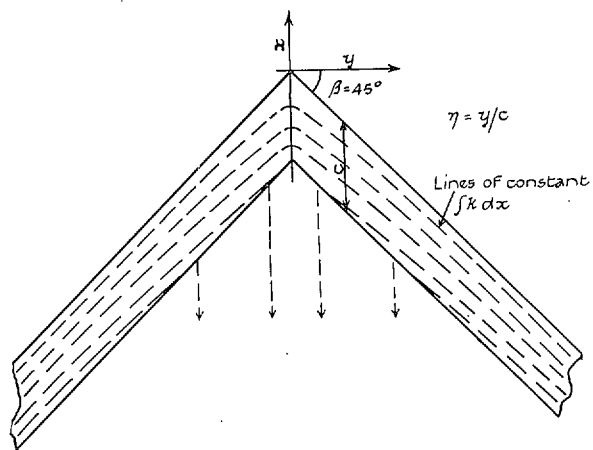


FIG. 1. Infinite V-wing: constant chord.

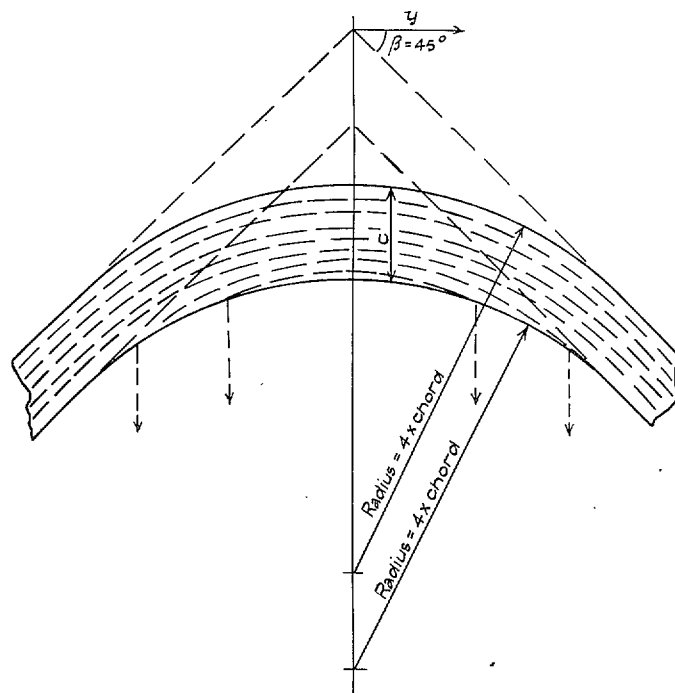
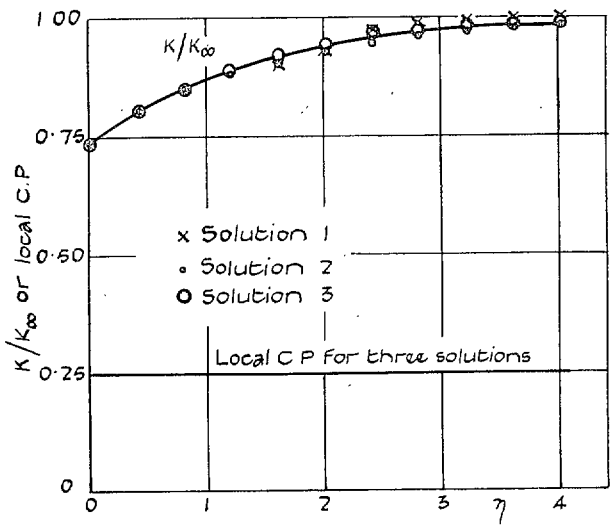
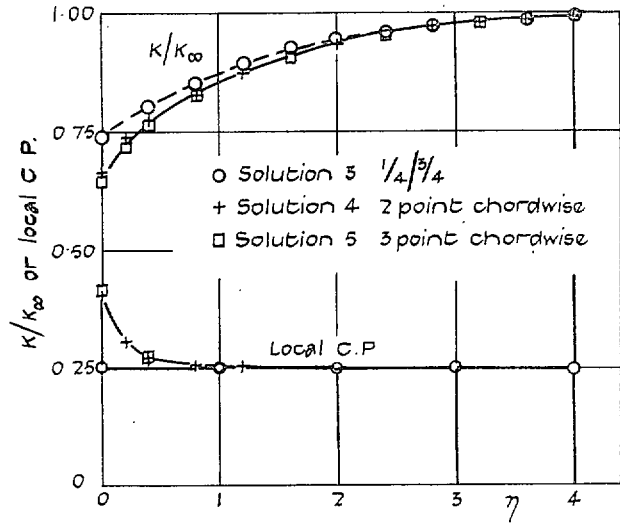


FIG. 2. Infinite wing: constant chord rounded-off.



$\frac{1}{4}/\frac{3}{4}$ Solutions for infinite V-wing.



Solutions for infinite V-wing.

FIG. 3.

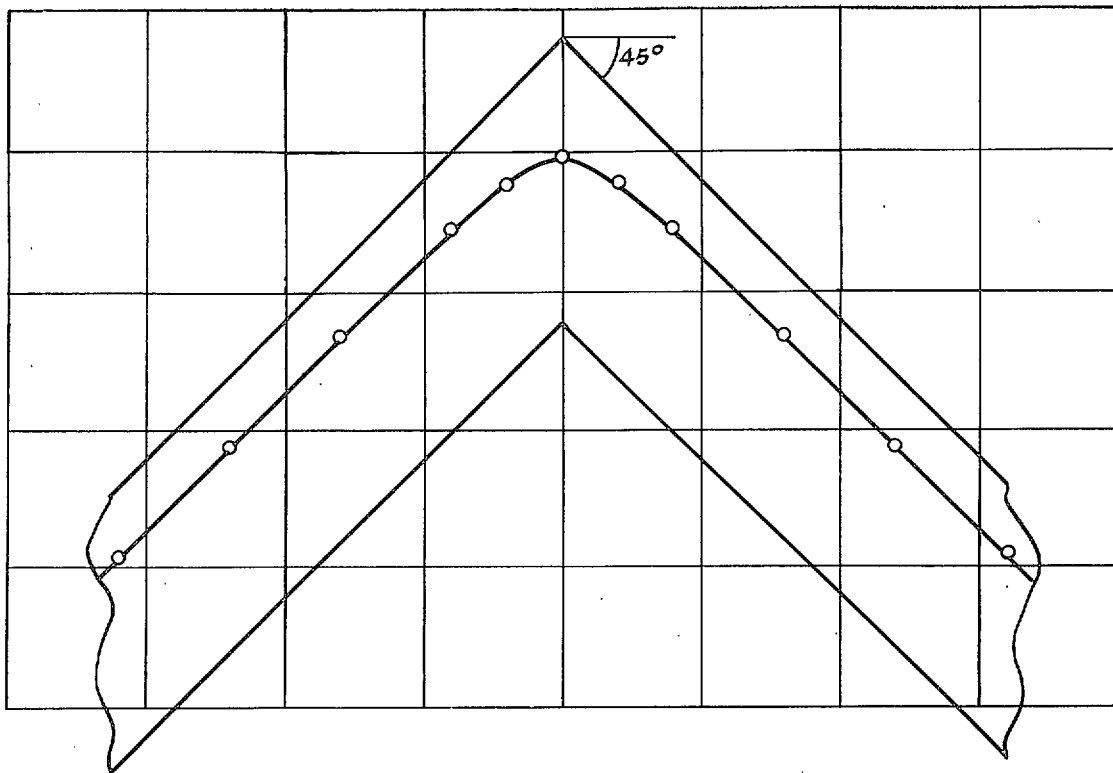


FIG. 4. Infinite V-wing: locus of local C.P.

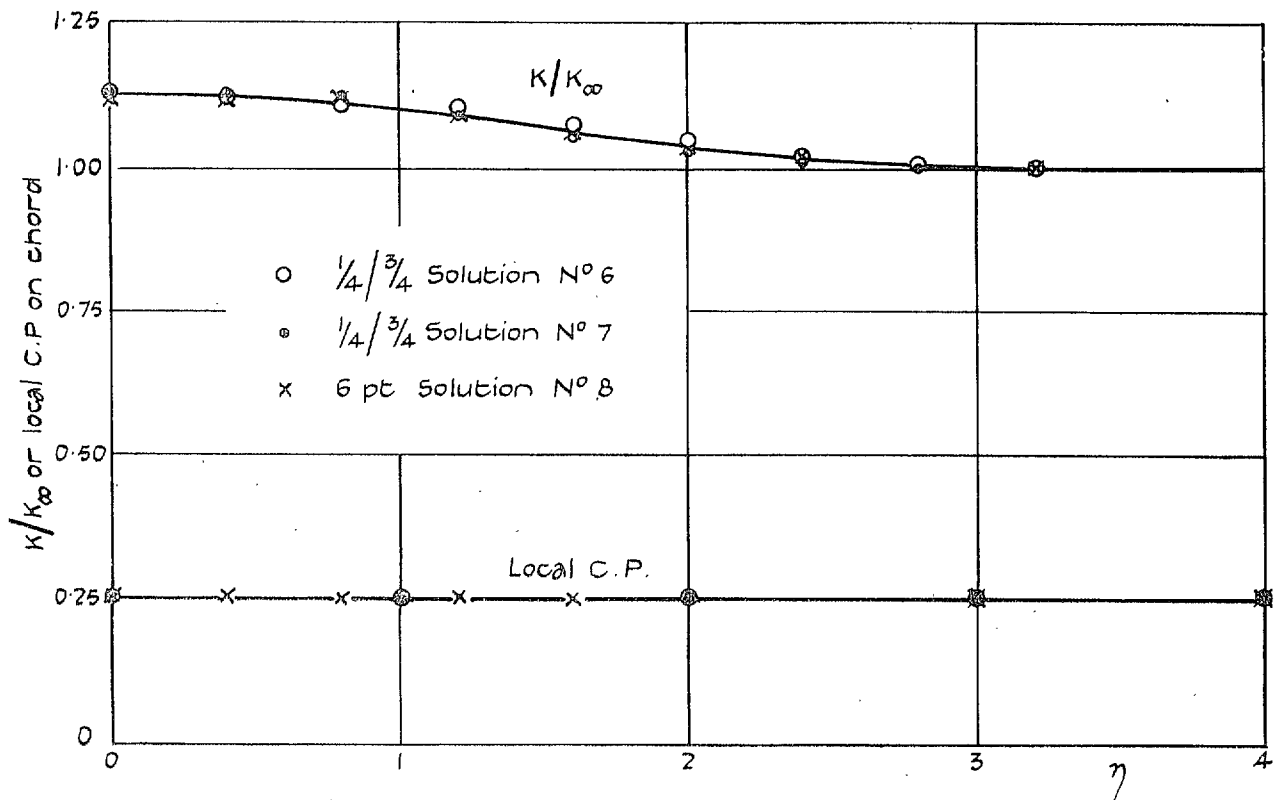


FIG. 5. Solutions for rounded-off V-wing.

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)—

- 1934-35 Vol. I. Aerodynamics. *Out of print.*
Vol. II. Seaplanes, Structures, Engines, Materials, etc. 40s. (40s. 8d.)
- 1935-36 Vol. I. Aerodynamics. 30s. (30s. 7d.)
Vol. II. Structures, Flutter, Engines, Seaplanes, etc. 30s. (30s. 7d.)
- 1936 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40s. (40s. 9d.)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 50s. (50s. 10d.)
- 1937 Vol. I. Aerodynamics General, Performance, Airscrews, Flutter and Spinning. 40s. (40s. 10d.)
Vol. II. Stability and Control, Structures, Seaplanes, Engines, etc. 60s. (61s.)
- 1938 Vol. I. Aerodynamics General, Performance, Airscrews. 50s. (51s.)
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30s. (30s. 9d.)
- 1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50s. (50s. 11d.)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63s. (64s. 2d.)
- 1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50s. (51s.)

Certain other reports proper to the 1940 volume will subsequently be included in a separate volume.

ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

1933-34	1s. 6d. (1s. 8d.)
1934-35	1s. 6d. (1s. 8d.)
April 1, 1935 to December 31, 1936.	4s. (4s. 4d.)
1937	2s. (2s. 2d.)
1938	1s. 6d. (1s. 8d.)
1939-48	3s. (3s. 2d.)

INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS, AND SEPARATELY—

April, 1950 R. & M. No. 2600. 2s. 6d. (2s. 7½d.)

INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850. 1s. 3d. (1s. 4½d.)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950. 1s. (1s. 1½d.)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050. 1s. (1s. 1½d.)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150. 1s. 3d. (1s. 4½d.)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250. 1s. 3d. (1s. 4½d.)

Prices in brackets include postage.

Obtainable from

HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, LONDON, W.C.2 423 Oxford Street, LONDON, W.1
P.O. Box 569, LONDON, S.E.1
13a Castle Street, EDINBURGH, 2 1 St. Andrew's Crescent, CARDIFF
39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1
2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST

or through any bookseller.